378. INNOVATIVE MECHANICAL SYSTEMS WITH ROTARY PIEZOMOTOR

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Abstract: Piezomotors can be classified according to their operating principles: traveling wave type and standing wave type motors. Matching of dynamic characteristics of traveling wave type piezomotors requires a special study for acquiring information about drive force, rotation velocity, etc. The type of generated waves (are they traveling waves) depends on the position of rotary piezomotors in respect to an elastic body. Analysis of traveling wave processes in ring actuators has indicated solution design operational area of piezomotors. In many cases it has proved to be more reasonable than usage of linear (rod) actuators. Rotary piezomotors with ring actuators are reliably connected to a rotor, thus ensuring the possibility of the accurate automatic control and increased torque.

Keywords: traveling wave type, piezomotor ring actuators, rotor, holographic interferometry method

Introduction

An important feature of mechanisms containing links made from active materials (e.g. piezoceramics) is that very often various functions may be performed by the same transducer. This enables the development of methods for designing adjustable or adaptive mechanisms so that multiple tasks can be performed by the same mechanism, including the ability to accommodate manufacturing and part-positioning errors in order to expand applicability.

The paper deals with the selection of maximal physical parameters of rotary piezomotor having ring actuators, their theoretical investigation and various design solutions [1].

The type of generated waves depends on the position of electromechanical actuators with respect to an elastic body. The analysis of wave processes in ring actuators has indicated the design solution to open the way for employing their entire operational area. In many cases it has proved to be more reasonable than the use of linear (rod) actuators. Rotary piezomotor with ring actuators are reliably connected with a rotor, thus ensuring the accurate automatic control and increasing the torque [2].

Experimental investigation of piezosystems by means of holographic interferometry enables one to obtain appreciably larger amounts of information about the vibrating surface in comparison with traditional methods. This investigation deals with the consideration of methods for determination of the vibrational characteristics of rotary piezomotors with ring actuators from holographic interferograms of linked analysis of these characteristics carried out by using numerical techniques based on the theories of piezosystem vibration and holographic interferometry.

Analysis of theoretical investigations

Section of a piezoring of small curvature is considered for a theoretical analysis (Fig.1).

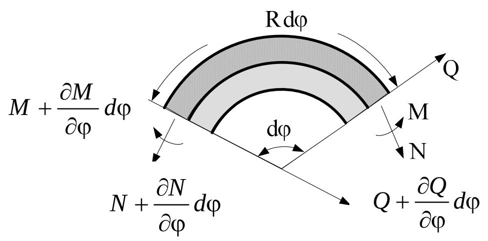


Fig. 1. Section of a piezoring

Tangential and radial wave processes are expressed by differential equations of ring motion:

$$\frac{\partial^6 w}{\partial Y^6}+2\frac{\partial^4 w}{\partial Y^4}+\frac{\partial^2 w}{\partial Y^2}+h\left(\frac{\partial^4 w}{\partial t^2\partial Y^2}-\frac{\partial^2 w}{\partial t^2}=\frac{R^3\partial P}{EJ\partial Y}\right) \qquad (1)$$

$$\frac{\partial^2 F}{\partial Y^2}+\frac{\partial T}{\partial Y}=\frac{\gamma AR}{g}\frac{\partial^3 U}{\partial t^2\partial Y} \qquad (2)$$

where $h = \frac{A\gamma R^4}{gEJ}$, Y is angular coordinate corresponding to the counterclockwise rotation, t is time, w is tangential coordinate (of displacement), A is cross-section area of a ring, E is modulus of elasticity, g is acceleration of gravity,
I is moment of inertia, R is radius, γ is density, U is radial coordinate, T is tangential coordinate.

Excitation force $P(Y,t) = f(Y) sin\omega t$, where:

$$f(Y) = A_0 + \sum_{k=1}^{\infty}(A_0 \cos kY + B_k \sin kY).$$

The solutions of equations (1) and (2) are written as:

$$w(Y,t) = \frac{R^3 \sin \omega t}{EJ} \sum_{k=1}^{\infty} \frac{k(B_k \cos kY - A_k \sin kY)}{h\omega^2(k^2+1) - k^2(k^2-1)^2} \quad (3)$$

$$U(Y,t) = -\frac{R^3 \sin \omega t}{EJ} \sum_{k=1}^{\infty} \frac{k^2(A_k \cos kY + B_k \sin kY)}{h\omega^2(k^2+1) - k^2(k^2-1)^2} \quad (4)$$

When the exciting forces are subjecting the points N, the following expression is obtained:

$$P(Y,t) = \sum_{j=1}^{N} c_j \delta_\varepsilon (Y - Y_j) sin(\omega t - \theta_j) \quad (5)$$

The case of N points with amplitude c_j and phases displacement θ_j are applied to the solution of equations (1) and (2):

$$v = \frac{R(k^2-1)}{\sqrt{h(k^2+1)}}; \qquad \omega = \frac{k(k^2-1)}{\sqrt{h(k^2+1)}} \quad (6)$$

The set of motion equations (7) describing a rotary piezomotor with a ring actuator has been analyzed.

$$\begin{cases} m_0 R \frac{\partial^2}{\partial t^2}\left(\frac{\partial x_1}{\partial \varphi} + x_2\right) + \frac{1}{R}\frac{\partial}{\partial \varphi}\left(\frac{\partial^2 M}{\partial \varphi^2} + M\right) = 0 \\ M = \frac{EI}{R^2}\left(\frac{\partial x_2}{\partial \varphi} - \frac{\partial^2 x_1}{\partial \varphi^2}\right) \\ x_1 = \frac{\partial x_2}{\partial \varphi} \end{cases} \quad (7)$$

These equations describe a piezoring segment with radius R. Displacement of the cross-section center is marked by angular coordinate, which can be decomposed into radial and longitudinal components. The force of inertia is shown in Fig. 1. Curvature moment is expressed by moment of inertia and the modulus of strength. Expression of Eq. (8) fully reflects the revolution of the rotor.

$$I_p \frac{du}{dt} = fBR^2 \int_0^{2\pi} (v - Ru)(c - x_1) d\varphi \quad (8)$$

Having Eq.(8) the following is obtained:

$$\frac{du}{dt} + \left[f \frac{BR^2}{I_p} \int_0^{2\pi} (c - x_1) d\varphi \right] u = \frac{fBR^2}{I_p} \int_0^{2\pi} v(c - x_1) d\varphi \quad (9)$$

Where $u-$ velocity of rotor revolution, I_p – moment of inertia of rotor, $f-$ coefficient of friction, $B-$pressing force, $v-$ velocity of a traveling wave in a piezoring. The following expressions are introduced:

$$P(t) = \frac{fBR^2}{I_p} \int_0^{2\pi} (c - x_1) d\varphi$$

$$Q(\tau) = \frac{fBR^2 v}{I_p} \int_0^{2\pi} (c - x_1) d\varphi$$

Eq. (9) is rewritten and equation $\frac{du}{dt} + P(t)u = Q(t)$ is obtained, which solution is:

$$u = e^{-\int_0^t p(\tau)dt} \left[c_1 + \int_0^t Q(\tau) e^{\int_0^t p(\varsigma)d\varsigma} d\tau \right] \quad (10)$$

In this case it is assumed that $u(0) = 0, c_1 = 0, Q = \frac{Pv}{R}$, and velocity of rotor revolution can be calculated by:

$$u(t) = \frac{v}{R} + c_2 e^{-Pt} \quad (11)$$

It is evident that rotational velocity of rotor depends on the arising wave deformations in a piezoring and not on the rotation of the driving part. It is considered that at $\varphi_i (i = 1,2...)$ points force $A\delta(\varphi)\sin(\omega t - \theta_i)$ is acting. Here ω - angular velocity $0 \le \theta_i \le 2\pi$, $\omega t = \tau, x_2 = x$ and function $\partial(\varphi)$:

$$\delta(\varphi) = \begin{cases} 0, \varphi \notin (\varphi_{i-\varepsilon;\varphi_{i+\varepsilon}}) \\ 1, \varphi \in (\varphi_{i-\varepsilon}; \varphi_{i+\varepsilon}) \end{cases}$$

Having estimated all conditions and having solved equations, where $a-$number of waves, the following is obtained:

$$x_1 = \frac{2B}{\pi} \sum_{i=1}^{n} \sum_{k=1}^{\infty} \frac{k \sin k(\varphi + \alpha_i) \sin(\tau + \theta_i)}{a(k^2+1) + b(k^6 - 2k^4 + k^2)} \quad (12)$$

In the first case, when a piezoring is subjected to the force at two points $\alpha_1 = 0, \alpha_2 = \frac{\pi}{2}$, and phase displacements are $\theta_1 = 0, \theta_2 = \frac{\pi}{2}$. Equation (12) will be transformed into:

$$x_1 = \frac{2B}{\pi}\left[\frac{\cos(\varphi-\tau)}{2a} + \frac{2\sin 2\varphi(\sin\tau - \cos\tau}{5a+36b} + \ldots\right] \tag{13}$$

In the second case, when a piezoring is subjected to the force at four points $\alpha_1 = 0, \alpha_2 = \frac{\pi}{4}, \alpha_3 = \frac{\pi}{2}, \alpha_4 = \frac{3\pi}{4}$ and phase displacements are $\theta_1 = 0, \theta_2 \frac{\pi}{4}, \theta_3 = -\frac{\pi}{2}, \theta_4 = -\frac{\pi}{4}$. Equation (12) will be transformed into:

$$x_1 = \frac{2B}{\pi}\left[\frac{\sqrt{2}\sin(\varphi+\tau-\frac{\pi}{4})}{2a} + \frac{\sqrt{2}\sin 2\varphi\cos(\tau-\frac{\pi}{4}) + \cos 2\varphi\cos\tau}{5a+36b} + \ldots\right]$$

As mentioned above, the developed deforma-tions (on the basis of a traveling wave) in a ring drive causes the rotation of the rotor. We assume $y(t)$ as the angular speed of the rotor (at moment *t*), *v* is as the linear speed of a traveling wave in the ring drive. To set up a differential equation for expressing the piezoring speed we make some assumptions. Force component $d\tau$ with which the ring surface element acts at the corresponding rotor element is linearly proportional to the magnitudes of:

- initial pressure *B* between the unit ring drive element in the initial equilibrium state at the corresponding rotor part,
- contact area $aRd\varphi$ where a is thickness of the ring drive.

Difference $C - U(\varphi, T)$, corresponds to the dynamic clamping of the ring drive contact surface on the piezoring, C is experimental constant, $C \geq max\, U(t,\varphi)$. Coefficient of proportionality *f* depends on the rotor mass. Then we obtain:

$$\tau = faR\int_0^{2\pi}(v - Ry)(c-u)d\varphi \tag{14}$$

and expressing force τ by linear acceleration of the rotor surface $R\frac{dy}{dt}$ the expression becomes:

$$IR\frac{dy}{dt} = faR\int_0^{2\pi}(v - Ry)(c-u)d\varphi \tag{15}$$

$$\frac{dy}{dt} = \frac{fa}{I}\int_0^{2\pi}(v - Ry)(c-u)d\varphi \tag{16}$$

The angular speed of the rotor is:

$$y(t) = \frac{v}{R}\left(1 - e^{-Pt}\right) \tag{17}$$

We assume that the force of viscous friction is proportional to rotor speed $y(t)$. The viscous friction force between the rotor and the ring drive proportional to the difference of traveling wave speed $v(t)$ and rotor $Ry(t)$ is obtained by equalizing to the force which creates rotor acceleration $\frac{dy}{dt}$. Then we obtain:

$$\frac{dy}{dt} + \eta y + \eta_1(v - Ry) = k\int_0^{2\pi}(v - Ry)(c-u)d\varphi \tag{18}$$

where η and η_1 are coefficients of viscous friction and $b < 2\pi ck$, in this way:

$$\frac{dy}{dt} = (2\pi ck - \eta)\left[v - \left(R + \frac{\eta_1}{2\pi ck - \eta}\right)y\right] \tag{19}$$

Equation (16) yields where I is coefficient of proportionality. The final expression is:

$$y = \frac{v}{1 + \frac{\eta_1}{R(2\pi ck - \eta)}} \tag{20}$$

The rotational speed can be said to depend on the wave deformation of the driving link. It should be noted that reduction of the friction force tends to recur the wave processes.

Experiments and validation

Experimental testing has confirmed that results of theoretical investigation are correct [3]. Selection of piezomaterial and number of experiments have been made depending on temperature effect, magnitude of the load, piezomotor material, etc. As a result of the research a number of piezomotors have been developed and proposed for application in mechanical systems.

Rotary piezomotor consists of holder 1, piezoring 2, stator 3 and rotor 4 (Fig.2).

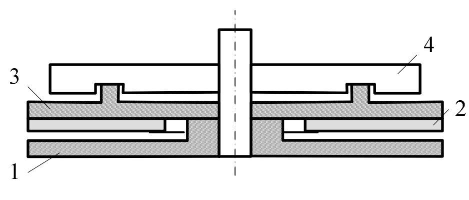
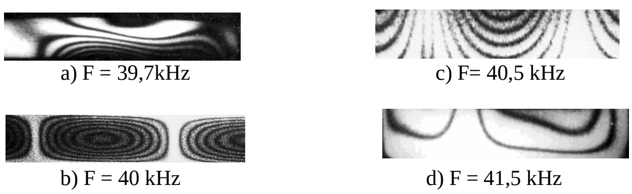


Fig. 2. Rotary piezomotor: 1- holder, 2 - piezoring, 3 - stator, 4 - rotor



a) F = 39,7kHz
c) F= 40,5 kHz
b) F = 40 kHz
d) F = 41,5 kHz

Fig. 3. The holographic interferograms of the links of the piezoring

The experimental investigation has been performed by a holographic interferometric method enabling to watch the picture of deformations in a piezoring drive which practically come to its conclusions.

The experimental investigation of precision vibrosystems by means of holographic interferometry enables one to obtain appreciably larger amounts of information about the vibrating surface in comparison with traditional methods.

The paper deals with the consideration of methods for determination of the vibrational characteristics of precision mechanical systems from the holographic interferograms of linked analysis of these characteristics obtained by using numerical techniques based on the theories of mechanical system vibration and holographic interferometry. A multipurpose device has been developed for storing the holographic interferograms. It allows the application of various methods of holographic interferometry in order to obtain interferograms of excellent quality. When analyzing the performance of the wave systems it is necessary to investigate the wave characteristics of the input member, its influence on the other elements of the system. It is very important to calculate the normal and tangential components of the amplitudes of the surface points of the input member. The determination of their values enables to use the traditional laws of the classical mechanical vibration theory, where the initial data for the theoretical calculations are taken from the holographic interferograms. The obtaining of holographic interferograms enables to optimize the design of the working regimes of the mechanisms with ring actuators, to obtain supplementary data which assists in the development of device designs. Select the conditions of transfers vibration of the ring actuator that are least affected by rotational inertia. In the holographic interferogram of Fig. 3 (a, c, d) the case when the pressure force of piezoring does not fully ensure the motion of the rotor is presented. In the case of Fig. 3 (b) the holographic interferogram is presented that indicates normal operation mode of the piezodrives.

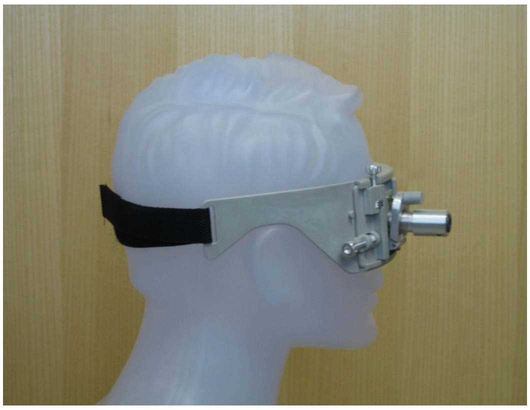


Fig. 4. General view of the frame for fixing an observation device on the patient (medical equipment)

Determination of rotary piezomotor values enables researchers to apply the traditional laws of the classical mechanical vibration theory, where the initial data for the theoretical calculations are taken from a holographic interferograms which for selecting the conditions of transferring vibrations of a piezoring that are least affected by the rotational inertia are used. Holographic interferogram of a case when pressure force of a piezoring does not fully ensure motion of the rotor is presented in [4]. Experimental studies of rotary piezomotors with piezoelectric elements has revealed possibilities to optimize design and materials for obtaining maximum displacements [5]. Mechanical systems with piezoelectric ring are used for implementing an accurate angle displacement. Development of new structures makes it possible to apply rotary piezomotors in design of new mechanical systems [6] such as medical equipment illustrated in Fig. 4.

Conclusions

Analysis of various parameters of rotary piezomotors enabled optimization of their structures and determination of their capabilities under different mechanical loads. Dynamic process of mechanisms with piezoelectric elements, their investigation and attempts to select best versions for applying them in mechanical structures in practice have been the objective of this study.

Presented theoretical and experimental study makes it possible to draw the following conclusions. The expressions of differential equations have been obtained for describing dynamic variations in ring drives. Relationships of friction and tension force in a kinematic pair has been analyzed and it confirms the fact that reducing the friction force magnitude the wave processes become recurring processes in the ring drive. The expressions of linear and rotational speeds allow to calculate and apply the deformations to the optimal extent in piezoactuators.

A holographic interferometry method used in the experimental work has confirmed results of theoretical analysis of differential equations and allowed formulation of conclusions of the research. The experimental results are easy to access and they are applied to develop a tool for loosening rigid tightenings, eliminating corrosion impurity in machining and tool adjustment, medical equipment.

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