436. Elastic instabilities in polymer tube of resonant sound absorbers under hydrostatic pressure

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Abstract. The solution of the problem of hydrostatic compression of an elastomeric tube is based on the nonlinear theory of elasticity for incompressible materials employing the method of Ritz variations and the ANSYS finite-element solutions. The main purpose of the investigations is to obtain the loading diagrams in the total range of elastomeric tube thicknesses, without any kinematical and physical restrictions on the behavior of elastomer, e.g., neglecting the Kirchhoff and Timoshenko hypotheses or Hooke's law.

Keywords: circular tube, elastomer, hydrostatic pressure, elastic potential, critical load, postcritical loading diagram

Introduction

A tube made of an elastomer of arbitrary thickness under the action of hydrostatic pressure is considered. Due to the highly elastic material, such a tube can be deformed without failure up to the total loss of its inner cavity. These tubes can be used in a wide range of applications, for example, as elements of resonance sound-proof coverings of transportation facilities. In the present study, these objects are investigated from the viewpoint of static deformation by an external pressure in the region of high strains. The circular symmetry of the tube and load leads to two deformation stages. The precritical stage is characterized by the symmetric form of compression with a simultaneous increase in the load and deformation (the Lame problem). After reaching the critical loading value, the deformation acquires an elliptical form. Depending on the relative thickness of the tube, variants of growing or falling loading diagrams can be realized. The aim of this study is to obtain the loading diagrams in the total range of elastomeric tube thicknesses, without any kinematical and physical restrictions on the behavior of elastomer, e.g., without Kirchhoff and Timoshenko hypotheses or Hooke's law. Thus, the problem is solved in the general statement of the nonlinear theory of elasticity. The resolving equations in a cylindrical coordinate system for the case of plane deformation will be presented below. The solution obtained is compared with the known results from the shell theory and linear elasticity theory, so that to estimate the bounds of their applicability. Along with the semi-analytical method, this problem is examined within the framework of the finite-element method (ANSYS program) for different types of shell elements and in the statement of the problem of plane deformation for a two-dimensional element.

L. Leybenzon (1913) [1] was the first to consider the problem of compression of a long circular tube by external pressure (plane deformation) in the classical statement of the (geometrically and physically) linear theory of elasticity. Later, this problem was analyzed by S. Lubkin (1957) [2] and E. Zeldich (1978) without essential differences in the statements. In each study, the finite dependence for the value of critical pressure was compared with the Bryan formula from the theory of thin shells [3]. The problem of pressure compression of a circular cylindrical tube was also investigated in the nonlinear statement based on the method of superposing small deformations on finite ones [4, 5, 6]. In that case, the existence of an analytical solution for the precritical deformed state (the Lame problem) made it possible to deduce a system of differential equations for increments, which was further analyzed numerically. The similar scheme of solution was employed by C. Sensenig (1964) [7] for a semi-linear John material, where the effect of Poisson's ratio on the magnitude of critical pressure was examined. In addition, it was shown that the transformation from the hollow cylinder to a solid one with particular features (zero radial stresses) retained on the cylinder axis led to the finite value of critical pressure. This result principally differs from the stable behavior of solid (compressible and incompressible) bodies at any magnitude of hydrostatic pressure proved by L. Zubov [8] and A. Guz' [9]. An approach, similar to that described in [7], was developed in later studies [10, 11, 12] for a neo-Hookean material. Among these studies, of particular interest is the investigation of A. Wang and A. Ertepinar (1972) [12], where the problem in hand was considered more thoroughly. In particular, the numerical results were compared with the experiments performed by the authors over the range of cylinder wall thicknesses up to 0.5 of the outside radius, where a good agreement with the Bryan formula was observed. D. Haughton and R. Ogden (1979) [13, 14] discussed the relations between the magnitude of critical load and the type of elastic potential of deformations. It was demonstrated that the neo-Hookean potential does not allow one to obtain an extremum on the loading diagram experimentally observed in [15, 16].

The problem of compression of a long circular tube under external pressure can be solved within the framework of the finite-element method. A number of universal programs, such as ANSYS, NASTRAN, ABAQUS, etc. contain elements for large deformations and rotations suitable for calculating elastomers. Unfortunately, the present authors are unaware of such solutions. Therefore, in what follows, we present our own results of calculating the problem in view using the ANSYS program for different types of shell elements.

Solution Algorithm Based on the Ritz Method

The solution of the elasticity problem of a circular tube under external pressure is based on the principle of stationarity of the total potential energy for small deformations. The functional of the internal potential energy Π^{ε}, associated with the finite deformations of an ideally elastic isotropic incompressible body, is written in the form of the integral of undeformed volume V_0:

$$\Pi^{\varepsilon} = \mu \iiint [W + S(\Theta - 1)]\, dV, \qquad (1)$$

where μ is the shear modulus at small strains, W is the function of a specific internal potential energy (elastic potential), and $\Theta = V / V_0$ is the relative change in the volume of the elastic body with the Lagrangian multiplier S.

The functional of the hydrostatic pressure Π^q is determined from the purely geometrical representation, namely:

$$\Pi^q = q\, \Delta V, \qquad (2)$$

where ΔV is the volume enclosed between the deformed and undeformed positions of the loaded surface.

Now, applying the procedure of Ritz method to the functional of the total potential energy $\Pi = \Pi^{\varepsilon} + \Pi^q$, we can pass from the variation condition of stationarity $\delta\Pi = 0$ to the system of nonlinear algebraic equations:

$$F(\beta, q) = \partial \Pi(\beta, q) / \partial \beta = 0, \qquad (3)$$

where β is the vector of unknown coefficients in the approximation of displacements and the hydrostatic pressure function S.

Let us reduce the solution of Eqs. (3) to a step procedure of continuation with respect to the natural parameter of loading q, with an iteration refinement at each step according to the scheme of the Newton–Kantorovich method:

$$q = q_k, \quad \beta_k^{\,0} = \beta_{k-1},$$
$$J(\beta_k) * \Delta\beta_k^{\,m+1} = -F(\beta_k^{\,m+1}), \qquad (4)$$
$$\beta_k^{\,m+1} = \beta_k^{\,m} + \Delta\beta_k^{\,m+1},$$

where k is the step number; m is the iteration number; * is the symbol of scalar product; $\Delta\beta_k$ is an increment of the vector of sought-for coefficients; $J(\beta) = \{\partial\, \Pi^2(\beta) / \partial\beta_i\, \partial\beta_j\}$ is the Jacobi matrix of second derivatives.

After several solution steps with respect to the load q, which is necessary for determining the initial approximation, the parameter of continuation is changed. In this case, the previous continuation parameter joins the number of varied quantities, namely $\alpha = \{\beta, q\}$. System (3) is supplemented with a new equation suggested in [17], which links the generalized parameter of continuation t with the vector of unknown α:

$$F(\beta, q) = 0, \qquad (5)$$
$$d\alpha_{k-1} * \alpha - (d\alpha_{k-1} * \alpha_{k-1} + t) = 0,$$

where the expression of the total differential $d\alpha_{k-1}$ can be given in finite differences:

$$d\alpha_{k-1} = (\alpha_{k-1} - \alpha_{k-2})/((\alpha_{k-1}-\alpha_{k-2})*(\alpha_{k-1}-\alpha_{k-2}))^{1/2}. \qquad (6)$$

The Jacobian of an extended system of equations (5) does not degenerate in the neighborhood of local extrema of the loading parameter q:

$$J_0 = \begin{vmatrix} J & \partial F/\partial q \\ d\,\beta^T_{k-1} & dq_{k-1} \end{vmatrix} \qquad (7)$$

The step procedure of the solution to Eqs. (5), based on the Newton–Kantorovich iteration process, has the following form:

$$\alpha_k^{\,0} = \alpha_{k-1} + t\, d\alpha_{k-1},$$
$$J_R(\alpha_k^{\,m}) * \Delta\alpha_k^{\,m+1} = -F(\alpha_k^{\,m}), \qquad (8)$$
$$d\,\alpha_{k-1} * \Delta\alpha_k^{\,m+1} = t - d\alpha_{k-1} * (\alpha_k^{\,m} - \alpha_{k-1}),$$
$$\alpha_k^{\,m+1} = \alpha_k^{\,m} + \Delta\alpha_k^{\,m+1}, \text{ where } J_R = \{J, \partial F/\partial q\}.$$

It should be noted that Eq. (5), contrary to Eq. (3), is not symmetrical any more.

Potential of Elastic Finite Deformations

One of the most universal forms of introducing the function of specific internal potential energy (elastic potential) W is the generalized form suggested by Ogden [18]:

$$W = \sum \gamma_k (\lambda_1^{\nu 1} + \lambda_2^{\nu 2} + \lambda_3^{\nu 3} - 3), \quad k = 1,2,3, \qquad (9)$$

where ν_k and γ_k are constants of the Ogden potential, and λ_i are the main elongation ratios bound by the condition of incompressibility $\lambda_1 \lambda_2 \lambda_3 = 1$.

Practically all the known potentials used earlier for solving applied problems of elastomers follow from Eq. (9) as special cases (Table 1). The parameter ξ_0 takes an arbitrary value from the interval [-1, +1].

Table1. Potentials of Elastic Finite Deformations

Potential	γ_k	ν_k
Treloar [19]	0.5	2
Bartenev, Khazanovich [20]	2.0	1
Mooney, Rivlin [21]	$0.25(1+\xi_0)$ $0.25(1-\xi_0)$	2 -2
Chernykh, Shubina [22]	$1+\xi_0$ $1-\xi_0$	1 -1

The selected potential (9) allows us to write all the necessary physical relations, for example, basic values of the Cauchy–Green stress tensor:

$$\sigma_i = \sum \gamma_k \nu_k \lambda_i + S, \qquad k = 1,2,3. \qquad (10)$$

For the following calculations, we introduce the cylindrical coordinates r, φ, and z and the corresponding components of the displacement vector u, v, and w. The main elongation ratios λ_i can be expressed in terms of components of the displacement vector only in the problems on plane and axisymmetric deformations:

$$\lambda_1 = g_1^{1/2}, \; \lambda_2 = g_2^{1/2}, \; \lambda_3 = 1+ \chi\, u / r, \qquad (11)$$

where

$$g_1 = 0.5\,(G + (G^2 - 4\theta\,)^{1/2}),$$

$$g_2 = 0.5\,(G - (G^2 - 4\theta\,)^{1/2}).$$

The parameter χ takes the values 0 and 1 in the case of the plane and axisymmetric deformations, respectively.

Now, we will focus our attention on the problem of plane deformation. For this case, we have:

$$G = (1 + \partial u/\partial r)^2 + (\partial v/\partial r)^2 + (\,(v - \partial u/\partial \varphi)^2 + (\,r + u + \partial v/\partial \varphi)^2)\,/\,r^2\,,$$

$$\theta = (\,(\,1+ \partial u/\partial r\,)\,(\,r + u + \partial v/\partial\varphi\,) + \partial v/\partial r\,(\,v - \partial u/\partial\varphi\,)\,)\,/\,r\,. \qquad (12)$$

Potential of Hydrostatic Loading

The value ΔV in the expression for the functional of hydrostatic pressure (2) can be presented through the respective potential Q as an integral of the smooth surface R (φ, z):

$$\Delta V = \iint Q\, R\, d\varphi\, dz. \qquad (13)$$

The general expression of specific work of hydrostatic pressure (potential) in cylindrical coordinates for the case of large displacements and rotations can be obtained from consideration of the small surface element before and after the loading. For the particular case of a straight round cylinder $R(\varphi,z) = R_0$ and plane deformation, we can write the following expression:

$$Q = u + 0.5(u(u+\partial v/\partial\varphi) + v(v-\partial u/\partial\varphi))/R_0. \qquad (14)$$

Approximation of Displacements and the Function of Hydrostatic Pressure

The region occupied by the cylindrical body is presented in the form:

$$r_1 \le r \le r_2, \qquad \varphi_1 \le \varphi \le \varphi_2, \qquad z_1 \le z \le z_2.$$

Let us introduce the dimensionless coordinates varying in the interval [-1, +1]:

$$\xi = (2 r - r_1 - r_2) / (r_2 - r_1),$$

$$\psi = (2 \varphi - \varphi_1 - \varphi_2) / (\varphi_2 - \varphi_1),$$

$$\eta = (2 z - z_1 - z_2) / (z_2 - z_1). \qquad (15)$$

The approximation of displacements and the hydrostatic pressure function is shown in the following form, with separation of variables:

$$u = \sum\sum\sum A_{ijk} T_i(\xi)\Phi_j(\psi)T_k(\eta), \quad i=1..n_{11},\ j=1..n_{12},\ k=1..n_{13},$$

$$v = \sum\sum\sum B_{ijk} T_i(\xi)\Phi_j(\psi)T_k(\eta), \quad i=1..n_{21},\ j=1..n_{22},\ k=1..n_{23},$$

$$w = \sum\sum\sum C_{ijk} T_i(\xi)\Phi_j(\psi)T_k(\eta), \quad i=1..n_{31}, j=1..n_{32}, k=1..n_{33},$$

$$S=\sum\sum\sum D_{ijk} T_i(\xi)\Phi_j(\psi)T_k(\eta), \quad i=1..n_{41},\ j=1..n_{42},\ k=1..n_{43}, \qquad (16)$$

where A_{ijk} , B_{ijk} , C_{ijk}, and D_{ijk} are unknown constants forming the earlier-introduced vector of unknowns α; T_i and T_k are the Chebyshev polynomials of first kind.

Such approximation allows us to consider two classes of elastic bodies, namely cylindrical panels and cylindrical tubes. In the first case, for the approximation in terms of an angular coordinate, we can also take the Chebyshev polynomials. The fixity conditions can be satisfied, for example, by multiplying into the function $(1 - \psi^2)$. In the second case, owing to the closed contour, we select an expansion in terms of trigonometrical functions $\sin(j\varphi)$ and $\cos(j\ \varphi)$.

Imperfection of the Geometry

Approximation (16) allows us to introduce the imperfection of the circular cross-section, for example, in the form of small ellipticity:

$$r_1 = r_{01} f(\varphi),\ r_2 = r_{02} f(\varphi),\ f(\varphi)=(1- \cos 2\varphi)^{-1} \qquad (17)$$

where e is the parameter of imperfection.

The introduction of imperfection is governed by the fact that the nonlinear solution has to pass through the bifurcation point, which separates the precritical and postcritical branches of the solution.

Results of Calculating an Elastomeric Tube

Let us consider the plane deformation of a circular tube with external $r_1 = 1$ and internal $r_2 = \rho$ radii. The elastomeric material is regarded as an incompressible (Poisson's ratio 0.5) elastic material with a shear modulus μ.

The loading diagram of a circular cylindrical tube under an external hydrostatic load contains three sections (Fig. 1):

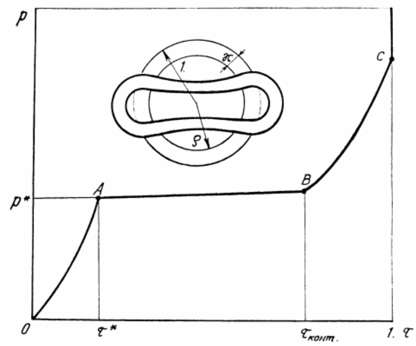

Fig. 1. Loading diagram p - τ

- section OA (precritical branch) of the axisymmetric deformation;
- section AB (postcritical branch), which after e appearance of a contact in the tube cavity;
- section BC, after the formation of internal contacts up to the complete vanishing of cavity volume.

Here, we introduce the dimensionless load parameter $p = q/\mu$ and the dimensionless parameter of relative change in the volume of inner cavity τ, for the critical values of which the designations p^* and τ^* are selected. The precritical deformation is determined by the known solution of Lame problem :

$$q = \int_{\lambda_a}^{\lambda_b} (\partial w / \partial \lambda) / (\lambda^2 - 1) \, d\lambda, \qquad (18)$$

where λ is the relative multiplicity of the radial elongation: $\lambda_a = \lambda (\rho)$ and $\lambda_b = \lambda (1)$. The latter parameters are connected by the condition of incompressibility:

$$\lambda_b^2 = 1 - (1 - \lambda_a^2) \rho^2. \qquad (19)$$

Relation (18) for a neo-Hookean material (Treloar potential) allows for an analytical integration:

$$p = q / \mu = 0.5 \, (1/ \lambda_a^2 - 1/ \lambda_b^2) - \ln (\lambda_a / \lambda_b). \qquad (20)$$

The load diagrams for axisymmetric loading and compression of elastomeric tubes for different types of elastic potential are presented in Fig. 2a,b. The influence of the type of the potential is clearly traced in the case of loading the tube, beginning with increase in volume by 100%; however, in the problem of compression, practically all over the total range of deformations, such influence is insignificant. This allows us to restrict our calculations for the compression problem to the Treloar potential.

Formula (20) makes it possible to investigate the convergence of the result, depending on the number of terms of a series in the radial direction in approximation (16).

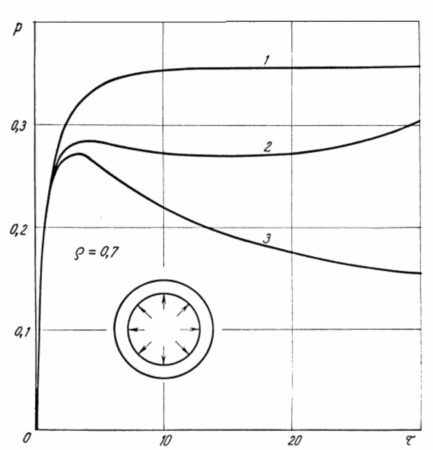

Fig. 2a. Loading diagrams: 1 – Treloar, 2 – Ogden, 3 – Bartenev- Khazanovich

From the calculation results for a rather thick tube, ρ = 0.4 (Table 2), it follows that the four first Chebyshev polynomials in the radial direction give at least a 0.4% accuracy in the deformations.

Using the results of calculating the tubes of different thickness, the relation between the necessary number of series terms in the radial direction and the relative thickness æ = 1 - ρ is constructed (Fig. 3).

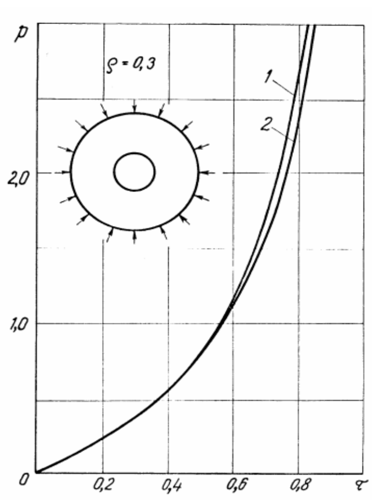

Fig. 2b. Loading diagrams: 1 – Treloar, 2 – Bartenev-Khazanovich

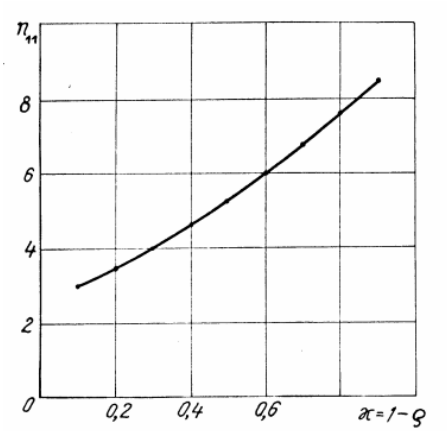

Fig. 3. Necessary number of series terms

For calculating the critical load, in the theory of thin shells (within the framework of Kirchhoff hypothesis), the Bryan formula exists [3], which, for the Poisson's ratio of 0.5, in the designations assumed above, takes the form:

$$p_{BR}^{*} = 8\left((1- \rho) / (1+ \rho)\right)^{3} . \qquad (21)$$

Similar formula in the linear theory of elasticity was obtained by L. Leybenzon [1]. For the case of the Poisson's ratio of 0.5, we can write the following expression:

$p_{LB}^{*} = 0.5\ (1/\rho^2 - 1)((2(1+\rho^4))^{0.5} - (1+\rho^2))$. (22)

These relations are used below for predicting the initial step of loading and for estimating the bounds of applicability of the classical theory of shells and the linear theory of elasticity to the problem considered, by comparison with numerical results obtained without the use of Kirchhoff (or Timoshenko) kinematic hypotheses for a neo-Hookean material.

The dimensionless critical load p* and the corresponding relative change in the volume of inner cavity τ^* (point A on the loading diagram in Fig. 1) were determined according to the following scheme (Fig. 4).

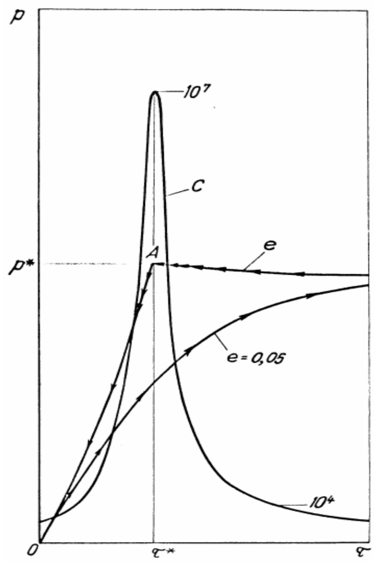

Fig. 4. The resonance in the parameter of singularity condition

The solution was performed directly for a relatively significant imperfection of circular form of the tube: e = 0.05 in relations (19). Then, the imperfection in the postcritical region was eliminated (e = 0), and the reverse step with a sequential decrease in the parameter of continuation was realized. On each step, the singularity condition of the resolving system of equations (8) was controlled. The resonance increase in the parameter of singularity condition, calculated by the procedure described in G. Forsythe, C. Moler [28], indicated that the critical point A on the loading diagram had been passed. The calculation results for the tubes of different thickness according to the scheme suggested are presented in Table 2. The elastic potential was presented by the Treloar potential (neo-Hookean material). This table shows also the values of the approximation formula obtained by the minimization method of least squares:

$p_{apr}^{*} = \exp(\ 5.95 + 6.18\ x^2 - 5.21\ x^4 + 1.06 x^8 - 7.50\ x^{-0.25})$,

where $x = 1.18$ æ. (23)

The small discrepancy between the approximation and reference values of delta = $(1 - p_{apr}^*/p^*)\ 100 < 2\%$ allows us to recommend formula (23) for calculating the critical pressure of compression of elastomeric tubes of arbitrary thickness.

The results of numerical calculation, presented in Table 2, are compared with experimental data given in Wang A.S.D., Ertepinar A (Fig. 5).

Table 2. Results for the tubes of different thickness.

P	p*	p_{apr}^{*}	delta (%)
0.90	0.00117	0.00117	0.0
0.80	0.0112	0.0113	0.9
0.70	0.0466	0.0460	-1.3
0.60	0.139	0.138	-0.7
0.50	0.342	0.343	0.3
0.40	0.688	0.690	0.3
0.30	1.11	1.11	2.0
0.20	1.49	1.47	-1.3

Here, for comparison, the own calculated values of the authors of the above-mentioned study are shown, which were obtained by the method of superposition of small disturbances on the finite deformations of a neo-Hookean tube. We should point to the good coincidence in the considered range of relative thicknesses 0.2 ≤ æ ≤ 0.5. Unfortunately, no data in the range æ > 0.5 were found in the literature.

The results obtained are also compared with formula (21) of the theory of thin shells and formula (22) of the linear theory of elasticity (Fig. 6).

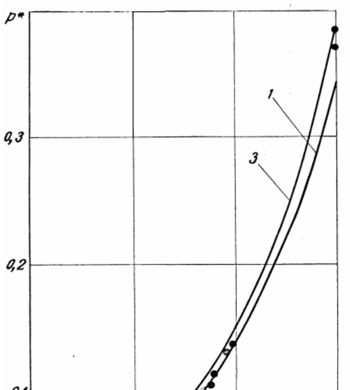

Fig. 5. Loading diagrams: 1 – present result, 2 – result [12], 3 – experiment [12]

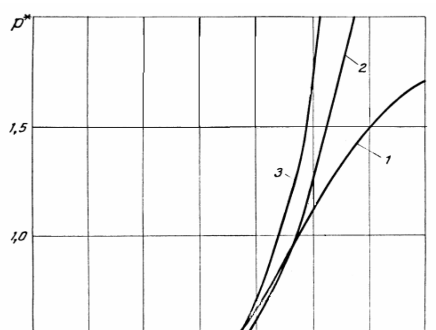

Fig. 6. Loading diagrams: 1 – present result, 2 – formula (21), 3 – formula (22)

The best agreement was achieved in the case of Bryan formula (21). In the range of relative thicknesses æ ≤ 0.65, the discrepancy did not exceed 10%. This is quite an unexpected result, since, within the range indicated, the hypotheses of the theory of thin shells formally are not applicable. The amplitude of coincidence with Leybenzon formula (22) is slightly smaller: æ ≤ 0.6.

The calculation in the postcritical region of the solution raises a question on the number of terms in the approximation of required displacements and function of hydrostatic pressure in the angular direction necessary for a satisfactory convergence of the solution. The calculations carried out for different values of the parameter $n_{12} = n_{22} = n_{42}$ from Eqs. (16) indicate that the fraction of trigonometric terms of the series above the fifth harmonics does not exceed 2% (Fig. 7).

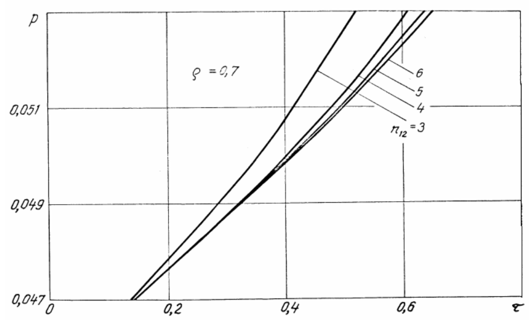

Fig. 7. Loading diagrams for different values n_{12}=n_{22}= n_{42}

For an expansion in Chebyshev polynomials in the radial direction, we can assume the dependence presented earlier in Fig. 3, which determines the parameter n11. For the remaining functions, we assume that $n_{21} = n_{41} +1 = n_{11}$.

Fig. 8 illustrates the numerical calculations of postcritical branches of the loading diagram for elastomeric tubes of different thickness.

Two groups of tubes can be distinguished here. For the first group (ρ > 0.6), the load in the postcritical region of the solution grows, which agrees with the results of the theory of thin shells. The second group (ρ ≤ 0.6) is characterized by a drop in the load in the postcritical region. The difference in these two groups is also manifested in the form of deformed sections (Figs. 9 and 10).

In the first group, by the moment of origination of the inner contact zone, a noticeable (about 30% of the initial) volume is still retained. In the second group, upon reaching the critical pressure, a sudden vanishing of the inner volume is observed.

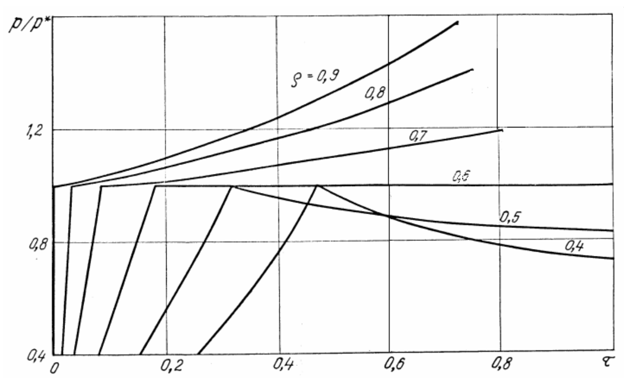

Fig. 8. Loading diagrams: different values ρ

This result, in particular, can explain such a known phenomenon as the heart attack at a "collapse" of a blood vessel with sclerotic depositions as a result of a sharp gradient of internal pressure. As a first approximation, the relative thickness of blood vessels exceeding æ = 0.4 can be considered dangerous.

The imperfection of the circular geometry of elastomeric tubes is associated both with the precision of manufacture and with the rheological properties of the material revealing during the long-term storage.

Figs. 9-10 indicate that even a 5%-imperfection of the circular form leads to a noticeable drop in the magnitude of critical pressure.

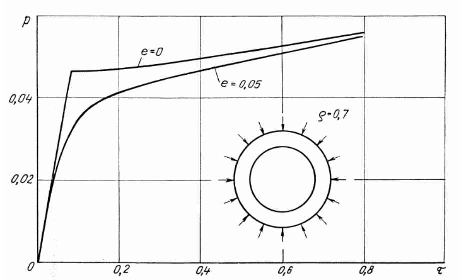

Fig. 9. Loading diagrams: different values e

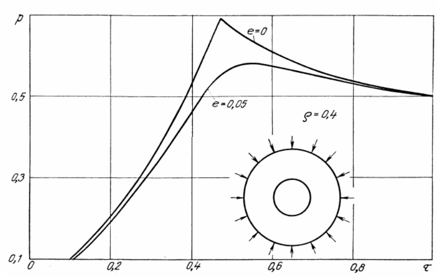

Fig. 10. Loading diagrams: different values of e

To a lesser degree, such a defect affects the postcritical behavior, which agrees with the results of the theory of thin shells.

Finite-Element Method

It is of interest to compare the results obtained above within the framework of the Ritz method with the solution from the finite-element simulations. For this purpose, we will use a widely-used ANSYS program. First, we will consider shell-type elements SHELL63, SHELL93, SHELL181, and SHELL281. To solve the nonlinear problem by the finite-element method, a macros-program was elaborated, which specifies the geometry, physical law, and properties of the material, boundary conditions, loading, division into finite elements, and the step scheme of the solution. The scheme is realized for assigning a geometrical imperfection in the first buckling mode at the first linear stage of the solution, which is then added, with a certain term, to the initial geometry. At the second stage, the Newton–Raphson step procedure with the choice of a generalized step of continuation of the solution is implemented.

After a number of numerical experiments, minimum values of the geometrical imperfection, when the transition to the adjacent form of equilibrium occurs, are found. The critical values of the load, determined for different relative thicknesses, are presented in Table 3. The results obtained earlier (Table 2) are also given for comparison.

Upon comparing the results obtained from the finite-element method for different types of finite-elements with the solution from the Ritz method (Fig. 11), we should point to an unexpectedly good agreement with the simplest SHELL63 element.

Table 2. Critical values of the load p*.

æ	3-D	SHELL63	SHELL93	SHELL181	SHELL281
0.1	0.00117	0.00116	0.001145	0.001145	0.001145
0.2	0.0112	0.011	0.0107	0.0107	0.0107
0.3	0.0466	0.045	0.043	0.042	0.042
0.4	0.139	0.13	0.12	0.11	0.107

We should note that the Bryan formula also gave the same good agreement. Obviously, this can be connected with the Kirchhoff–Love hypothesis, based on which the SHELL63 element was obtained, and with the Bryan solution. It can be assumed that, for rather thick elastomeric tubes, the hypothesis of straight normals is fulfilled quite well. Apparently, this is associated with the incompressibility of the elastomeric material.

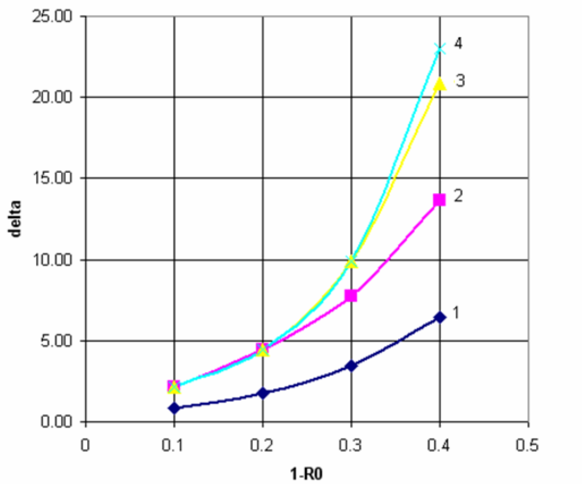

Fig. 11. Comparing the results obtained from the finite-element method: 1 –SHELL63, 2 –SHELL93, 3 – SHELL181, 4 – SHELL281

Finally, we present the results of calculating the compression of an elastomeric tube by the finite-element method as a problem of plane deformation of elasticity theory (a PLANE183 8-nodal element) (Figs. 14-16). We should note that the earlier-obtained effect of descending postcritical sections of loading diagrams for tubes of thickness æ > 0.4 proves to be true. In comparing the critical loads (Table 4), the coincidence between the results obtained by the classical Ritz method and the finite-element method lies within the limits of a 5% accuracy.

Conclusions

The nonlinear problem of hydrostatic compression of an elastomeric tube is considered within the framework of the nonlinear elasticity of an incompressible material. Based on the Ritz method with an expansion of displacements into Chebyshev polynomial series in thickness and trigonometrical functions in angle, the critical loads and the postcritical branches of loading diagrams of elastomeric tubes of arbitrary thickness are determined. These results are compared with the known results obtained from the shell and elasticity theories, with experimental data available in the literature, and with calculations within the framework of superposition of small deformations on the finite ones. For completeness of comparison, the present authors deliver their own calculations carried out based on the finite-element method with several types of shell elements, in the statement of the plane problem of elasticity theory. As a result, an analytical formula for calculating the critical load is constructed, and the effect of descending postcritical sections of loading diagrams for tubes of thickness æ > 0.4 is determined.

References

[1] **Лейбензон Л. С.** Теория упругости. – Собр.трудов, М.:Изд.-во АН СССР, 1951, т.1, с. 68 – 81.

[2] **Lubkin S**. Determination of buckling criteria by minimization of total energy. – Inst. Math. Sci. N.Y. Univ., 1957, Rep. IMM – NYU241.

[3] **Тимошенко С. П.** Устойчивость упругих систем. – Л.- М. : Гостехиздат, 1946. -532 с.

[4] **Гузь А.Н**. Устойчивость упругих тел при конечных деформациях. – Киев: Наукова думка, 1973, - 270 с.

[5] **Зубов Л. М.** Вариационные принципы нелинейной теории упругости. Случай наложения малой деформации на конечную. – ПММ, 1971, т. 35, №5, с.849-852.

[6] **Green A. E., Rivlin R. S., Shield R. T.** General theory of small elastic deformation superposed of finite elastic deformations. – Proc. Roy. Soc., A, v.211, 1952, pp.128 – 154.

[7] **Sensenig C. B.** Instability of thick elastic solids. – Communs. Pure and Appl. Math., v. 17, 1964, pp. 451- 491.

[8] **Зубов. Л. М.** Об условиях единственности в малом состояния гидростатического сжатия упругого тела. –ПММ, 1980, т. 44, вып. 3, с. 497 – 506.

[9] **Гузь А. Н.** Устойчивость упругих тел при всестороннем сжатии.–Киев:Наукова думка,1979.- 144 с.

[10] **Nowinski J. L., Shahinpoor M.** Stability of an elastic circular tube of arbitrary wall thickness subjected to an external pressure. – Int. J. Non-Linear Mech., v.4, №2, 1969, pp. 143 – 158.

[11] **Shahinpoor M., Novinski J.L.** Exact solutions to the problem of forced large amplitude radial oscillations of a thin hyperelastic tube. Int. J. Non-Linear Mech., v.6, 1971, pp.193-207.

[12] **Wang A. S. D., Ertepinar A.** Stability and vibrations of elasticthick-walled cylindrical and spherical shells subjected to pressure. – Int. J. Non-Linear Mech., v.7, №5, 1972, pp.539-555.

[13] **Haughton D. M., Ogden R. W.** Bifurcation of inflated circular cylinders of elastic materials under axial loading. – Membrane theory for thin walled tubes. - J. Mech. Phys. Solids, v.27, №, 1979, pp. 179 - 212.

[14] **Haughton D. M., Ogden R. W.** Bifurcation of inflated circular cylinders of elastic materials under axial loading. – I I. Exact theory for thick-walled tubes. - J. Mech. Phys. Solids, v.27, №5-6, 1979, pp. 179 - 212.

[15] **Charrier J. M., Li Y. K.** Large elastic deformations theories unstable of rubber structures: thick tubes. – Trans. Soc. Rheology, v.21, №3, 1977, pp. 301-325.

[16] **Skala D. P.** Modified equations of rubber elasticity applied to the inflation mehanics of a thick-walled rubber cylinder. Rubber Chemistry and Technology, v.43, 1973, pp. 745 – 757.

[17] **Григолюк Э. Н., Шалашилин В. Н.** О некоторых формах продолжения по параметру в нелинейных задачах теории упругости. – ПМТФ, 1980, №5, с. 158-161.

[18] **Ogden R. W.** Large deformation isotropic elasticity – on the correlation of theory and experiment for incompressible rubberlike solids. – Proc. Roy. Soc., London, 1972, A 326, pp. 565-584.

[19] **Трелоар Л**. Физика упругости каучука. М., ИЛ, 1953. – 240 с.

[20] **Бартенев Г. М., Хазанович Т. Н.** О законе высокоэластичных деформаций сеточных полимеров. – Высокомолекулярные соединения, 1960, т.2, №1, с.20-28.

[21] **Ривлин Р. С.** Большие упругие деформации – В кн.: Реология. Теория и приложения / Под ред. Эйриха Ф.М, ИЛЛ, 1962, с.421-459.

[22] **Черных К. Ф., Шубина И. М.** Законы упругости для несжимаемых материалов. – Механика эластомеров, 1977, т.1, с.54-64.

[23] **Форсайт Дж., Малькольм М., Моулер К.** Машинные методы математических вычислений. – М.: Мир, 1980. – 279 с.

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