269. Investigation of Vibrations of a Sheet of Paper in the Printing Machine

Edmundas Kibirkštis[1,a], Asta Kabelkaitė[1,b], Artūras Dabkevičius[1,c], Liutauras Ragulskis[2,d]
[1] Department of Graphic Communication Engineering, Kaunas University of Technology, Studentų 56, LT-51424 Kaunas, Lithuania
[2] Department of Informatics, Vytautas Magnus University, Vileikos 8-702, LT-44404, Kaunas, Lithuania
E-mail: [a] *edmundas.kibirkstis@ktu.lt,* [b] *asta.kabelkaite@ktu.lt,* [c] *arturas.dabkevicius@ktu.lt,* [d] *l.ragulskis@if.vdu.lt.*

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Abstract. It is assumed that a sheet of paper performs transverse vibrations as a plate having additional stiffness due to static tension in its plane. The first eigenmodes are determined.
The setup for experimental investigation of the nodal lines of the standing waves and for the determination of eigenmodes using projection moire is created for the analysis of unsymmetric loading of the paper. The comparison of the theoretical and experimental results of investigations is performed.
The obtained results could be used in the process of design of the elements of the printing device.

Keywords: vibrations, paper, printing, eigenmode, plate, finite elements

Introduction

The model for the analysis of vibrations of a sheet of paper in a printing device is proposed on the basis of the models described in [1, 2]. It is assumed that a paper performs transverse vibrations as a plate having additional stiffness due to static tension in its plane. Thus the analysis consists of two stages:

1. the static problem of plane stress by assuming the displacements at the boundary of the analyzed paper to be given is solved;
2. vibrations of the investigated paper as of a plate with additional stiffness due to static tension determined in the previous stage of analysis are analyzed (the first eigenmodes are determined).

Moire techniques are widely employed for the investigation of vibrations: shadow moire [3], geometric moire [4, 5] is also used for the visualisation of periodic dynamic processes; stochastic moire [6] is used for the identification of plane vibrations. In this paper the analysis of vibrations of a sheet of paper is performed using the techniques of projection moire [7]. In [8] vibrations of a sheet of paper under symmetric loading are investigated.

The purpose of this paper is to present the investigations of the vibrations of the paper under non-symmetric tension.

The obtained results could be used in the process of design of the elements of the printing device.

Model for the analysis of vibrations of the paper

Further *x*, *y* and *z* denote the axes of the orthogonal Cartesian system of coordinates. First the static problem of plane stress is analyzed. The element has two nodal degrees of freedom: the displacements *u* and *v* in the directions of the axes *x* and *y* of the orthogonal Cartesian system of coordinates.

It is assumed that the displacements at the boundary of the analyzed paper are given and they produce the loading vector. Thus the vector of displacements is determined by solving the system of linear algebraic equations.

In the second stage of the analysis the eigenproblem of the plate is solved. The element has three nodal degrees of freedom: the transverse displacement of the paper *w* and the rotations Θ_x and Θ_y about the axes of coordinates *x* and *y*. The displacements due to bending *u* and *v* in the directions of the axes *x* and *y* are expressed as $u=z\Theta_y$ and $v=-z\Theta_x$. The additional stiffness due to static tension in the plane of the paper is taken into account.

The mass matrix has the form:

$$\left[\overline{M}\right]=\int\left[N\right]^T\begin{bmatrix}\rho h & 0 & 0\\ 0 & \dfrac{\rho h^3}{12} & 0\\ 0 & 0 & \dfrac{\rho h^3}{12}\end{bmatrix}\left[N\right]dxdy, \qquad (1)$$

where ρ is the density of the material of the paper, *h* is the thickness of the paper and:

$$\left[N\right]=\begin{bmatrix}N_1 & 0 & 0 & \dots\\ 0 & N_1 & 0 & \dots\\ 0 & 0 & N_1 & \dots\end{bmatrix}, \qquad (2)$$

where N_i are the shape functions of the finite element.

The stiffness matrix has the form:

$$[\overline{K}] = \int \left(\begin{matrix} [\overline{B}]^T [\overline{D}][\overline{B}] + [\overline{\overline{B}}]^T [\overline{\overline{D}}][\overline{\overline{B}}] + \\ + [G]^T [M][G] \end{matrix} \right) dxdy, \tag{3}$$

where:

$$[\overline{B}] = \begin{bmatrix} 0 & 0 & \frac{\partial N_1}{\partial x} & \dots \\ 0 & -\frac{\partial N_1}{\partial y} & 0 & \dots \\ 0 & -\frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} & \dots \end{bmatrix}, \tag{4}$$

$$[\overline{D}] = \frac{h^3}{12} \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{E\nu}{1-\nu^2} & 0 \\ \frac{E\nu}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & \frac{E}{2(1+\nu)} \end{bmatrix}, \tag{5}$$

where E is the modulus of elasticity and ν is the Poisson's ratio and:

$$[\overline{\overline{B}}] = \begin{bmatrix} \frac{\partial N_1}{\partial y} & -N_1 & 0 & \dots \\ \frac{\partial N_1}{\partial x} & 0 & N_1 & \dots \end{bmatrix}, \tag{6}$$

$$[\overline{\overline{D}}] = \frac{Eh}{2(1+\nu)1.2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{7}$$

$$[G] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & 0 & \dots \\ \frac{\partial N_1}{\partial y} & 0 & 0 & \dots \end{bmatrix}, \tag{8}$$

$$[M] = h \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix}, \tag{9}$$

where the stresses σ_x, σ_y, τ_{xy} are determined from the static problem of plane stress.

Thus the eigenmodes of transverse vibrations of the analyzed paper can be controlled by the static tension in the plane of the paper.

Analysis of vibrations of the paper for the case of non-symmetric loading

The square piece of paper is analyzed. On the upper and lower boundaries all the generalised displacements are assumed equal to zero, except on the upper boundary linear variation of the displacement v is assumed with v=0.5 on the left side of the boundary and v=1.5 on the right side of the boundary.

Contour plot of the transverse displacement for the first eigenmode is presented in Fig. 1, for the second eigenmode in Fig. 2, …, for the ninth eigenmode in Fig. 9.

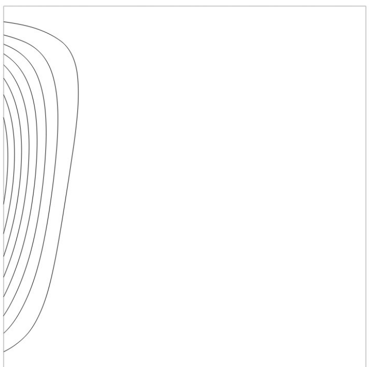


Fig. 1. The first eigenmode

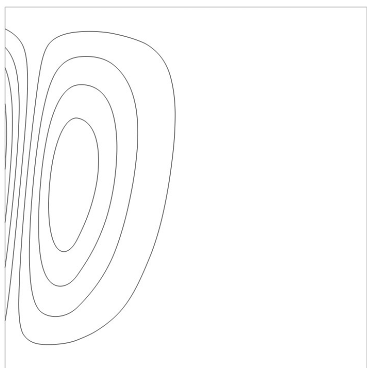


Fig. 2. The second eigenmode

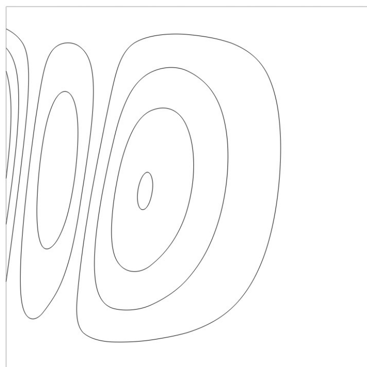


Fig. 3. The third eigenmode

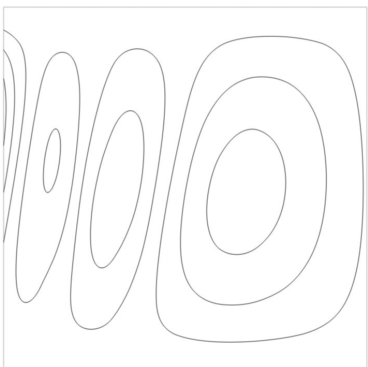


Fig. 4. The fourth eigenmode

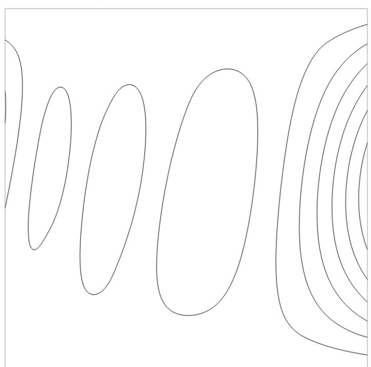


Fig. 5. The fifth eigenmode

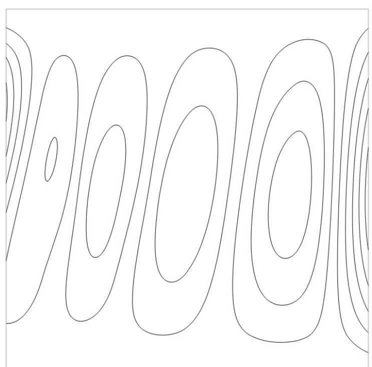


Fig. 6. The sixth eigenmode

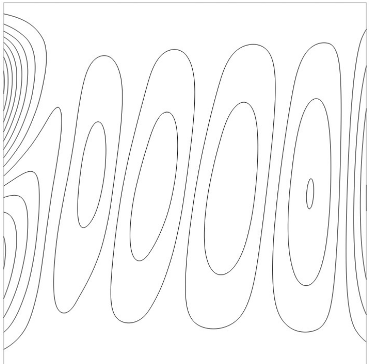


Fig. 7. The seventh eigenmode

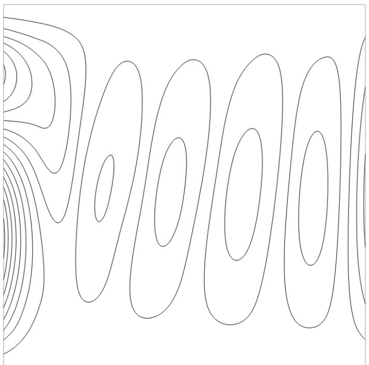


Fig. 8. The eighth eigenmode

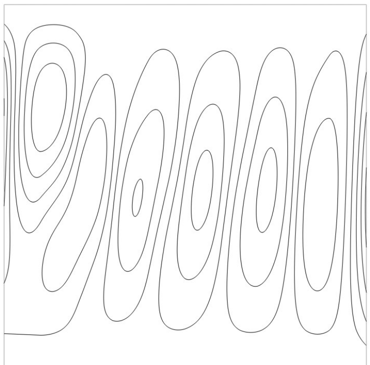


Fig. 9. The ninth eigenmode

Experimental setup, the method of investigation and analysis of the obtained results

During the process of printing the road of transportation of the paper in the printing device undergoes several regions of processing of the paper (in some devices up to 20 m.): first of all is the region of control of the tension of the paper, then the region of printing on the paper, and later the regions of drying, bending and transverse cutting of the printed paper (see Fig. 10). In the regions of tension and printing of the paper the process of equalisation of the paper is extremely important. In all previously mentioned regions the loading of the paper is to be distributed symmetrically, because an insufficiently precisely equalised paper and the vibrations generated in it have a negative effect to the quality of the printing process. For the investigation of the dynamical characteristics of the paper for the case of unsymmetric loading of the paper a special experimental setup was designed and produced (see Fig. 11). For the symmetric loading of the paper a similar setup was developed and the results were investigated in [8]. Similarly to the previous paper [8] here for the visualisation of standing waves two methods are used: 1) grain of silicium carbide (150-180 μm); 2) projection moire.

The created setup consists of: the vibroexciter S_1, the amplifier S_2, setup for investigation of vibrations S_3, the investigated material S_4, digital camera *DC* (Canon A700), personal computer *PC*, ink printer *PR* (HP Deskjet 920c) and the equipment of projection moire (not indicated in the figure).

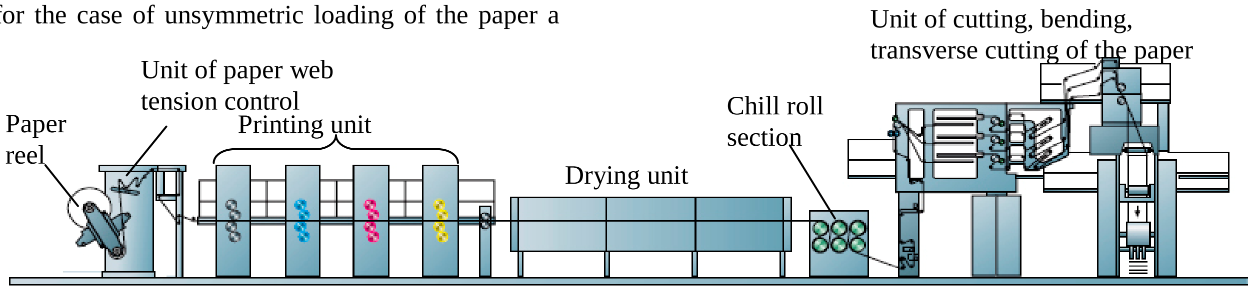


Fig. 10. Structure of the web offset printing machine

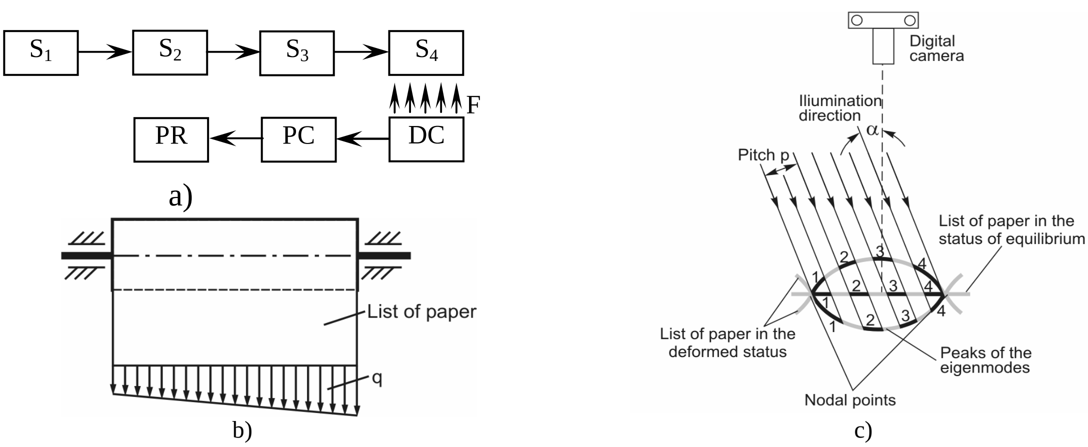


Fig. 11. Structural diagram of the experimental setup: S_1 – signal generator; S_2 – amplifier; S_3 – vibroexciter; S_4 – the investigated material; DC – digital camera; PC – personal computer; PR – printer; F– light flux of the digital camera; b) diagram of unsymmetric loading *q* of the investigated object; *q* – tension force of the list of paper; c) diagram of the projection moire

In the process of investigation sinusoidal longitudinal vibrations of the chosen frequency were generated in the setup for analysis of vibrations S_3 which generated standing waves in the paper. In the first method for the visualisation of the nodal lines of the standing waves, a grain of silicium carbide was put on the surface of the investigated material. In the second method for the generation of eigenmodes the grid of step p was projected on the surface of the investigated material by using a light flux emitted by a monochromatic source at a definite angle [7, 8]. Using the first method of experimental investigation the shapes of the nodal lines of the standing waves are obtained, while in the second method the eigenmodes are registered by a digital camera and observed in the monitor of a personal computer.

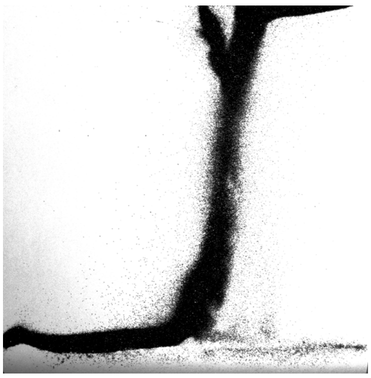
In the experiments projection moire was obtained by projecting thin parallel lines of high contrast on a vibrating paper by using the flux of light.

The created experimental setup enables to change:

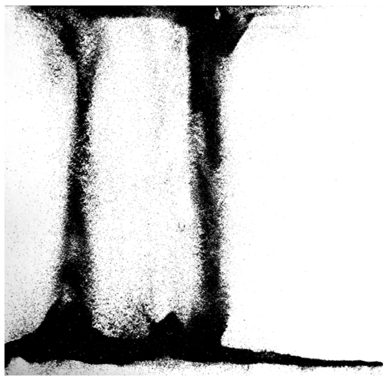
- the loading of the investigated material q;
- to generate longitudinal vibrations of the investigated material in the frequency interval (8 Hz up to 2,2 MHz).

The paper Plano Plus 80g/m^2 was chosen for investigations.

The obtained results of experimental investigations under non-symmetric tension of the paper are presented in Fig. 12-14. The image of the nodal lines of the standing waves of the first eigenmode of the paper Plano Plus obtained by using the grain of silicium carbide is presented in Fig. 12 a, of the second eigenmode – in Fig. 12 b, of the third eigenmode – in Fig. 13 a, of the fourth eigenmode – in Fig. 13 b. When using projection moire the image of the first eigenmode is presented in Fig. 14 a, of the second eigenmode – in Fig. 14 b, of the third eigenmode – in Fig. 14 c.

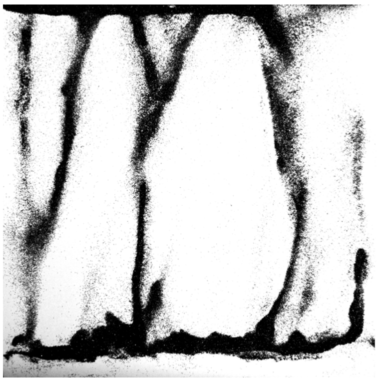


a)

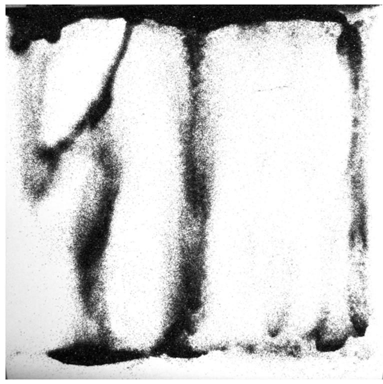


b)

Fig. 12. The nodal lines of the standing waves of the unsymmetrically loaded paper Plano Plus 80 g/m^2: a) the first eigenmode (frequency of vibrations 15,4 Hz, amplitude $1{,}5\times10^{-5}$ m); b) the second eigenmode (frequency of vibrations 20 Hz, amplitude 5×10^{-5} m)

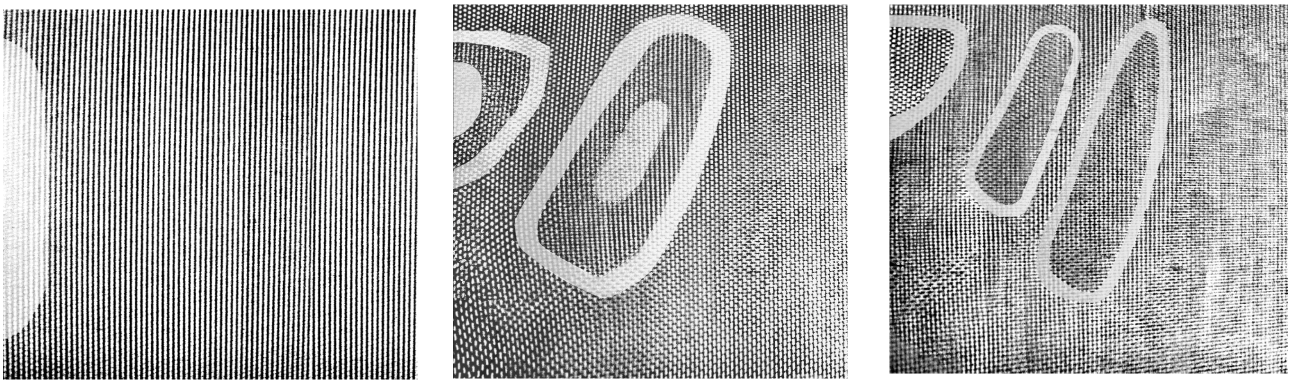


a)



b)

Fig. 13. The nodal lines of the standing waves of the unsymmetrically loaded paper Plano Plus 80 g/m^2: a) the third eigenmode (frequency of vibrations 30 Hz, amplitude $2{,}2\times10^{-5}$ m); b) the fourth eigenmode (frequency of vibrations 40 Hz, amplitude $2{,}2\times10^{-5}$ m)



a) b) c)

Fig. 14. The eigenmode of the unsymmetrically loaded paper Plano Plus 80 g/m^2: a) the first eigenmode (frequency of vibrations 21 Hz, amplitude 8×10^{-5} m); b) the second eigenmode (frequency of vibrations 26 Hz, amplitude $4{,}2\times10^{-5}$ m); c) the third eigenmode (frequency of vibrations 29 Hz, amplitude 5×10^{-5} m)

Analysing the obtained results of experimental investigations under non-symmetric loading of the paper, one can note that the shape of the nodal lines of the standing waves changes and it more corresponds to the shape of the rectangle. Thus by observing the shape of the nodal lines of the standing waves of the loaded paper it is possible to judge if the paper in the region of transportation of the printing device and also in the other regions is sufficiently equalized. Also this simple method may be used for the analysis of eigenmodes of the paper. This method has lower precision and thus it is usable in preparatory investigations, which do not require very high precision.

By using the second method, that is projection moire, the deformations of the paper under unsymmetric loading were investigated. The analysed eigenmodes are similiar to the nodal lines of the standing waves obtained by the first method. The method of projection moire is more precise. Thus it is widely applied for the investigation of the dynamics of the paper and also for the control of the uniformity of tension of the paper in the printing device.

Conclusions

The proposed model for the analysis of vibrations of a sheet of paper in a printing device is based on the assumption that a sheet of paper performs transverse vibrations as a plate having additional stiffness due to static tension in its plane. The static problem of plane stress by assuming the displacements at the boundary of the analyzed paper to be given is solved. Then the vibrations of the investigated paper as of a plate with additional stiffness due to static tension determined previously are analyzed.

It is shown that the eigenmodes of transverse vibrations of the analyzed paper are substantially influenced by the static tension in the plane of the paper.

The setup for experimental investigations has been created with the help of which the nodal lines of the standing waves of the paper Plano Plus (80 g/m^2) and their eigenmodes under unsymmetric loading have been determined by using the grain material and the method of projection moire.

The shape of the nodal lines of the standing waves and also the eigenmodes obtained by projection moire characterize the quality of equalisation of tension of the paper in the printing device and the uniform loading of the paper.

The obtained results could be used in the process of design of the printing device and its elements.

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