424. Sound-insulation of ellipsoidal shells

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Abstract In the introduction of this paper, the importance of the work in the period of the development of new equipment and technology is formulated when optimum constructions are sought not only in terms of resistance or firmness but also imparting additional properties to them. One of the more important properties of the constructions under development is acoustical ones, i.e. constructions that may to reduce to a maximum the sound (noise) entering the environment.

The work deals with an analysis of sound insulation properties of ellipsoidal shells, comparing them with the same properties of other forms. Examples for the use of the shells of such forms and their efficiency in the environmental noise reduction are provided.

Keywords : protection against noise, sound insulation, shells.

Introduction

With the rapid development of technologies and equipment of a new stage, with their use for means of transport and industry, constructors face the specified requirements in respect of consumers. One of such requirements is the characteristic of noise excited by the means (device), according to which it is possible to calculate the level of noise to be radiated into the environment.

In this paper, we present the sound insulation qualities of the ellipsoidal shell (body), within which the sound propagates, i.e. the efficiency of the shell (the body of the aggregate) by reducing the level of sound propagating into the environment.

In the published works [1, 2, 3], the sound insulation of cylindrical and semi-cylindrical shells was investigated. In the papers, it is shown that calculated sound insulation efficiency of those constructions is obtained when their form is geometrically regular.

Here it was noticed that with the change of cylindricity (incorrect form of cylinder), the sound insulation values and the process of sound transfer through walls are also subject to change. Therefore, it is possible to draw a conclusion that in applying the forms of shell surfaces other than cylindrical ones in the constructions, the ways of solution of other type should be applied.

In this paper we study the sound insulation theory of the ellipsoidal shell.

Theory

We will study the sound insulation of the cylindrical shell, which according to the line of direction is ellipsis (see Fig. 1).

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1 \qquad (1)$$

Instead of the system of coordinates (x, y, z) (or alongside with it) we shall consider the elliptical system of coordinates (σ, τ, z), where the cylindrical surface (1) would be with the coordinate surface. Transfer to the elliptical coordinates is realized by means of the following transformation

$$x = a, \sigma, \tau, \quad y^2 = a^2\left(\sigma^2 - 1\right)\left(1 - \tau^2\right), \qquad (2)$$
$$z = z,$$

where coordinates σ, τ, z, and get changed within the following limits

$$1 \le \sigma < \infty, \; -1 \le \tau \le +1, \; -\infty < z < +\infty . \qquad (3)$$

Coordinate surfaces form the system of confocal ellipses and hyperbolas

$$\frac{x^2}{a^2\sigma^2}+\frac{y^2}{a^2(\sigma^2-1)}=1;\quad \frac{x^2}{a^2\tau^2}+\frac{y^2}{a^2(\tau^2-1)}=1$$

;($\tau = Const$ forms the hyperbolic cylinder)

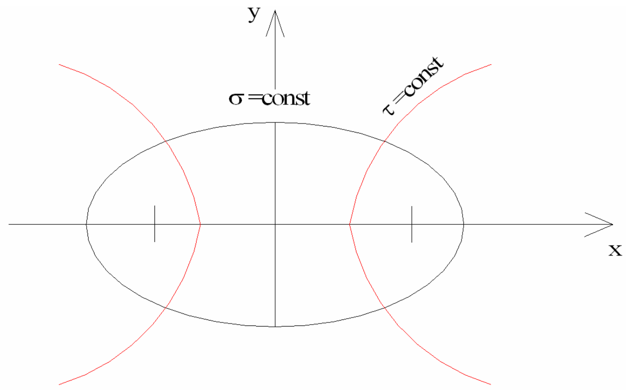

Fig. 1. The system of coordinates of the shell surface of ellipsoidal form (σ,τ,z)

Now it is possible to require that the surface of the given elliptical cylinder (1) be one of the coordinate surfaces in the system of coordinates (σ,τ,z). This means that such coordinate σ_0 must exist where two surfaces

$$\frac{x^2}{A^2}+\frac{y^2}{B^2}=1 \quad \text{and}$$

$$\frac{x^2}{a^2\sigma_0^2}+\frac{y^2}{a^2(\sigma_0^2-1)}=1 \qquad (4)$$

coincide. This is possible if

$$A^2=a^2\sigma_0^2,\quad B^2=a^2(\sigma_0^2-1).$$

In other words, the elliptical system of coordinates must be such that

$$a=\sqrt{A^2-B^2},\quad \sigma_0=A/\sqrt{A^2-B^2}\,. \qquad (5)$$

Finally, we shall notify that coordinates σ and τ have the fully concrete physical meaning.
Since at $\sigma >> 1$ we have

$$x^2=(a\sigma\tau)^2,\quad y^2=(a\sigma)^2(1-\tau^2), \qquad (6)$$

a conclusion may be drawn that coordinate σ is in the essence a dimensionless polar radius, since

$$x^2+y^2\cong a^2\sigma^2, \sigma\cong r/a$$

Hence
($\sigma = Const$ forms the elliptical cylinde

$$\frac{x^2}{x^2+y^2}\to\frac{(a\sigma\tau)^2}{(a\sigma)^2}=\tau^2$$

coordinate τ corresponds to the cosine of the polar angle

$$\tau\cong\cos\varphi$$

A problem on deformations of an arbitrary curvilinear (shallow) shell in arbitrary curvilinear coordinates α_1 and α_2 is related, according to the fundamental works of Goldenveizer [4], with the following system of equations

$$\frac{Eh^3}{12(1-\upsilon^2)}\Delta\Delta u-\Delta_1\Psi=z,\quad \Delta_1 u+\frac{1}{Eh}\Delta\Delta\Psi=0 \qquad (7)$$

Here u is shell deflection, Ψ is the function, reflecting the plane stressed condition, z is normal to the surface external loading, E, υ, h parameters (Young's modulus, Poisson's ratio, thickness). Differential operators Δ and Δ_1 are written by means of coefficients A_1, A_2, R_1, R_{12}, and R_2, reflecting the geometry of shells, in the following form:

$$\Delta_1=\left\{\begin{array}{l}\frac{\partial}{\partial\alpha_1}\frac{A_2}{A_1R_{22}}\frac{\partial}{\partial\alpha_1}+\frac{\partial}{\partial\alpha_2}\frac{1}{R_{12}}\frac{\partial}{\partial\alpha_1}+\\ +\frac{\partial}{\partial\alpha_1}\frac{1}{R_{12}}\frac{\partial}{\partial\alpha_2}+\frac{\partial}{\partial\alpha_2}\frac{A_1}{A_2R_{11}}\frac{\partial}{\partial\alpha_2}\end{array}\right\}\frac{1}{A_1A_2}$$

$$\Delta=\left\{\frac{\partial}{\partial\alpha_1}\frac{A_2}{A_1}\frac{\partial}{\partial\alpha_1}+\frac{\partial}{\partial\alpha_2}\frac{A_1}{A_2}\frac{\partial}{\partial\alpha_2}\right\}\frac{1}{A_1A_2} \qquad (8)$$

Thus, for the formulation of the problem, it is necessary, primarily, to identify all metrical coefficients, included in this equation as applicable to the elliptical system of coordinates, where we present the problem at the given stage in the process of progression to the result, identifying coordinate $\alpha_1 c$ with coordinate z, and α_2 coordinate τ with elliptical coordinate .

A problem on the combined vibrations of the elliptical shell and the surroundings may be considered exactly in the elliptical coordinates with the Mathieu's functions and Mathieu functions of third class, reflecting the outgoing waves in the elliptical coordinates, however, such approach is of low productivity, as the Mathieu's functions are extremely insignificantly studied in the mathematical literature and its is difficult to achieve the numerical results at a given stage. Therefore it is expedient to apply the theory of shallow shells (using the terminology of L. A. Goldenveizer [4]), which differs insignificantly from the circular cylinder, for which this theory, as shown from practice, leads to good results [5, 6].

For formulation of an equation of movement of any shell, it is necessary to select the metrical coefficients, reflecting the geometrical properties of curvilinear medial surface of that shell. Primarily, it is necessary to know the expression of the so-called first square form. The differential of the arc dl on the surface of the elliptical shell is given in the following square form

$$(dl)^2 = a^2\left(\frac{\sigma_0^2 - \tau^2}{1-\pi^2}\right)(d\tau)^2 + (dz)^2 \cdot \qquad (9)$$

If to identify coordinate z with coordinate α_1 , used in Goldenveizer's theory of shells, coordinate τ will be coordinate α_2, and coefficients of that square form would be

$$A_1^2 = 1, \quad A_2^2 = a^2\left(\frac{\sigma_0^2 - \tau^2}{1-\tau^2}\right). \qquad (10)$$

Now it is necessary to find radiuses of curvature of the shell. Only curvature of ellipsis will be different from zero and since

$$R_{22} = \frac{\left(B^4x^2 + A^4y^2\right)^{\frac{3}{2}}}{A^4B^4} \qquad (11)$$

the radius, written by means of shell parameters, introduced above, i.e. at

$$A^4 = (a\sigma_0)^4, B^4 = a^4\left(\sigma_0^2 - 1\right),$$
$$x^2 = (a\sigma_0\tau)^2, y^2 = a^2\left(\sigma_0^2 - 1\right)\left(1-\tau^2\right)$$

will have the following form

$$R_{22} = \frac{a\sigma_0^2}{\sqrt{\sigma_0^2 - 1}}\left\{1 - \frac{\tau^2}{\sigma^2}\right\}^{\frac{3}{2}}, \qquad (12)$$

in addition, it is necessary to write

$$R_{11} = R_{12} = \infty$$

Thus all initial parameters for writing of an equation of a shallow shell will be found.

Equations of a shell have the form [4]:

$$\frac{Eh^3}{12\left(1-\upsilon^2\right)}\Delta\Delta u - \Delta_1\Psi + \rho h\frac{\partial^2 u}{\partial t^2} = (P_1 - P_2), \quad \Delta_1 u + \frac{1}{Eh},$$
$$\Delta\Delta\Psi = 0, \qquad (13)$$

where u - is deflection, Ψ - is function, reflecting the plane stressed state in the shell, h - is thickness, ρ - is density, E - is Young's modulus, P_1 and P_2 - are sound pressures, forming the drop of pressures on both sides of the shell surfaces
(P_1 inside the shell, P_2 outside the shell), operators Δ and Δ_1 essence

$$\Delta = \frac{1}{A_2}\left\{\frac{\partial}{\partial z}A_2\frac{\partial}{\partial z} + \frac{\partial}{\partial\partial\tau}\frac{1}{A_2}\frac{\partial}{\partial\tau}\right\}, \quad \Delta_1 = \frac{1}{R_{22}}\frac{\partial^2}{\partial z^2}\cdot \qquad (14)$$

Pressure is determined from the wave equation, which in coordinates (σ, τ, z) have the form

$$\frac{1}{a^2\left(\sigma^2 - \tau^2\right)}\left[\begin{matrix}\sqrt{\sigma^2 - 1}\dfrac{\partial}{\partial\sigma}\sqrt{\sigma^2 - 1}\dfrac{\partial P}{\partial\sigma} + \\ + \sqrt{1-\tau^2}\dfrac{\partial}{\partial\tau}\sqrt{1-\tau^2}\dfrac{\partial P}{\partial\tau}\end{matrix}\right] + \frac{\partial^2 P}{\partial z^2} = \frac{1}{c_0^2}\frac{\partial^2 P}{\partial\tau^2}\cdot \qquad (15)$$

On the surface of the shell and radiator, located in the centre of the system, the boundary conditions are formulated

$$\frac{\partial\rho}{\sigma\partial} = -\frac{a\sqrt{\sigma^2 - \tau^2}}{\sqrt{\sigma^2 - 1}}\rho_0\frac{\partial^2 u}{\partial t^2}, (npu\sigma = \sigma_0) \qquad (16)$$

where c_0 is sound velocity, ρ_0 is density of the surroundings,
σ_1 and σ_0 coordinates (corresponding to radial coordinates in the polar system of coordinates) of the radiator and of the shell, v - is preset function, describing the movement of the radiator.

Further the sound insulation will be calculated for the traveling wave

$$\left.\begin{aligned} u &= u(\tau)e^{inz - i\omega t} \\ \Psi &= \Psi(\tau)e^{ikz - i\omega t} \\ P_2 &= P_2(\tau,\sigma)e^{ikz - i\omega t} \\ P_1 &= P_1(\tau,\sigma)e^{ikz - i\omega t} \\ v_0 &= v_0(\tau)e^{ikz - i\omega t}\end{aligned}\right\} \qquad (17)$$

In this case we obtain the following system of equations and boundary conditions

$$\frac{Eh^3}{12\left(1-\upsilon^2\right)}\Delta\Delta u + \frac{k^2\Psi}{R_{22}} = \rho h\omega^2 u = P_1(\tau,\sigma_0) - P_2(\tau,\sigma_0)$$
$$\Delta\Delta\Psi = \frac{k^2Eh}{R_{22}}u$$
$$u^2\left(\sigma^2 - \tau^2\right)\left\{\sqrt{\sigma^2 - 1}\frac{\partial}{\partial\sigma}\sqrt{\sigma^2 - 1}\frac{\partial P}{\partial\sigma} + \sqrt{1-\tau^2}\frac{\partial}{\partial\tau}\sqrt{1-\tau^2}\frac{\partial P}{\sigma\tau}\right\} +$$
$$+\left(\frac{\omega^2}{c_0^2} - k^2\right)\cdot P = 0,$$

$$\frac{\partial P}{\partial\sigma}\cdot\frac{\sqrt{\sigma^2 - 1}}{a\sqrt{\sigma^2 - \tau^2}} = \rho_0\omega^2 u, \frac{\partial P}{\partial\sigma}\cdot\frac{\sqrt{\sigma^2 - 1}}{a\sqrt{\sigma^2 - \tau^2}} = \rho_0\omega^2 v_0. \qquad (18)$$

The approximated method by means of the theory of variable rigidity.

If the length of a sound wave radiated by the shell is considerable as compared to the eccentricity of the shell, the elliptical shell of the sound field is fully identical with the shell of variable rigidity. On the other hand, the circular shell (including also of variable rigidity) form the following fields inside the shell

$$P_2 = \int_{-\pi}^{+\pi} G_{11}(\varphi-\varphi_1, r)v(\varphi_1)d\varphi_1 + \int_{-\pi}^{+\pi} G_{12}(\varphi-\varphi_1 r)u(\varphi_1)d\varphi_1, \tag{19}$$

where

$$G_{11} = \frac{\rho_0\omega^2}{2\pi\sqrt{\frac{\omega^2}{c_0^2}-k^2}} \sum_{n=-\infty}^{n=+\infty} e^{in(\varphi-\varphi_1)} \left\{ \frac{H_n^{(1)}\left(\sqrt{\frac{\omega^2}{c_0^2}-k^2 r}\right)\dot{H}_n^{(2)}\left(\sqrt{\frac{\omega^2}{c_0^2}-k^2 R}\right) - H_n^{(2)}\left(\sqrt{\frac{\omega^2}{c_0^2}-k^2 r}\right)\dot{H}_n^{(1)}\left(\sqrt{\frac{\omega^2}{c_0^2}-k^2 R}\right)}{\dot{H}_n^{(2)}\left(\sqrt{\frac{\omega^2}{c_0^2}-k^2 R}\right)\dot{H}_n^{(1)}\left(\sqrt{\frac{\omega^2}{c_0^2}-k^2 a}\right) - \dot{H}_n^{(2)}\left(\sqrt{\frac{\omega^2}{c_0^2}-k^2 a}\right)\dot{H}_n^{(1)}\left(\sqrt{\frac{\omega^2}{c_0^2}-k^2 R}\right)} \right\}$$

$$G_{12} = \frac{\rho_0\omega^2}{2\pi\sqrt{\frac{\omega^2}{c_0^2}-k^2}} \sum_{-\infty}^{+\infty} e^{in(\varphi-\varphi_1)} \left\{ \frac{H_n^{(2)}\left(\sqrt{\frac{\omega^2}{c_0^2}-k^2 r}\right)H_n^{(1)}\left(\sqrt{\frac{\omega^2}{c_0^2}-k^2 a}\right) - H_n^{(1)}\left(\sqrt{\frac{\omega^2}{c_0^2}-k^2 r}\right)H_n^{(2)}\left(\sqrt{\frac{\omega^2}{c_0^2}-k^2 a}\right)}{\dot{H}_n^{(1)}\left(\sqrt{\frac{\omega^2}{c_0^2}-k^2 a}\right)\dot{H}_n^{(2)}\left(\sqrt{\frac{\omega^2}{c_0^2}-k^2 R}\right) - \dot{H}_n^{(2)}\left(\sqrt{\frac{\omega^2}{c_0^2}-k^2 a}\right)\dot{H}_n^{(1)}\left(\sqrt{\frac{\omega^2}{c_0^2}-k^2 R}\right)} \right\}$$

and outside the shell

$$P_1 = \int_{-\pi}^{+\pi} G(\varphi-\varphi_1, r)u(\varphi_1)d\varphi_1 \quad , \tag{20}$$

where

$$G(\varphi-\varphi_1, r) = \frac{\rho_0\omega^2}{2\pi\sqrt{\frac{\omega^2}{c_0^2}-k^2}} \sum_{n=-\infty}^{n=+\infty} \frac{H_n^{(1)}\left(\sqrt{\frac{\omega^2}{c_0^2}-k^2 r}\right)}{\dot{H}_n^{(1)}\left(\sqrt{\frac{\omega^2}{c_0^2}-k^2 R}\right)} e^{in(\varphi-\varphi_1)}$$

For the shallow shell, the coordinate τ may be identified with cosine of the polar angle φ. In this case

$$A_2 = a\left(\sigma_0^2 - \cos^2\varphi\right)^{\frac{1}{2}} / \sin\varphi,$$

$$R_{22} = a\left(\sigma_0^2 - \cos^2\varphi\right)^{\frac{3}{2}} / \sigma_0\sqrt{\sigma_0^2 - 1} \tag{21}$$

$$\Delta = \frac{1}{a^2}\left\{\left(\sigma_0^2 - \cos^2\varphi\right)^{-\frac{1}{2}} \frac{\partial}{\partial\varphi}\left(\sigma_0^2 - \cos^2\varphi\right)^{-\frac{1}{2}} \frac{\partial}{\partial\varphi} - (ka)^2\right\} \tag{22}$$

and, consequently, the equation of the shell movement is deduced to the following integro-differential equation with the variable coefficient

$$\frac{Eh^3}{12(1\upsilon^2)}\Delta\Delta u + \frac{k^2}{R_{22}}\Psi - \rho h\omega^2 u = \int_{-\pi}^{+\pi} G_{11}(\varphi-\varphi_1)v(\varphi_1)d\varphi_1 +$$
$$+ \int_{-\pi}^{+\pi} G_{12}(\varphi-\varphi_1)u(\varphi_1)d\varphi_1 - \int_{-\pi}^{+\pi} G(\varphi-\varphi_1)u(\varphi_1)d\varphi_1,$$

$$\frac{k^2 Eh}{R_{22}}u = \Delta\Delta\Psi \cdot \tag{23}$$

We shall determine the sound insulation of the wall of the ellipsoid through the relation of amplitudes of sound pressures P_1 / P_2, in the media inside and outside the shell:

$$R_{ins} = 10\lg\left|\frac{P_1}{P_2}\right|^2 \cdot \tag{24}$$

Sound pressures P_1 and P_2 are given in formulas (19 and 20).

Substituting here those values from equations (19, 20), we shall get the sound insulation of the walls of ellipsoid.

In Fig. 2 the diagrams of estimated values of sound insulation of walls of different metal articles in dB dependence on frequency are provided.

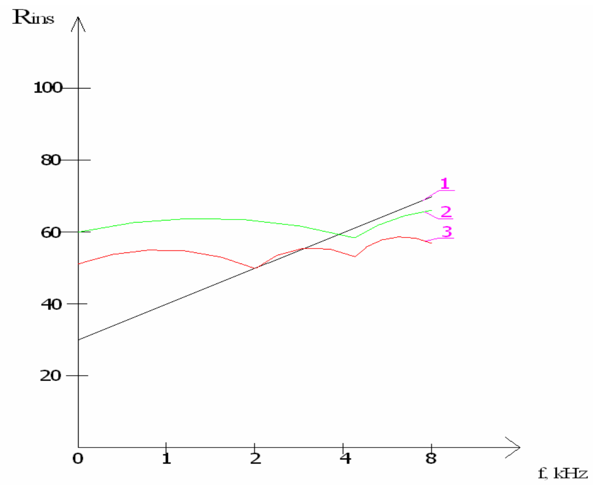

Fig. 2. Estimated values of sound insulation of the wall of the metal article, 3 mm in thickness: 1 – plates; 2 – of cylindrical shell; 3 – of ellipsoidal shell

After comparing the received diagrams, we see that sound insulation of the ellipsoidal shell reaches up to 55 d and 20 dB at low frequencies, but is about 10 dB lower than sound insulation of the cylindrical shell. At high frequencies, sound insulation of the indicated articles becomes the same.

Conclusions

1.The process of calculation of sound insulation of the walls of ellipsoidal shells, if compared to the calculation of sound insulation of plates, is complicated.
2.With the use of a computer program, after performance of concrete calculations of sound insulation of the ellipsoidal shell and comparing them with sound insulation of the shells of other forms, it is possible to state that:

- sound insulation of an ellipsoidal shell at low frequencies is considerably higher than that of the plates of the same thickness and reaches up to 55 dB;
- a characteristic of wall insulation in the ellipsoidal shell is equal to that of cylindrical shell, but the sound insulation of a shell is by10 dB higher due to the wall rigidity, dependent on the form a shell.

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