324. Calculation of Vibrations of a Single Degree of Freedom System

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Abstract. The three node numerical integration scheme with the displacement and velocity as the nodal variables for integration of the second order differential equation using finite elements in time on the basis of the method of least squares is proposed. The precision of integration is investigated and compared with the corresponding procedure based on the method of Galerkin. The error measure is introduced which shows the higher precision of the least squares technique.

Keywords: one degree of freedom, vibrations, finite elements, least squares.

Introduction

The use of finite elements in time for performing numerical integration is described in [1, 2, 3, 4]. The comparison of various numerical integration schemes for the second order differential equations is performed in [5].

The three node numerical integration scheme with the displacement and velocity as the nodal variables for integration of the second order differential equation using finite elements in time on the basis of the method of least squares is proposed. The precision of integration is investigated and compared with the corresponding procedure based on the method of Galerkin.

Numerical integration scheme

The dynamics of a single degree of freedom vibrating system is described by the equation:

$$m\frac{d^2x}{dt^2}+c\frac{dx}{dt}+kx=f, \tag{1}$$

where m, c, k and f are the mass, damping, stiffness and force; x is the displacement; t is the time variable.

The nodes of the finite element correspond to the values -1, 0 and 1 of the local coordinate ξ.

So the numerical integration is performed on the basis of the following matrix equation of second order:

$$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{Bmatrix} x_T \\ \dot{x}_T \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix}, \tag{2}$$

where T is the time step, the upper dot denotes differentiation with respect to time and:

$$k_{11}=\frac{m}{T}\int_{-1}^{1}W_5\frac{d^2N_5}{d\xi^2}d\xi+ \\ +c\int_{-1}^{1}W_5\frac{dN_5}{d\xi}d\xi+kT\int_{-1}^{1}W_5N_5d\xi, \tag{3}$$

$$k_{21}=\frac{m}{T}\int_{-1}^{1}W_6\frac{d^2N_5}{d\xi^2}d\xi+ \\ +c\int_{-1}^{1}W_6\frac{dN_5}{d\xi}d\xi+kT\int_{-1}^{1}W_6N_5d\xi, \tag{4}$$

$$k_{12}=m\int_{-1}^{1}W_5\frac{d^2N_6}{d\xi^2}d\xi+ \\ +cT\int_{-1}^{1}W_5\frac{dN_6}{d\xi}d\xi+kT^2\int_{-1}^{1}W_5N_6d\xi, \tag{5}$$

$$k_{22} = m\int_{-1}^{1} W_6 \frac{d^2 N_6}{d\xi^2} d\xi + + cT\int_{-1}^{1} W_6 \frac{dN_6}{d\xi} d\xi + kT^2 \int_{-1}^{1} W_6 N_6 d\xi, \tag{6}$$

$$f_1 = T\int_{-1}^{1} W_5 d\xi f - - \left(\frac{m}{T}\int_{-1}^{1} W_5 \frac{d^2 N_1}{d\xi^2} d\xi + + c\int_{-1}^{1} W_5 \frac{dN_1}{d\xi} d\xi + kT\int_{-1}^{1} W_5 N_1 d\xi \right) x_{-T} - - \left(m\int_{-1}^{1} W_5 \frac{d^2 N_2}{d\xi^2} d\xi + + cT\int_{-1}^{1} W_5 \frac{dN_2}{d\xi} d\xi + kT^2 \int_{-1}^{1} W_5 N_2 d\xi \right) \dot{x}_{-T} - - \left(\frac{m}{T}\int_{-1}^{1} W_5 \frac{d^2 N_3}{d\xi^2} d\xi + + c\int_{-1}^{1} W_5 \frac{dN_3}{d\xi} d\xi + kT\int_{-1}^{1} W_5 N_3 d\xi \right) x_0 - - \left(m\int_{-1}^{1} W_5 \frac{d^2 N_4}{d\xi^2} d\xi + + cT\int_{-1}^{1} W_5 \frac{dN_4}{d\xi} d\xi + kT^2 \int_{-1}^{1} W_5 N_4 d\xi \right) \dot{x}_0, \tag{7}$$

$$f_2 = T\int_{-1}^{1} W_6 d\xi f - - \left(\frac{m}{T}\int_{-1}^{1} W_6 \frac{d^2 N_1}{d\xi^2} d\xi + + c\int_{-1}^{1} W_6 \frac{dN_1}{d\xi} d\xi + kT\int_{-1}^{1} W_6 N_1 d\xi \right) x_{-T} - - \left(m\int_{-1}^{1} W_6 \frac{d^2 N_2}{d\xi^2} d\xi + + cT\int_{-1}^{1} W_6 \frac{dN_2}{d\xi} d\xi + kT^2 \int_{-1}^{1} W_6 N_2 d\xi \right) \dot{x}_{-T} - - \left(\frac{m}{T}\int_{-1}^{1} W_6 \frac{d^2 N_3}{d\xi^2} d\xi + + c\int_{-1}^{1} W_6 \frac{dN_3}{d\xi} d\xi + kT\int_{-1}^{1} W_6 N_3 d\xi \right) x_0 - - \left(m\int_{-1}^{1} W_6 \frac{d^2 N_4}{d\xi^2} d\xi + + cT\int_{-1}^{1} W_6 \frac{dN_4}{d\xi} d\xi + kT^2 \int_{-1}^{1} W_6 N_4 d\xi \right) \dot{x}_0, \tag{8}$$

where N_i are the shape functions and W_5, W_6 are the weighing functions.

For the method of Galerkin:

$$W_i = N_i, \tag{9}$$

and for the method of least squares:

$$W_i = \frac{m}{T^2}\frac{d^2 N_i}{d\xi^2} + \frac{c}{T}\frac{dN_i}{d\xi} + kN_i. \tag{10}$$

Investigation of precision of integration

The following equation is analysed:

$$\frac{d^2 x}{dt^2} + x = 0, \tag{11}$$

with the initial conditions:

$$x(0) = 1, \tag{12}$$

$$\frac{dx}{dt}(0) = 0. \tag{13}$$

The analytical and numerical time histories of motion when 5 steps in the period of oscillations are used for the method of Galerkin are presented in Fig. 1. The analytical time history is grey, while the numerical one is black.

The corresponding results for the method of least squares are presented in Fig. 2.

The values of the displacements after one period of oscillations as a function of the number of time steps are presented in Fig. 3. For the method of Galerkin they are grey, while for the method of least squares they are black.

The corresponding values of the velocities after one period of oscillations as a function of the number of time steps are presented in Fig. 4. For the method of Galerkin they are grey, while for the method of least squares they are black.

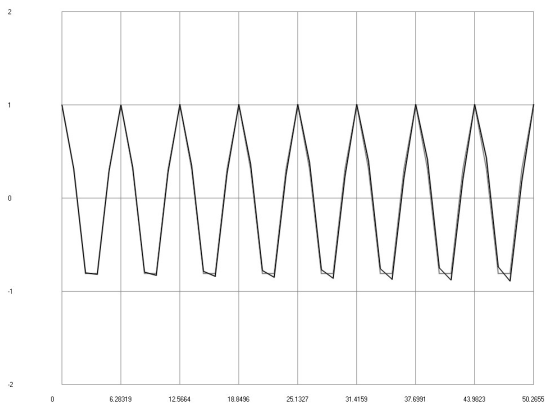

Fig. 1. Analytical and numerical time histories of motion for the method of Galerkin (the analytical time history is grey, while the numerical one is black).

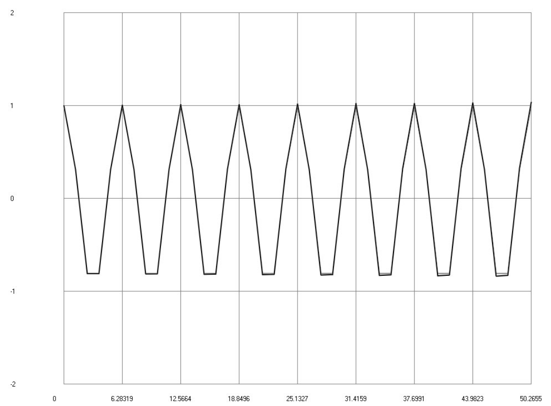

Fig. 2. Analytical and numerical time histories of motion for the method of least squares (the analytical time history is grey, while the numerical one is black).

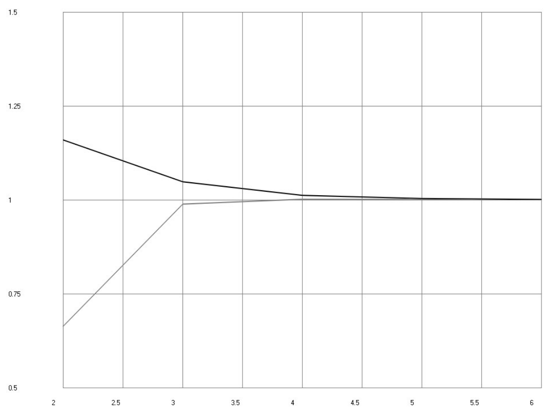

Fig. 3. Displacement after one period of oscillations as a function of the number of time steps (for the method of Galerkin in grey, while for the method of least squares in black).

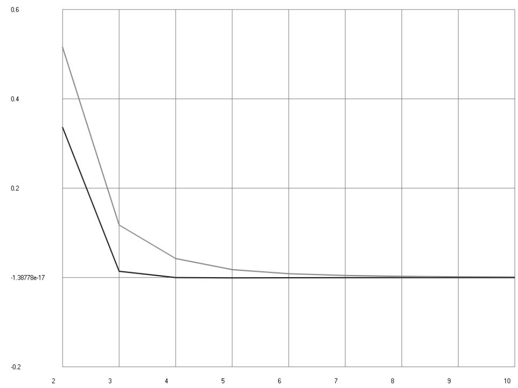

Fig. 4. Velocity after one period of oscillations as a function of the number of time steps (for the method of Galerkin in grey, while for the method of least squares in black).

The error is defined as:

$$E = \left| x\left(\tilde{T}\right) - 1 \right| + \left| \frac{dx}{dt}\left(\tilde{T}\right) \right|, \qquad (14)$$

where $\tilde{T}$ is the period of oscillations. The values of the error as a function of the number of time steps are presented in Fig. 5. For the method of Galerkin they are grey, while for the method of least squares they are black.

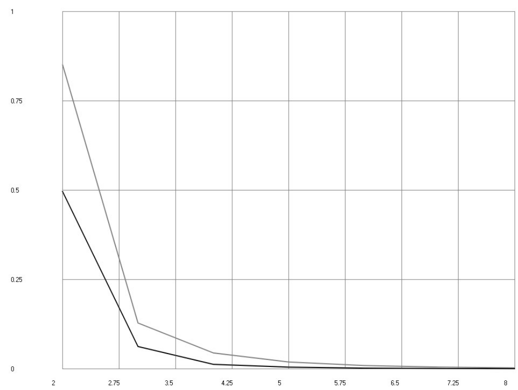

Fig. 5. Error as a function of the number of time steps (for the method of Galerkin in grey, while for the method of least squares in black).

Thus the introduced error measure evidently shows the higher precision of the least squares technique.

Conclusions

The three node numerical integration scheme with the displacement and velocity as the nodal variables for integration of the second order differential equation using finite elements in time on the basis of the method of least squares is developed in detail.

The precision of integration is compared with the corresponding procedure based on the method of Galerkin. The error measure is introduced which evidently shows the higher precision of the least squares technique.

The presented results are used in the process of analysis of vibrating systems with one degree of freedom.

References

[1] Zienkiewicz O. C., Morgan K. Finite Elements and Approximation. Moscow: Mir, 1986.

[2] Segerlind L. J. Applied Finite Element Analysis. Moscow: Mir, 1979.

[3] Zienkiewicz O. C. The Finite Element Method in Engineering Science. Moscow: Mir, 1975.

[4] Connor J. J., Brebbia C. A. Finite Element Techniques for Fluid Flow. Leningrad: Sudostroienie, 1979.

[5] Bathe K. J. Finite Element Procedures in Engineering Analysis. New Jersey: Prentice-Hall, 1982.