280. Research of variations of frequency of non-linear dynamic model of the muscle

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Abstract. The paper presents the methodic for research of non-linear muscular stiffness characteristics, operating with polynomial and Gaussian functions. The dynamical muscle model has been made which led to observe the variations of muscular amplitude-frequentative characteristics in loading duration and at different magnitudes of the load. It was appointed that in non-linear muscle system, in the beginning of muscle loading, not less 6 different frequency harmonics appeared and with longer loading duration the amount of them decrease. In marginal case when $t \to \infty$, the amount of frequencies approached to four. The regularities of muscle frequency characteristics in muscle loading duration explored analytically were compared to results got by experimental way.

Keywords: frequentative characteristics, muscle dynamic model, nonlinear characteristics

Introduction

The muscle is modeled by biomechanics, engineers, and scientists for the simple purpose – to find the best explanation of their behavior in the different situations. There are expressed in different ways simple models and sophisticated complex systems of the muscle.

Widely known muscle models have been made by Hill, Hatze, Horler, Huxley [1-5]. Hill-based model are founded upon experiments yielding parameters for visco-elastic series and parallel elements coupled with are contractile element. The results of such models are often used to predict force, length and velocity relationships describing muscle behavior. In literature Hill-type model are often chosen for the active stress [3, 6, 10].

Huxley based models are built upon biochemical, thermodynamic, and mechanical experiments that describe muscle at the muscular level. In a two state Huxley model cross- bridges are either attached or detached. These models are used to understand the properties of the microscopic contractile elements. There are some disadvantages of this model, especially for rapid events, which have been extensively discussed in the literature.

However, for relatively slow phenomena the model gives good results, compared with experimental data [9-11].

Muscular models could be categorized: a – empirical models (Chou and Hannaford, 1996 [8]; Medrano-Cerda and others 1995 [7]); b – models wherein emphases muscular geometry (Tondu etc. 1994; Cai and Yamaura 1996 [6]) and c – models where properties of muscle materials are highlighted (Chou and Hannaford, 1996 [8]; Shulte, 1961 [7]).

To describe the mechanical behavior of biological tissues and transport processes in biological tissues, conservation laws such as conservation mass, momentum and energy play a central role. Mathematically these are cast into the form of partial differential equations. Because of nonlinear material behavior, inhomogeneous properties and usually a complex geometry, it is impossible to find close-form analytical solutions for these sets of equations. The objective of the finite elements method is to find approximate solutions for these problems [6, 11].

Muscular models presented in the literature sources were analyzed and due to them were studied muscular mechanical characteristics. Unfortunately, having summarized presented research results following works haven't been found: were not researched the frequentative characteristics of muscle nonlinear dynamical systems and their alterations in muscle loading duration. Thus the aim of this work is to design a nonlinear muscular model and to study alterations of frequency characteristics in the way of loaded muscle.

Model description

For the short thumb abducent muscle (*m. abductor policis brevis)* were chosen following data: mass m = 4.54 g, damping coefficient c = 0.001, excitation force $F(t)$ was shifted from 12 to 18 N, excitation duration T_0 varied from 2 to 10 s. Due to presented [12] muscular stiffness expression, which was derived from experimental research results, the stiffness characteristic was approximated. This function was compared with other stiffness characteristics got by other scientists.

Taking into consideration the fact that the muscle receives rather precisely specified rather than random information, the variations of the muscle intensity could be expressed by way of a trigonometric row [14]. Therefore muscular stiffness could be calculated as the sum of accomplished works of trigonometric row single terms, where is possible the salutatory change of values in muscle loading duration.

By made research have been found that muscle stiffness dependence could be approximately expressed by 8^{th} degree polynomial function

$$k(t) = p_0 + p_1 t + p_2 t^2 + p_3 t^3 + p_4 t^4 + \\ + p_5 t^5 + p_6 t^6 + p_7 t^7 + p_8 t^8; \tag{1}$$

p_0 ,...p_8, are the coefficients of presented polynomial expression and their values are

$p_0 = 1398$, $p_1 = -72.12$, $p_2 = 1.65$, $p_3 = -1.926\cdot10^{-2}$, $p_4 = 1.261\cdot10^{-4}$, $p_5 = -4.821\cdot10^{-7}$, $p_6 = 1.069\cdot10^{-9}$, $p_7 = -1.271\cdot10^{-12}$, $p_8 = 6.29\cdot10^{-16}$.

Approximating stiffness by 3 term Gaussian function the polished characteristic of muscle stiffness has been found:

$$k(t) = a_1 \cdot e^{-\left(\frac{t-b_1}{c_1}\right)^2} + a_2 \cdot e^{-\left(\frac{t-b_2}{c_2}\right)^2} + a_3 \cdot e^{-\left(\frac{t-b_3}{c_3}\right)^2}, \tag{2}$$

when $0 \le t \le 60$, coefficients of Gaussian function are as follows: a_1 = 2781, a_2 = 696.7, a_3 = 182.9, b_1 = - 14.62, b_2 = - 51.1, b_3 = -379.2, c_1 = 14.79, c_2 = 75.4, c_3 = 608.7.

In the further calculations was used nonlinear dynamical model of the muscle as the stiffness is nonlinear function of the time:

$$m\ddot{x} + c\dot{x} + \int_0^t k(t)dt = F(t) \tag{3}$$

The force impulse was put in to two modes for analysis of transition processes of the system (fig. 1.):

$$F(t) = \begin{cases} 0, & when\ t = 0; \\ 12\ N, & when\ 0 < t \le T_0, \\ 0, & when\ t > T_0. \end{cases}$$

or

$$F(t) = \begin{cases} 0, & when\ \ t = 0; \\ 12\ \sin(\omega t), & when\ 0 < t \le T_0, , \\ 0, & when\ t > T_0. \end{cases} \tag{4}$$

where $\omega = \frac{2\pi}{T}$, t is the variable value of the time, T_0 stands for impulse duration, $T = 4\ T_0$. The magnitude of the load was operated from 12 to 18 N the same as have been used in the experimental research.

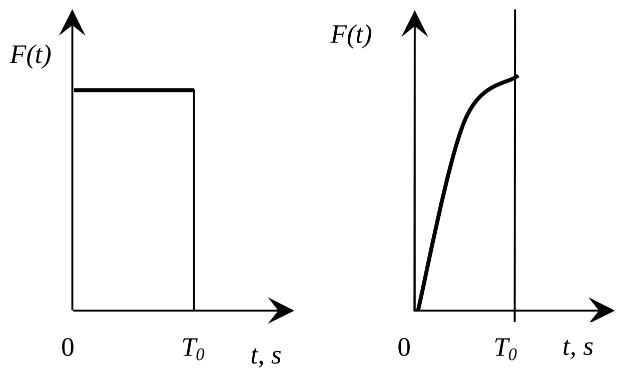
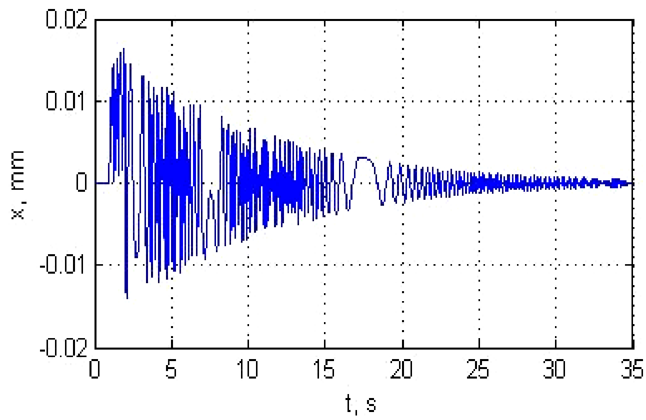


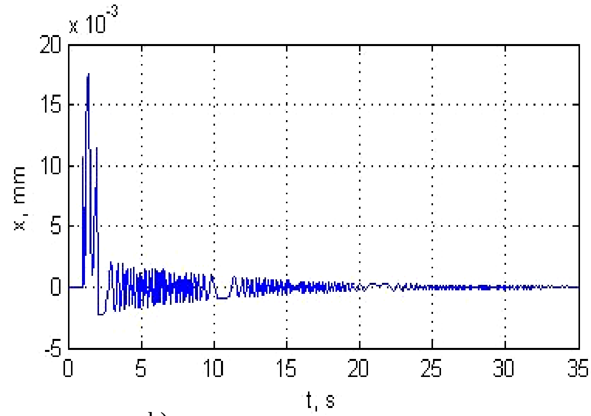
Fig. 1. Graphical interpretation of force impulse

Analysis and results

The expression (3) has been solved using the numerical Runge-Kutta method. During the calculations muscle stiffness has been approximated by polynomial and Gaussian functions. Besides the oscillations were studied in the time as muscle was affected by different magnitudes of loading and loading durations. Figures 2 and 3 present calculation results.



a)

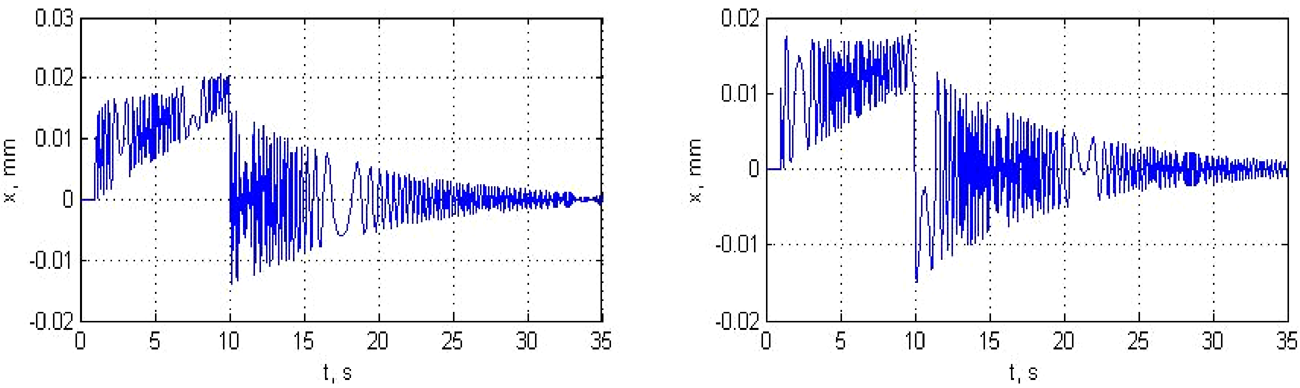


b)

Fig. 2. Muscular oscillations as the function of time (T_0 = 2 s, *F(t)* = 12 N), when stiffness was approximated by a) Gaussian function, b) polynomial function

Analysis of results presented in Fig. 3 has shown the influence of the magnitude of the excitation force to the amplitude of muscular oscillations. With the increase of excitation force the amplitude of oscillations become large and the character of oscillations stay the same.

By the way of approximating stiffness characteristic by Gaussian function irregularities of stiffness curve were polished, therefore with less frequency the lower amplitudes are got. However the polynomial function show up the stiffness characteristic as excitation impulse intensity is increasing.



a) b)

Fig. 3. Muscular oscillations as the function of time as T_0 = 10 s, when stiffness was approximated by a) Gaussian function, b) polynomial function.

For investigation of influence of force excitation on nonlinear system the force $F\sin(\omega t)$ has been used, where $\omega = 2\pi \cdot f$ and *F* stands for muscle force from 12 to 18 N.

Figure 4 represents calculation results in the way of force excitation into system.

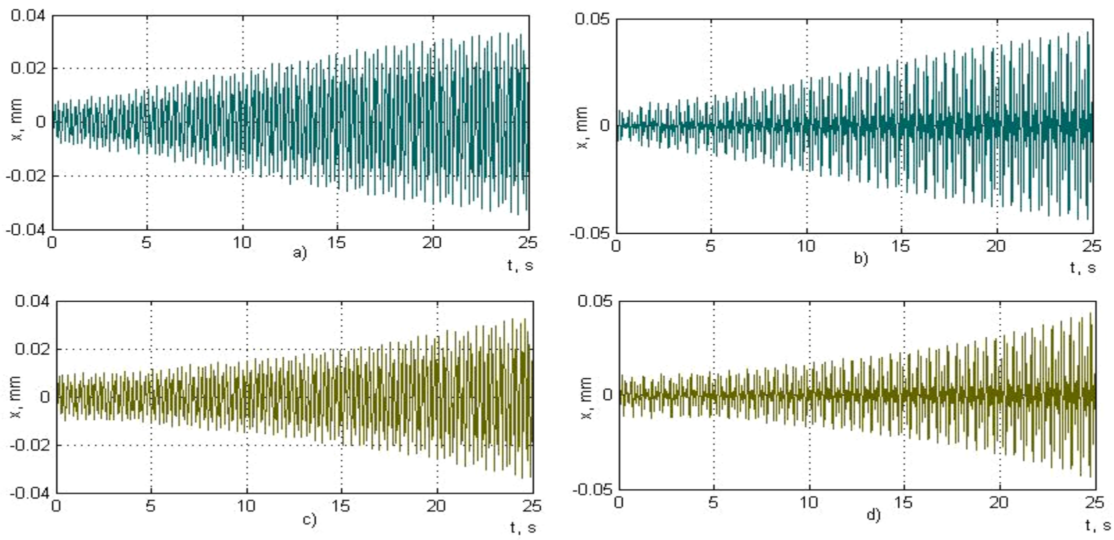


Fig. 4. Relationships of muscular oscillation amplitudes and time when stiffness was approximated by Gaussian function - a) *f* =12 Hz, b) *f* =24 Hz, when stiffness was approximated by polynomial function c) *f* =12 Hz, d) *f* =24 Hz

In order to explain variations of oscillations spectrum in muscle loading duration the spectral density was analyzed in time intervals. The realization of muscular oscillations (T_0 = 2 s, $F(t)$ = 12 N) was resolved in these time intervals: from 2 to 4 s, form 4 to 6 s, from 6 to 8 s, from 8 to 10 s, from 10 to 12 s and from12 to 14 s. Average values of spectral density were calculated in every range of time. Calculations were made by using the program Matlab. The results of spectral density calculations presented in figure 5.

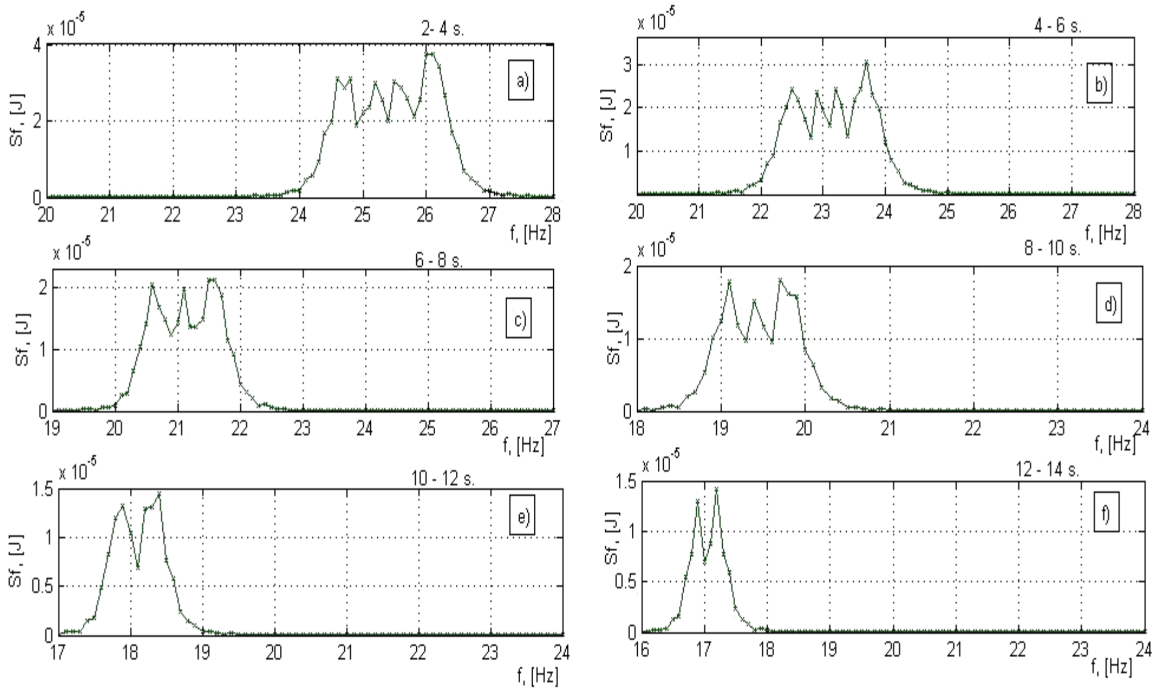


Fig. 5. The dependence of muscle amplitude - frequency spectrum on loading duration when T_0 = 2 s, $F(t)$ = 12 N, where a) 2-4 s, b) 4-6 s, c) 6-8 s, d) 8-10 s, e) 10-12 s, f) 12-14 s.

The analysis of the results of spectral density presented in Fig. 5 has shown that in the beginning of muscle loading less than 4 frequencies are dominating. However when loading duration is increasing, values of frequencies in spectral realization become larger and their number increase. As muscle get fatigued frequency values gradually approach to 0 and amount of them diminishing (number of harmonics decrease).

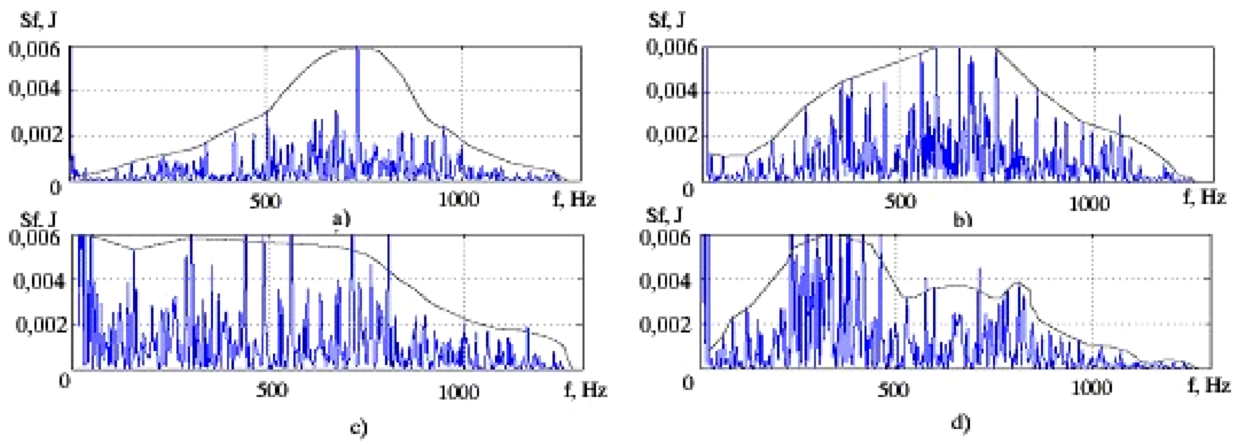


Fig. 6. A muscle biosignals spectrums dependency on loading duration when loading is 18 N, in male group as loading duration: a) 1 min; b) 3 min; c) 6 min; d) 10 min.

Spectrum of muscle biosignals was analyzed from experimental results (Fig. 6.). Analysis of results presented in figure 6 has shown that components of muscle biosignals spectrum enlarge with the increase of muscle loading duration. As the muscle is loaded longer the intensity of muscle biosignals is larger. Item with longer loading duration spectrum of muscle biosignals displace to a side of lower frequencies and approach to 0. Results got by experimental data prove above mentioned variations of spectral density and its distribution in the muscle loading duration.

Figure 7 presents alterations of dominate frequencies in muscle loading duration.

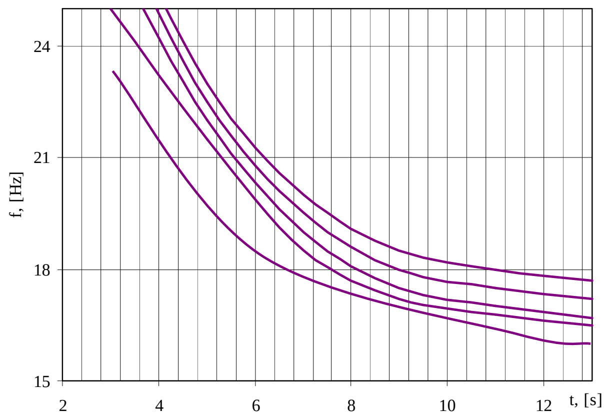


Fig. 7. The relationship of oscillation frequencies *f* and loading duration *t*

From figure 7 clearly seen that values of all dominate frequencies decrease due to nonlinear dependence with longer muscle loading. This appearance could be explained as the muscle stiffness characteristic likewise nonlinear decreasing as muscle loaded with fixed load.

Analytical research

Muscular frequentative characteristics were researched analytically. The expression was got using muscle accomplished work and biosignals energy conditions while evaluating inertial and damping forces of the muscle:

$$m\ddot{x} + c\dot{x} + F = \frac{U^2(t)}{R\dot{x}}. \qquad (6)$$

In the left side of expression is the sum of inertial, damping and muscle loading forces and in the right hand side of equation $\frac{U^2(t)}{R\dot{x}}$ also is the force.

Knowing that the muscle receives rather precisely specified rather than random information the variations of the muscle voltage *U (t)* could be expressed by way of a trigonometric row

$$U(t) = B_0 + \sum_{i=1}^{n} B_i \sin(\omega_i t - \varphi_i). \qquad (7)$$

where $\omega_i = i\omega_1$, ω_i is the lowest angular frequency of the lowest harmonic.

The systems could be assumed as linear if analyzing expression (6) for one harmonic excitation force $B_i \sin(\omega t)$ in short excitation time interval $t_{-\varepsilon} \le t \le t_{+\varepsilon}$ (where ε is small enough value). Then the solution of the equation (6) could have such a shape $y = B\sin(\omega t)$.

The solution of the equation (6) for one harmonic excitation force could be graphically interpreted (Fig. 8).

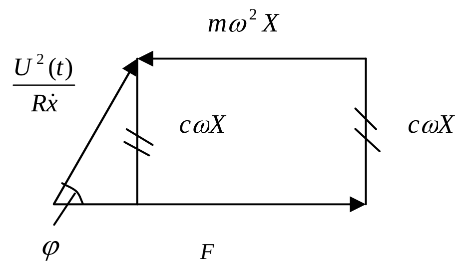


Fig. 8. Graphical interpretation of expression (6)

The scheme presented in figure 8 show the existing relation between inertial, damping, muscle loading and muscle biosignals

$$\left(F - m\omega^2 X\right)^2 + \left(c\omega X\right)^2 = \left(\frac{U^2(t)}{R\dot{x}}\right)^2. \qquad (8)$$

After reformations such the expression has been got:

$$m^2\omega^6 X^4 + c^2\omega^4 X^4 - 2Fm\omega^4 X^3 + \\ + F^2\omega^2 X^2 - \frac{U^4(t)}{R^2} = 0. \qquad (9)$$

Analysis of expression (9) was seen that it has four solutions X_1, X_2, X_3, and X_4. This system will oscillate by four different amplitudes but also will have six different frequencies.

Analysis into muscle deformation [12-14] has led to conclusion, that muscle elongation value is approximately proportional to muscle loading duration. Hereof muscle deformation velocity could be evaluated as approximately steady value $\omega X \approx const$. With longer muscle loading duration the value of $\frac{U^2(t)}{R}$ is increasing, so values of quantities R and $\dot{x}$ could be assumed as constant. In that way the scheme presented in figure 8 will change and will take a view presented in figure 9.

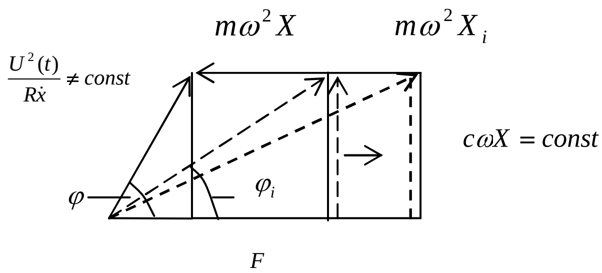


Fig. 9. Graphical interpretation of expression (6) after transformations

The scheme in figure 9 show that with longer muscle loading duration and given $c\omega X \approx const$, the value of force $\frac{U^2(t)}{R\dot{x}}$ increasing, the quantity of $m\omega^2 X_i$ diminish, because $c\omega X \approx const$ multiplied by decreasing value of ω, so product will get lower. Consequently the solution of equation (6) in the moment of time t_i could be calculated according to triangle displayed by dotted line. Herby marginal condition could be written:

$$\lim_{\substack{\omega\to 0\\ T\to\infty}} (F - m\omega^2 X)^2 + (c\omega X)^2 - \left(\frac{U^2(t)}{R\dot{x}}\right)^2 \to, \qquad (10)$$
$$\to F^2 + (c\omega\dot{x})^2 = \left(\frac{U^2(t)}{R\omega X}\right)^2$$

The size of phase angle φ_i could be calculated by this expression $\varphi_i \approx arctg\left(\frac{c\omega X}{F}\right)$.

From the condition (10) approximately we received

$$c^2\omega^2 X^4 + F^2\omega^2 X^2 = \frac{U^4(t)}{R^2}. \qquad (11)$$

The final expression (11) shows that, in marginal case when time t approach to infinity, the amount of oscillation frequencies decreasing. And in such case would be only 4 frequencies $\omega_1, \omega_2, \omega_3, \omega_4$.

Conclusions

Having summarized the research and calculation results the following conclusions can be made:

1. Presented methodic for research of non-linear muscular stiffness characteristics, operating with polynomial and Gaussian functions;
2. The dynamical muscle model has been made which led to analyze the variations of muscular amplitude-frequentative characteristics in loading duration and at different magnitudes of the load.
3. Appointed that in non-linear muscle system, in the beginning of loading, not less 6 different frequency harmonics appeared and with longer loading duration their amount decrease. In marginal case, when $t \to \infty$, the amount of frequencies approach to four;
4. Displayed, that values of frequencies decreased with loading duration get longer.
5. The regularities of muscle frequency characteristics in loading duration explored analytically are suitable to results got by experimental way.

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