# 412. Paradoxes of increasing linear damping in the nonlinear driven oscillators

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**Abstract.** The work is devoted to the systematic study of periodic and chaotic forced oscillations. Recently, on the basis of method of complete bifurcation groups new nonlinear effects were found in driven damped systems with various nonlinearities of elastic restoring forces. Construction of complete bifurcation groups is based on the method of stable and unstable periodic regimes continuation on parameter. Aim of the work – to study new nonlinear effects induced by varying linear dissipation in following dynamical systems with typical nonlinear restoring forces: symmetric trilinear and quadratic, bilinear, cubic with asymmetry, Duffing, pendulum. The work presents new qualitative and quantitative results of nonlinear dynamics in the systems with increasing linear dissipation.

**Keywords:** forced nonlinear oscillations, linear damping, unstable periodic infinitiums, rare attractors, chaotic attractors, complete bifurcation analysis.

## Introduction

In recent years much attention has been paid to studying new nonlinear effects which can be used in vibrotechnics. However the nonlinear effects in dynamical systems with linear dissipation haven't been studied sufficiently even for the simplest nonlinear systems [5-13].

At present the investigation of various dynamical natural and technical phenomena is closely connected with studying general regularities and nonlinear phenomena characterising the behaviour of dynamical systems. The basic approach to research of nonlinear effects is a method of complete bifurcation groups [1-4]. The work is devoted to the research of the forced oscillations and studying of nonlinear effects in dynamical systems with nonlinear restoring force and linear dissipation.

## Dynamical model and research methods

These models describing many important technical applications have the following form (Fig. 1):

$$m\ddot{x} + F_1(x) + F_2(\dot{x}) = H(t), \quad (1)$$

where m is a mass of the system, $F_1(x)$ is a restoring force with nonlinear characteristic, $F_2(\dot{x})=b\dot{x}$ is a linear damping force and $H(t)= h_1 \cos(\omega t+\varphi)$ is an external periodic force with period T. In this paper we use harmonic force with amplitude $h_1$, frequency $\omega$ and phase $\varphi$ ($\varphi=0$).

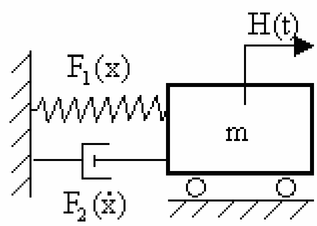


**Fig. 1.** Model of dynamical system

The analysis of system (1) is based on the method of complete bifurcation groups including the method of Poincaré points mapping, a method of parameter following of stable and unstable periodic regimes, of bifurcation diagrams construction with the analysis stability of periodic regimes, a contour mapping and constructing of periodic and chaotic regimes attraction domains. All results have been received by direct numerical simulation using software NLO [14] and SPRING [6].

## Nonlinear effects at the forced oscillations in dynamical systems with nonlinear elastic restoring force

We show that changing linear dissipation may generate nonlinear effects in the driven systems with nonlinear restoring forces. These phenomena can be seen even after increasing the level of linear dissipation. The most unexpected fact is birth of additional stable regimes after damping increasing. Some newly born regimes have greater amplitude for the region with large linear damping coefficient. Earlier existence of several nonlinear effects has been shown, for example, for piecewise linear dynamical systems with linear damping [9-11, 15]. A phenomenon has been discovered – the birth of periodic regimes of 1T and 3T different bifurcation groups through the unstable periodic infinitiums (UPI-1 and UPI-3) [13] under increasing dissipation (Fig. 2-3). There are several important but insufficiently learned elementary and complex typical bifurcation groups. Among them there are unstable periodic infinitiums (UPI), rare attractors (RA), protuberances, bifurcation groups with splitting, cascades and chains of subharmonic isles. In the work these conclusions are generalised on systems with different types of nonlinear restoring forces characteristics.

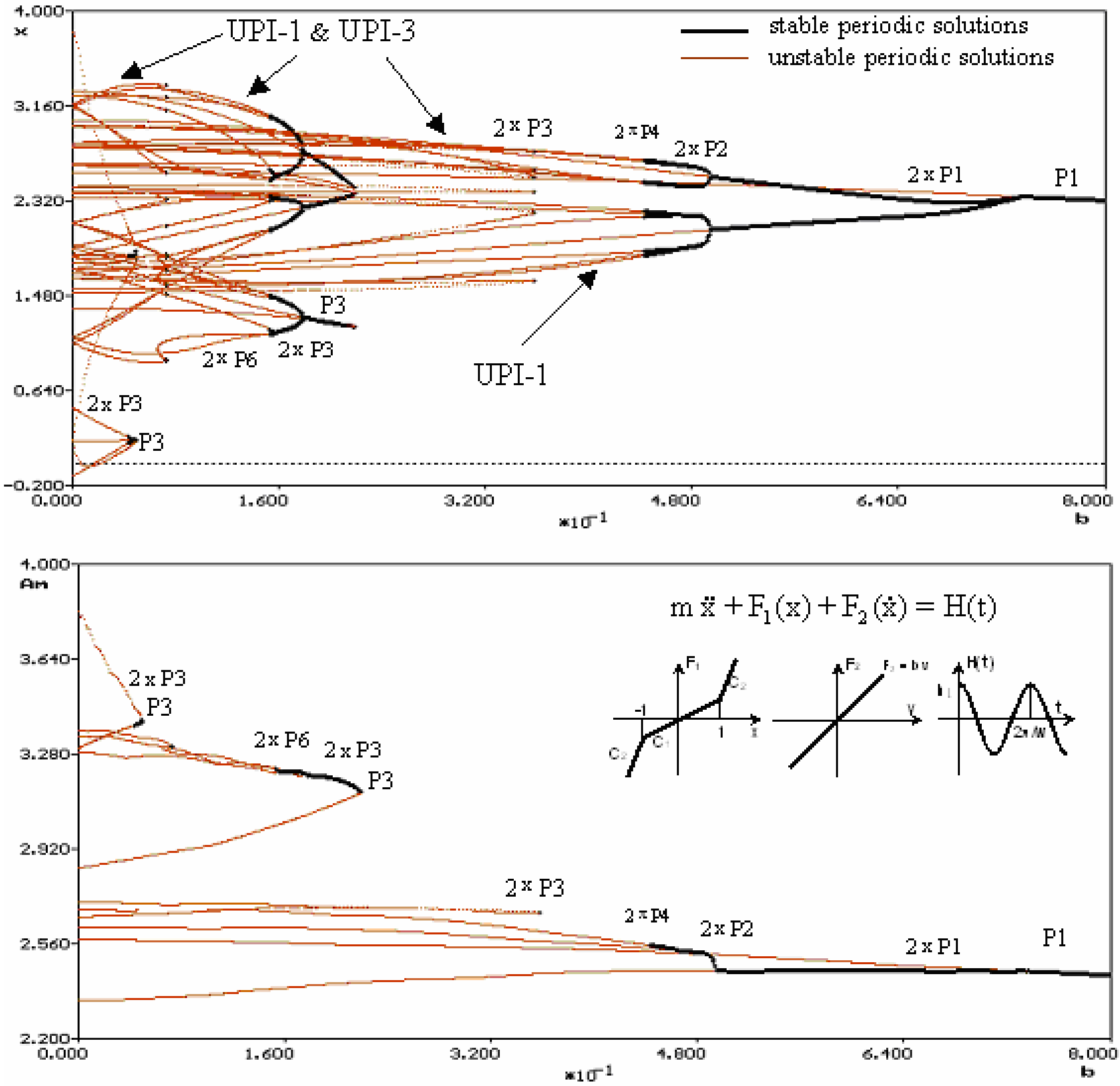


**Fig. 2.** Bifurcation diagrams of four bifurcation groups, 1T and three 3T with rare attractors and unstable periodic infinitiums (UPI-1 and UPIs-3), for symmetric system with trilinear restoring force F1(x), linear dissipation and harmonic excitation. Coordinate x of periodic regime fixed point and amplitude of oscillations Am vs linear dissipation coefficient b. System parameters: m = 1, c1 = 1, c2 = 9, Δ = 1, w = 1, h1 = 7, b = var.

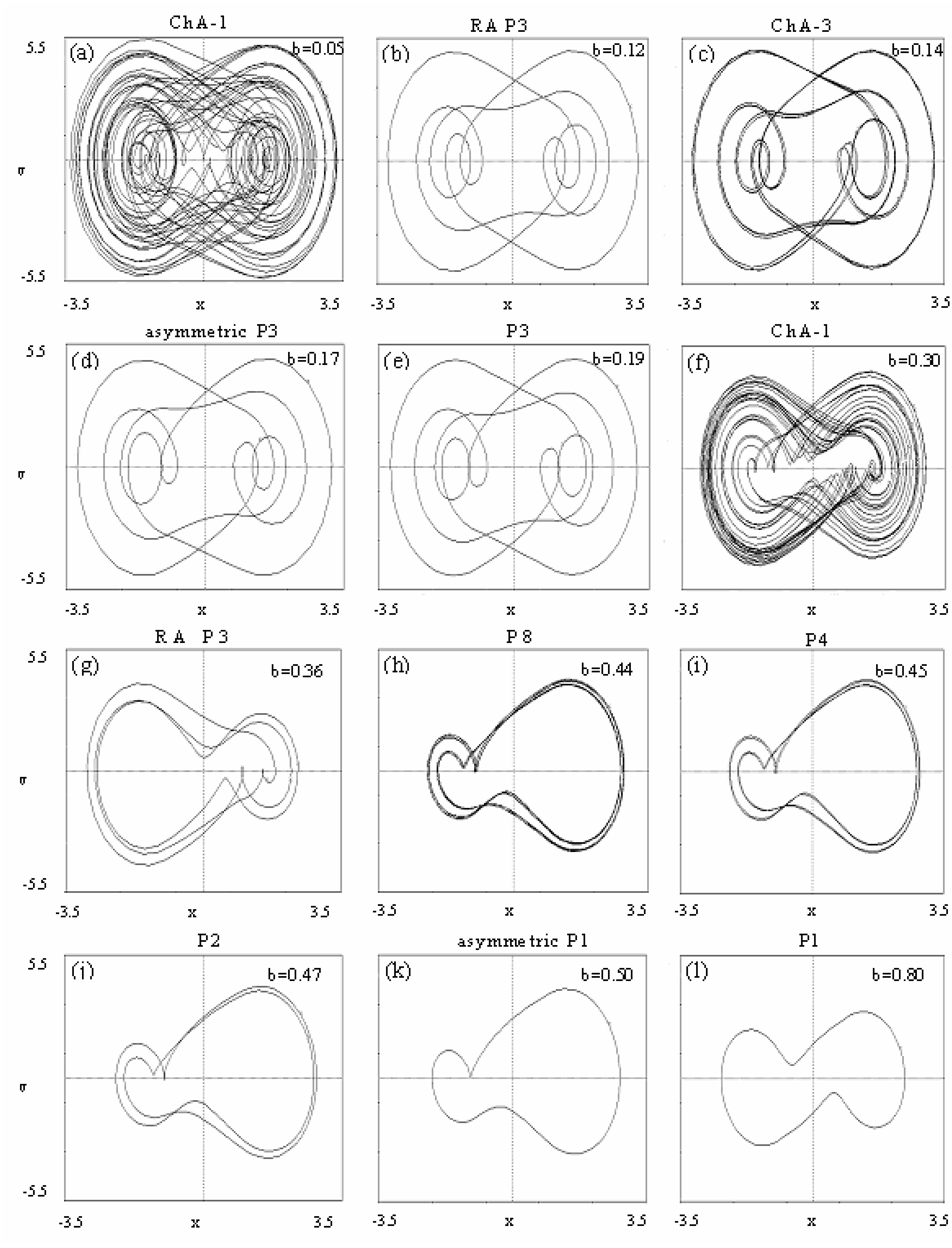


**Fig. 3.** Phase portrait of periodic regimes (d, e, h-l), rare attractors (b, g) and a chaotic orbits (a, c, f) of 1T and 3T bifurcation groups of the symmetric trilinear system (Fig. 2). Parameters: c1 = 1, c2 = 9, Δ = 1, w = 1, h1 = 7, b = var.

The results obtained in the trilinear symmetric system with linear damping and external harmonic forcing are used in studying regular and chaotic forced oscillations in nonlinear systems with different types of nonlinear restoring forces characteristics: Duffing forced system (Fig. 4-7), driven symmetrical system with quadratic restoring forces (Fig. 8) and pendulum with a horizontal harmonic excitation of the suspension point (Fig. 9-11).

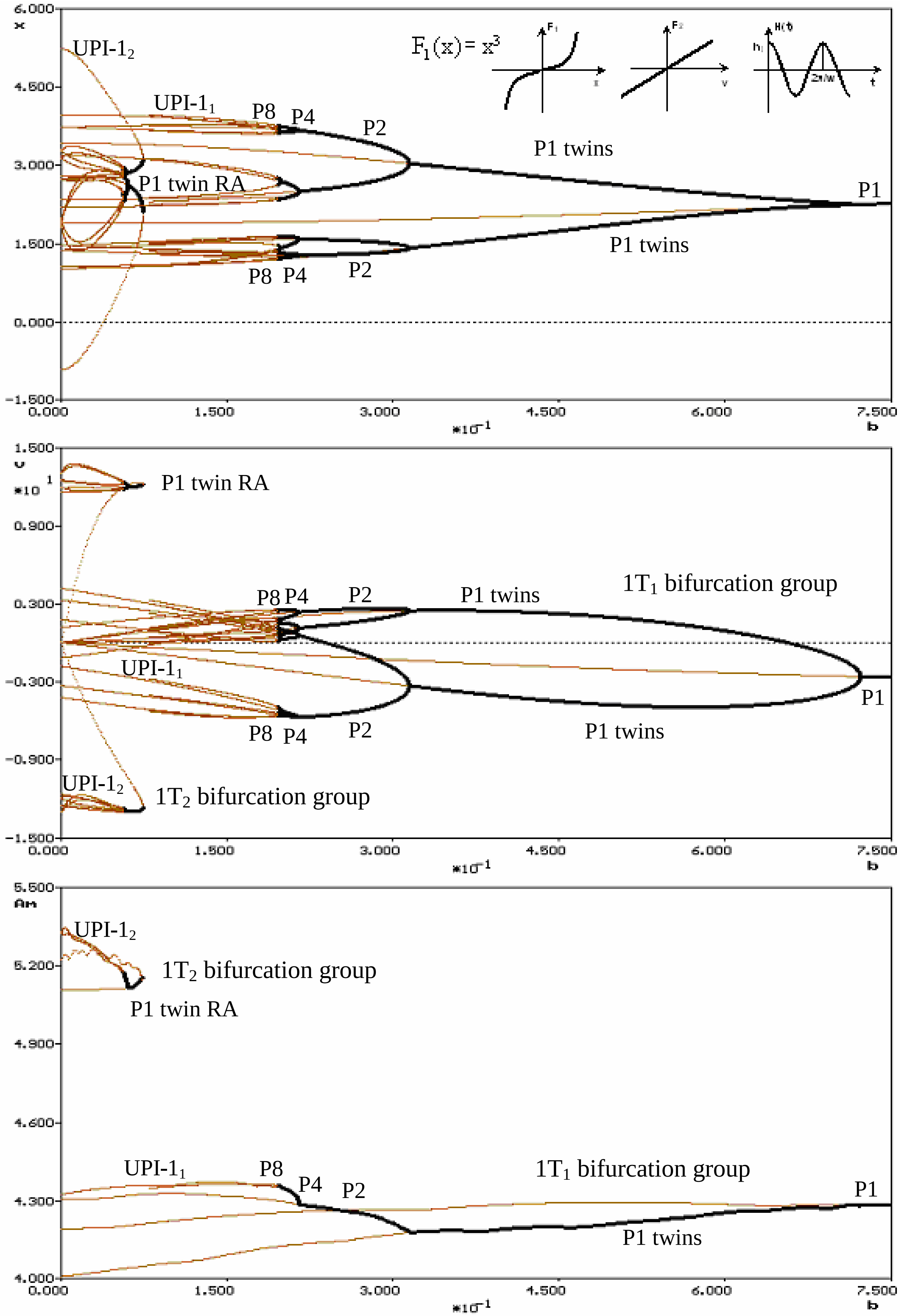


**Fig. 4.** Bifurcation diagrams of two bifurcation groups 1T both with its own period-doubling cascades and UPI-1 for the Duffing system with linear dissipation and harmonic excitation. Coordinates x, v of periodic regime fixed point and amplitude of oscillations Am vs linear dissipation coefficient b. System parameters: m = 1, w = 1, h1 = 34, b =var.

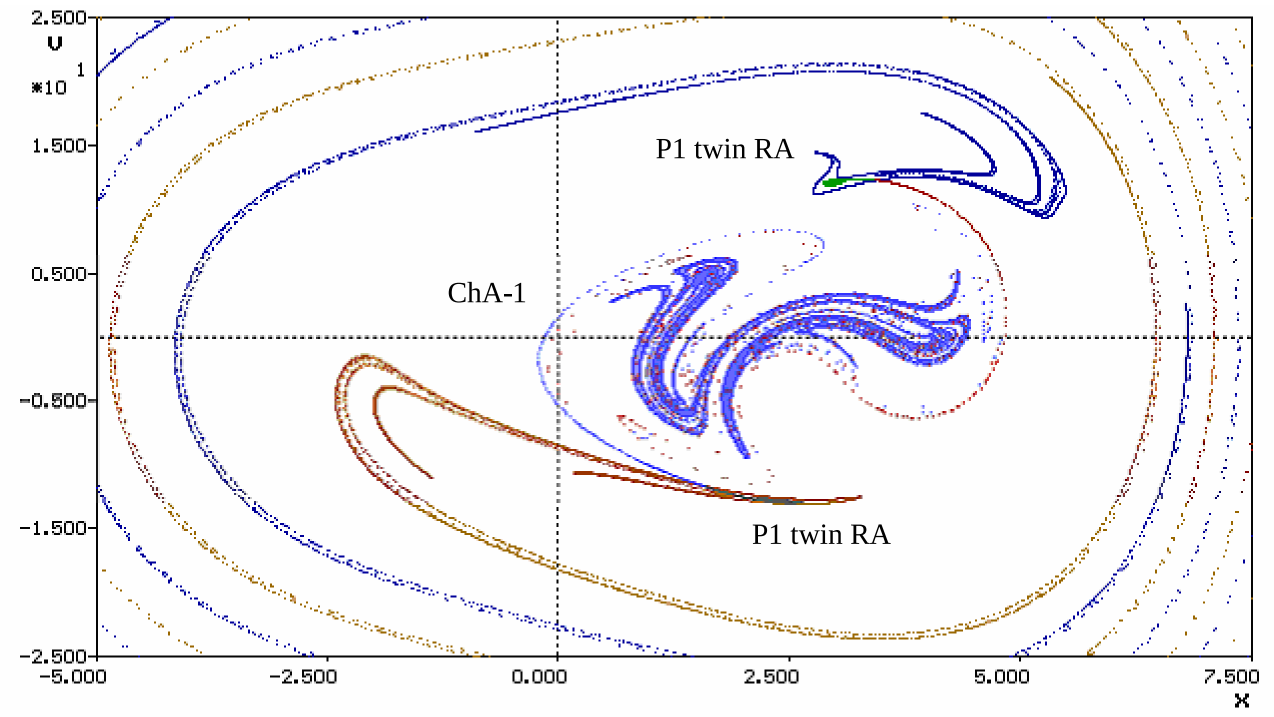


**Fig. 5.** Domains of attraction of two regimes, ChA-1 of $1T_1$ bifurcation group and P1 twin RA of $1T_2$ bifurcation group, for the Duffing system with linear dissipation and harmonic excitation (Fig. 4). System parameters: m = 1, w = 1, b=0.07, h1 = 34

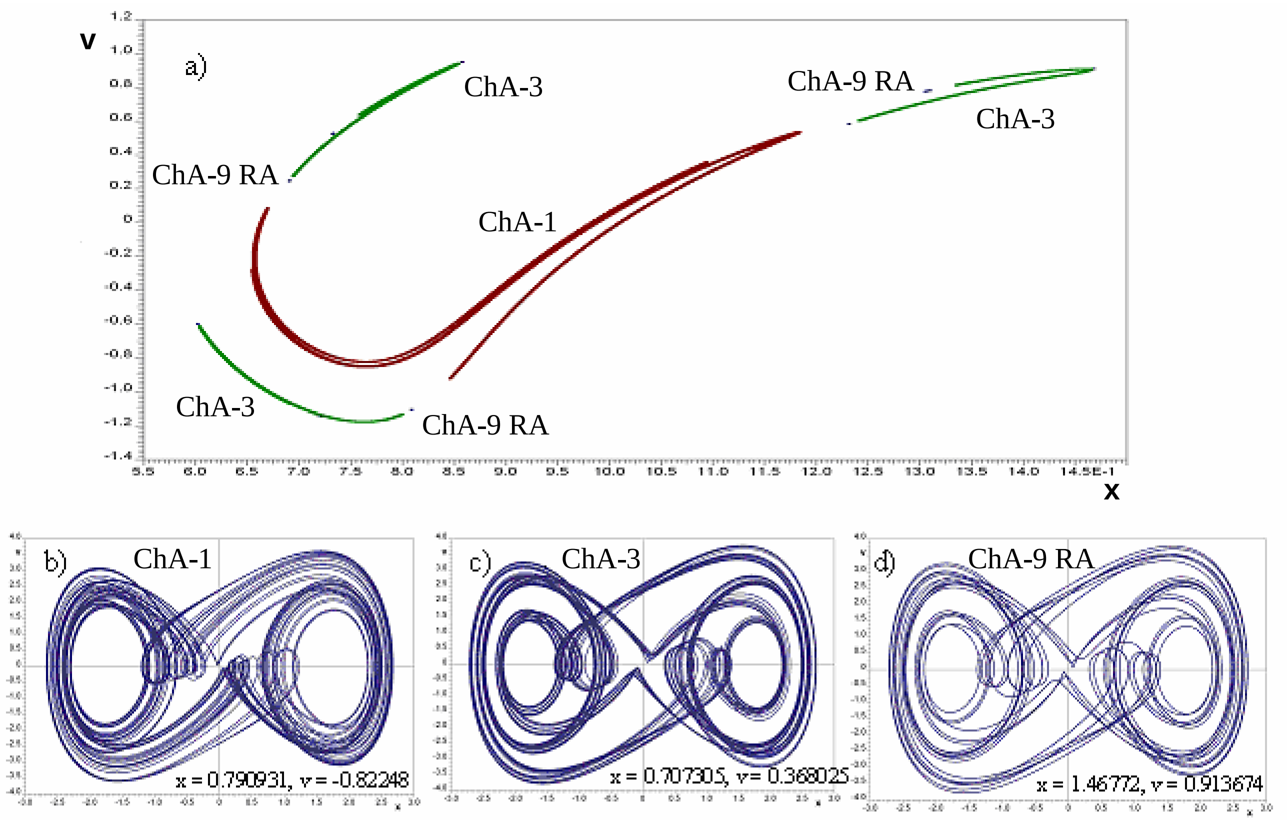


**Fig. 6.** Coexistence of three chaotic attractors, ChA-1, ChA-3 and ChA-9 RA, in the symmetric system with cubic restoring force, linear dissipation and external harmonic excitation. (a) Poincaré map, 25000 periods are shown; (b-d) phase trajectories projections, 25 periods are shown. System parameters: w = 0.5, b = 0.2, h1 = 6.05151

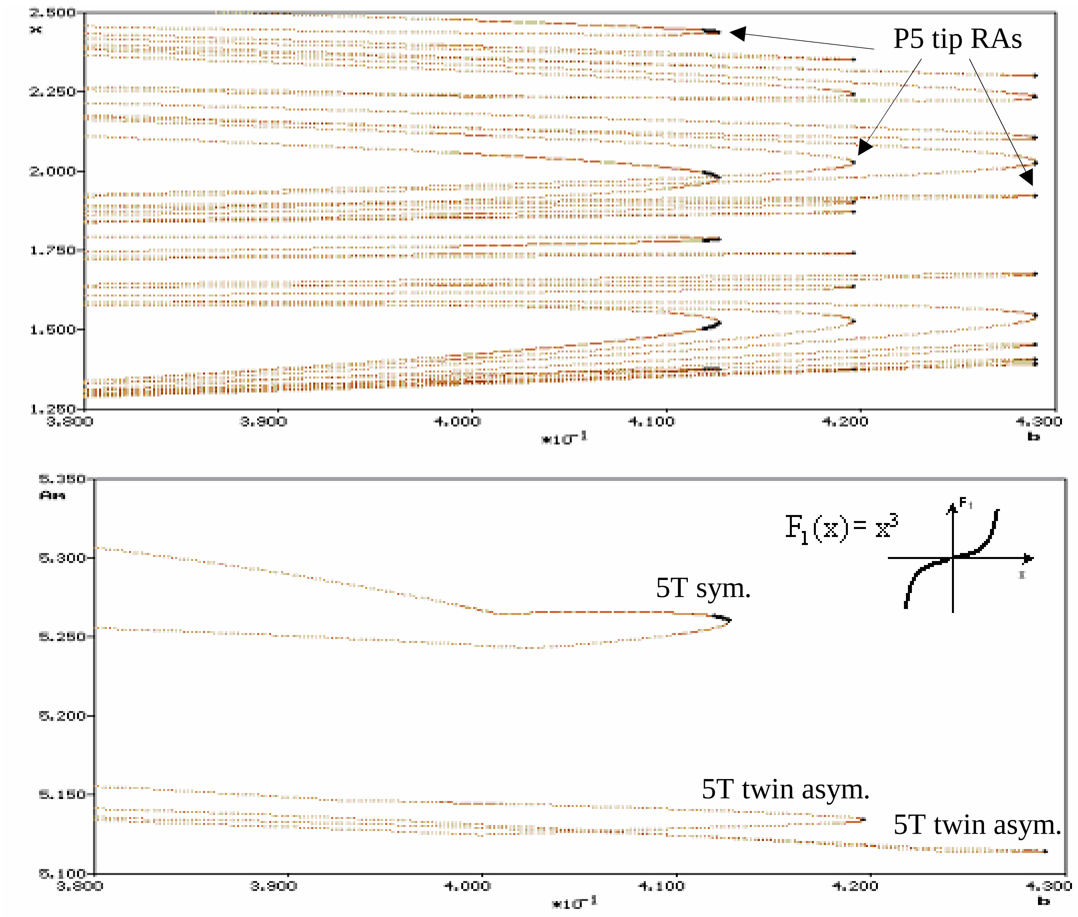


**Fig 7.** Bifurcation diagrams of three bifurcations groups 5T, one symmetric and two asymmetric, with tip rare attractors for the Duffing system with large linear dissipation and harmonic excitation. Coordinate x of periodic regime fixed point and amplitude of oscillations Am vs linear dissipation coefficient b. System parameters: w = 1, h1 = 47, b = var.

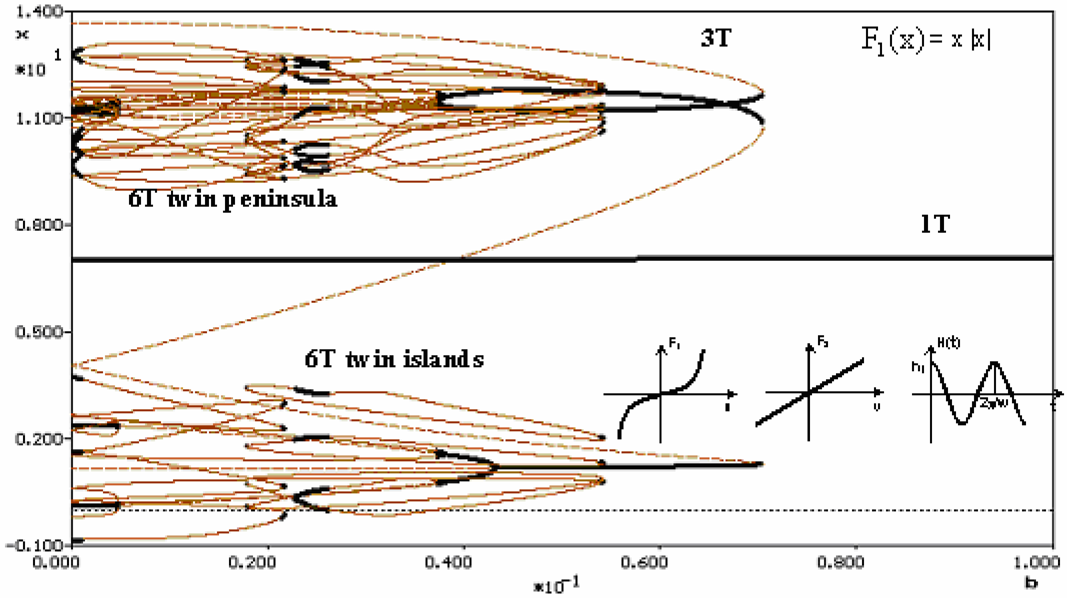


**Fig. 8.** Bifurcation diagrams of four bifurcation groups, 1T, complex 3T and 6T peninsula with tip rare attractors, and 6T island with rare tip and dumb-bell attractors, for symmetrical system with quadratic restoring force, linear dissipation and harmonic excitation. Coordinate x of periodic regime fixed point vs linear dissipation coefficient b. System parameters: m = 1, h1 = 89, w = 1, b = var.

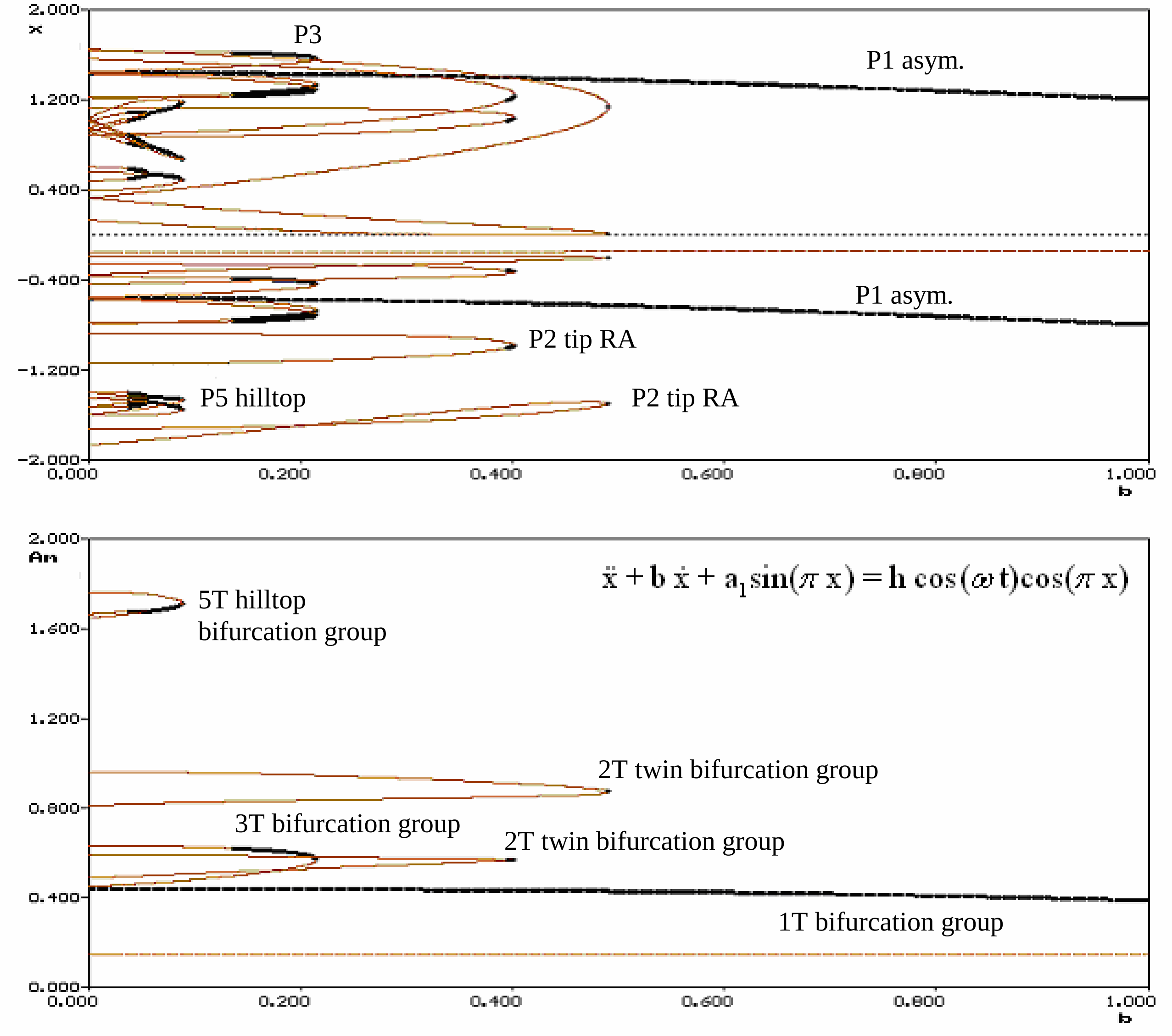


**Fig. 9.** Bifurcation diagram of five bifurcation groups, 1T, two 2T, 3T and 5T hilltop with large amplitudes, for the pendulum with a horizontal harmonic excitation of the suspension point. Coordinate x of periodic regime fixed point and amplitude of oscillations Am vs linear dissipation coefficient b. System parameters: a1 = -1, h = 1, w = 1.8, b = var.

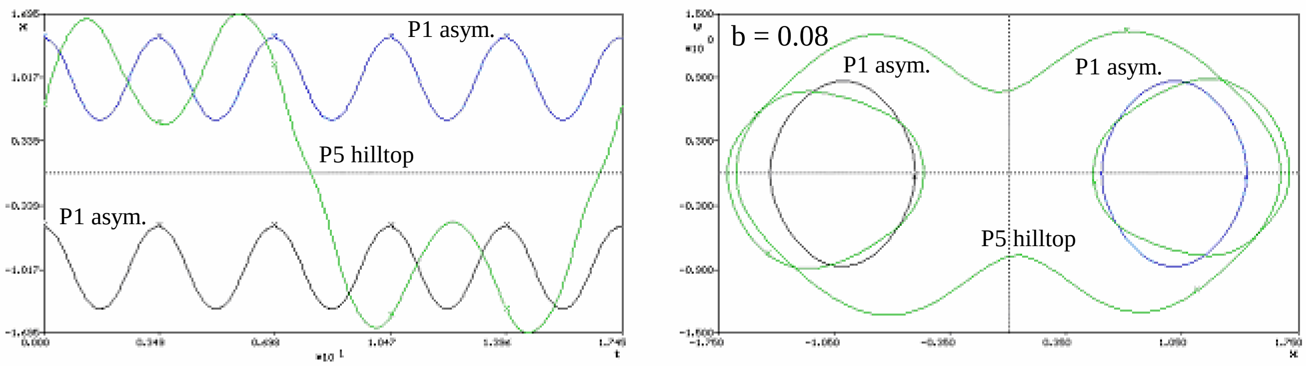


**Fig. 10.** Time histories and phase portraits of twin attractors P1 in the potential wells (fixed points: x = -0.563285, v = -0.057056 and x = 1.436715, v = -0.057056) and P5 hilltop attractor (fixed points: x = 0.712925, v = 1.335634) for pendulum with a horizontal harmonic excitation of the suspension point at b = 0.08 (Fig. 9). System parameters: a1=-1, b=0.08, h=1, w=1.8

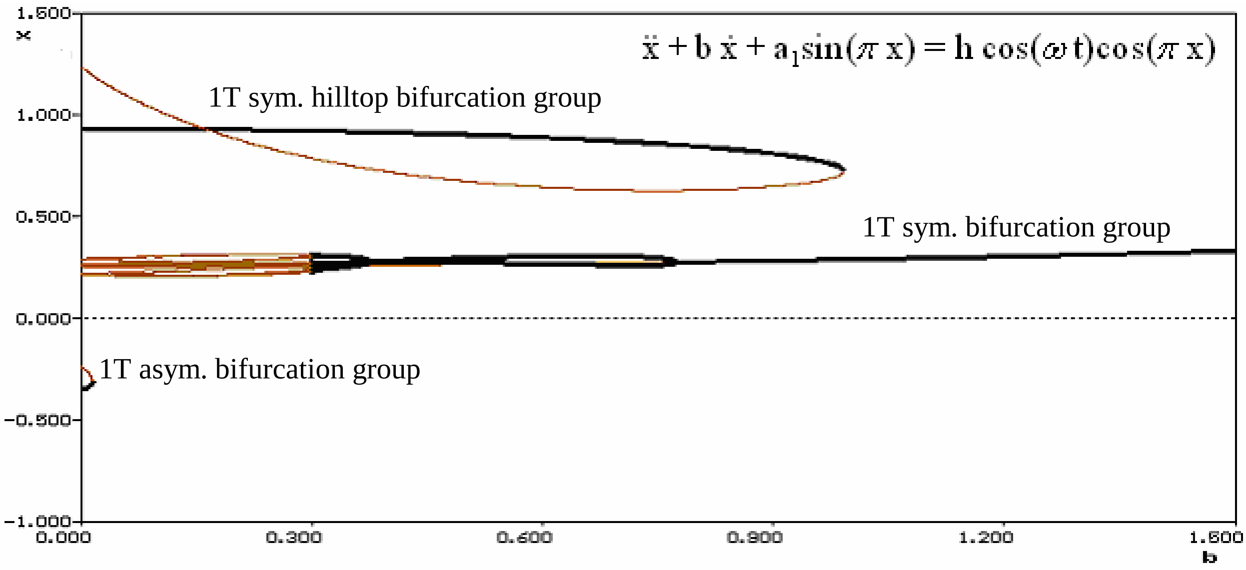


**Fig. 11.** Bifurcation diagram of three bifurcation groups, 1Tsym, 1Tasym and 1Tsym hilltop, for the pendulum with a horizontal harmonic excitation of the suspension point. Coordinate x of periodic regime fixed point and amplitude of oscillations Am vs linear dissipation coefficient b. System parameters: a1 = -1, h = 7, w = 1.8, b = var.

A nonlinear periodically driven oscillators with different types of nonlinear characteristics of restoring forces and linear damping has typical nonlinear effects at dissipation changing. There are rare attractors; multiplicity; periodic 1T, 2T, 3T, 5T ,… islands; period doubling bifurcation cascades, unstable periodic infinitiums (UPI) of different bifurcation groups and chaotic attractors.

**Paradoxes of linear dissipation in bilinear system**

New nonlinear effects in the simplest bilinear system with linear damping were discovered. Paradoxical influence of increasing dissipation: the amplitude of P1 oscillations decreases approximately in 1.5 times when coefficient b of linear dissipation goes through in the region 0.0958 < b < 0.098. A bifurcation diagram is shown in Fig. 12.

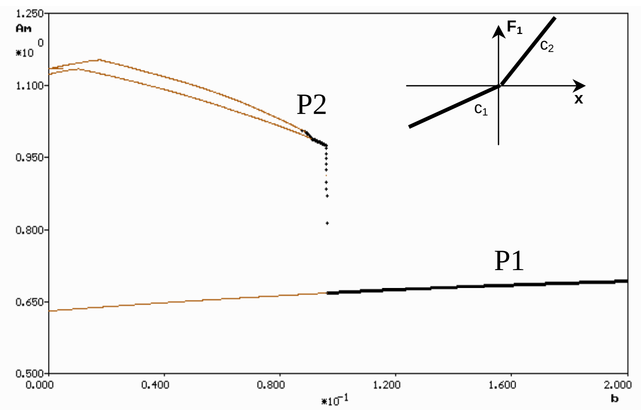


**Fig. 12.** Bifurcation diagram of bifurcation group 1T for bilinear system. Amplitude of oscillations Am vs linear dissipation coefficient b. Parameters: c1 = 1, c2 = 16, d = 0, h1 = 1, w = 0.9, b = var.

**Unusual bifurcations groups at linear damping coefficient varying**

The research reveals, that the use of linear dissipative driven oscillators leads to the whole spectrum of nonlinear effects formation, paradoxical influence of increasing and decreasing dissipation and unusual bifurcation groups.We illustrate in Fig. 13 the unusual bifurcation groups in the nonlinear driven oscillatory with different types of nonlinear restoring forces characteristics under the change of linear dissipation coefficient b.

**Conclusion**

The work presents bifurcation analysis of stable and unstable periodic regimes of different types of bifurcation groups in the nonlinear dynamical system with linear dissipative characteristic.

The analysis of nonlinear system with linear damping is based on the method of complete bifurcation groups. The work presents a systematic study of the mechanisms of birth of typical nonlinear effects at the forced oscillations in nonlinear system under the change of linear dissipation coefficient: loss of stability (birth of a basic regime); subharmonic oscillations; rare attractors; periodic nT (n=1,2,3,…) islands; period doubling bifurcation and unstable periodic infinitium UPI; unstable periodic infinitium of different bifurcation groups; chaotic oscillations; multiplicity (complex structure of phase space); unusual bifurcations groups; paradoxes of change coefficients of linear dissipation.

The typical nonlinear effects allow to predict motions and unexpected transitions (catastrophes) of nonlinear systems with linear damping.

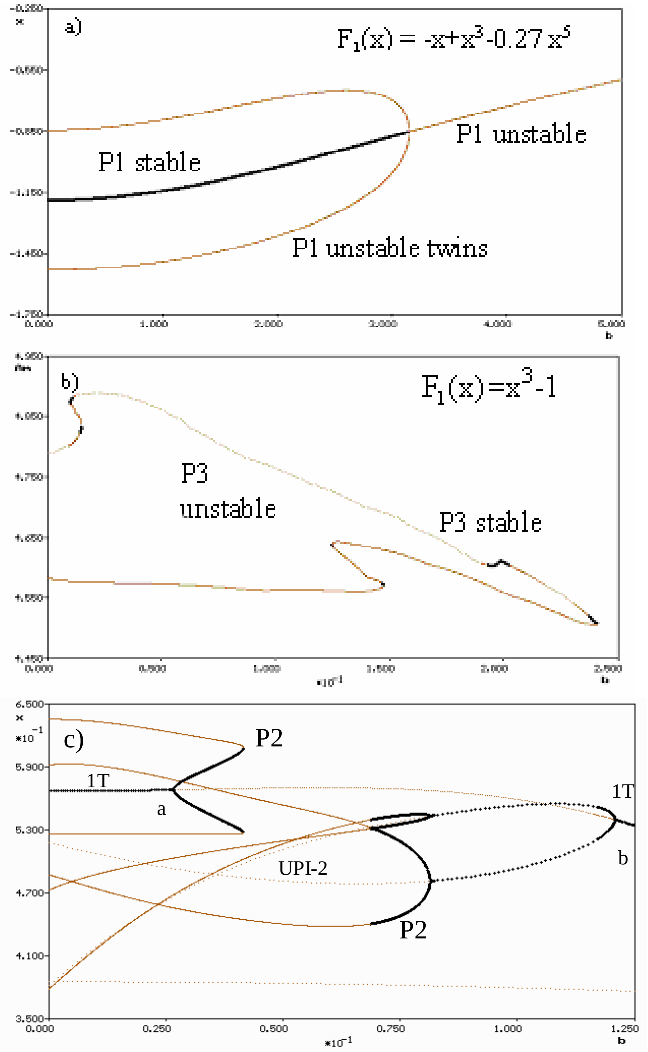


**Fig. 13.** Unusual bifurcations groups with different types of nonlinear characteristics of restoring forces at linear dissipation coefficient b varying: a) birth of a stable basic regime P1 in a nonlinear driven system with linear damping. Parameters: $F_1 = -x+ + x^3 - 0.27x^5$; h1 = 30, w = 5, b = var.; b) upper and lower branches have folds with 3T tip RAs in bifurcation diagram of 3T bifurcation group. System with cubic restoring force with asymmetry $F_1=x^3-1$. System parameters: w = 1, h1 = 27.8, b = var.; c) bifurcation group with P2 saddle-node bifurcation and co-existence of P2, UPI-2, and protuberance P2 from a to b. Bilinear system. System parameters: c1 = 1, c2 = 16, d = 0, h1 = 1, w = 1, b = var.

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