442. Creating of the mathematical model of a resilient support spring type element and its transfer function from the input and output impulse responses

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Abstract. Developing of the transfer function of the spring type element of the resilient bearing support by creating adequate mathematical models of processes for particular frequency intervals from signals obtained as impulse responses of the spring at its input and output measured in the stand under impact at the input is analysed in the article. Impulse responses are investigated by the Fourier method by separating components of particular frequencies. Mathematical models of the processes at the spring input and output are developed as the sum of models-formants corresponding to particular frequency ranges. Formants themselves are modeled by the sum of damped sine waves. Parameters of the model are estimated by the iterative Levenberg method. The mathematical model developed further is used for diagnostics of support bearings that are insulated from the external body by these resilient elements.

Keywords: mathematical model, failure diagnostics, resilient support, excitation, transfer function, frequency response

Introduction

Non-destructive diagnostic researches of the system for the evaluation of its operating state suitability or detection of signals of initial failures allowing to use preventive means to avoid them are widely spread in practice and are very effective. They are widely used for the evaluation of the suitability of high-speed rotor system bearings without stopping their operation and disassembly. Although there are systems in which such research is complicated due to construction particularities. One type of such systems are centrifugal systems whose bearing supports in most cases are connected with the external frame through resilient elements restricting the transmission to the frame of all row additives of the spectre of oscillation caused by bearing failures [1, 2]. For a proper evaluation of transfer properties of such a system it would be expedient to research the resilient element transfer function by creating its adequate mathematical model in frequency ranges which are expected to cover possible frequencies caused by bearing failures. For this aim, it would be expedient to determine the transfer function of the resilient element. This would allow to make a decision about the possibility of finding frequencies caused by the bearing failure by measuring oscillations of the external body. The spring element is a resilient element used in a centrifugal milk separating apparatus the diagnostic research of which is an important mean for a full and safe usage of the allowable working resource of it. It was chosen as the research object.

Experimental equipment

The experiment was carried out on the research stand shown in Fig. 1. The spring element was trapped between two resilient systems, insulated one from the other.

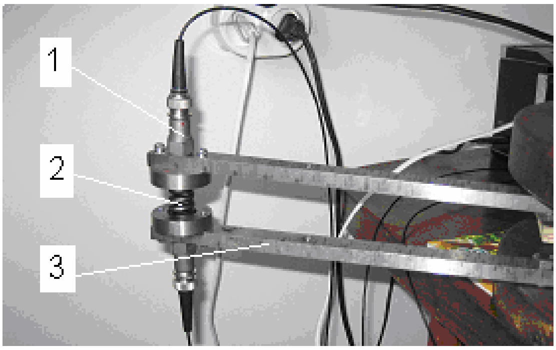


Fig. 1. Stand for researching of oscillations transfer properties of the spring: 1 – the piezoelectric accelerometer; 2 – the spring, 3 – the elastic band

The stand was composed by two massive plates with comparatively not rigid bands of rectangular cross section fixed to them. They imitate two separated systems of the centrifugal milk separating apparatus – the shaft and the body. These systems were insulated one from the other by resilient rubber supports.

Measuring sensors were piezoelectric accelerometers, fixed to both ends of the spring, and they registered accelerations of vibrations of both resilient systems at the points of the spring ends. Mostly, bearing failures cause a pulse type excitation accompanied by wide frequency spectre oscillations the appearance of which is the base for the diagnosis of bearing failures. So, a series of impacts was used as the excitation given to one of the resilient systems and transient processes (time signals) of both ends of the spring were measured. The response of one system to the excitation and its transfer to the other system was researched.

Research procedure

In order to research oscillations transmitting properties of the spring, it is necessary to create its adequate mathematical model allowing to calculate its transfer function, by the help of which it would be possible to estimate the influence of the resilient element to proper bearing diagnostics.

In order to create the mathematical model, a series of blows was submitted to the upper plate. The obtained input and output signals are shown in Fig. 2.

Realizations in the form of series of impacts allow to choose a better quality signal to be researched. The level of the output signal of the spring was approximately 20 times weaker than the input signal.

The standard voice signal digitising the procedure was used to digitise these analogical signals obtained from accelerometers. Then the signals were filtered from additives of higher than 5000 Hz and were integrated twice. The realizations obtained were processed by the FFT procedure, and spectres of frequencies of the input and output signals were analysed. The spectrum of the measured signals is very wide, and it is difficult or even impossible to model such a process in a rational form for all frequencies. The spectres of input and output signals of the spring are shown in Fig. 3.

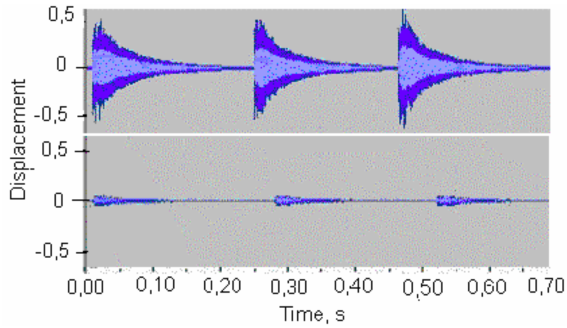
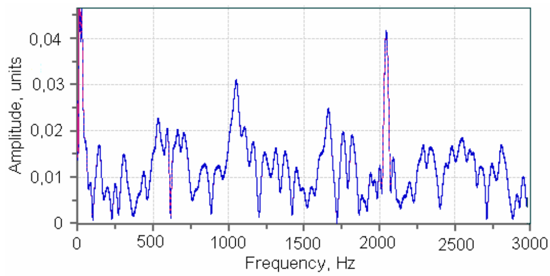
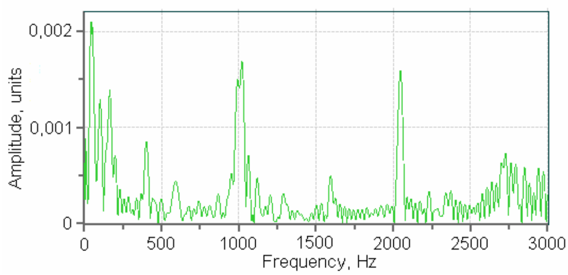


Fig. 2. Input (upper) and output (bellow) signals measured at ends of the spring



a)



b)

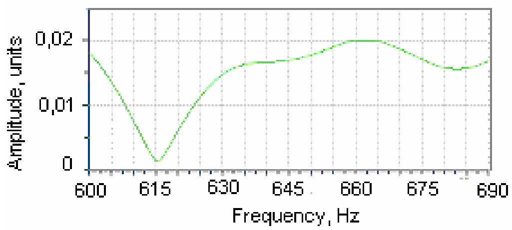
Fig. 3. Frequency spectres of the input (a) and output (b) signals

However, bearing failures cause oscillations of particular frequencies which are in particular ranges determined by the rotational frequency of the bearing, the number of rolling elements in the bearing, bearing work conditions [1, 2]. The main characteristic frequencies related with the bearing failures of the milk separating apparatus rotor system consisting of an investigated spring element are the following [1]: 46,4 Hz (the rotation frequency of the bearing retainer), 112,5 Hz (the rotor rotation frequency), 225,0 Hz (the doubled rotor rotation frequency), 602,5 Hz (the frequency of the run of balls by the outer race), 615,8 Hz (the rotation frequency of the ball), 859,7 Hz (the frequency of the run of balls by the inner race), 1205,5 Hz (the doubled frequency of the run of balls by the outer race), 1231,5 (the doubled rotation frequency of the ball).

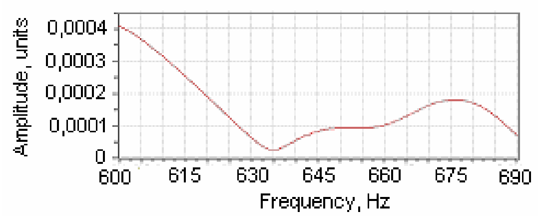
We found that for modelling of input and output signals, it is better to take a short frequency interval. The chosen interval should cover particular frequencies, which might occur due to the bearing particular failure. It is necessary to observe that sometimes the spectre of the particular frequency interval created in the stand upper system does not have a sufficient level of excitation amplitudes. This is the input excitation for the spring. In this case the dynamic characteristics of the stand upper system (stiffness or mass) should be changed to obtain some level of the input signal.

So, we attempted to model input and output processes in the following intervals of frequencies: 40-55 Hz, 110-190 Hz, 200-250 Hz, 480-600 Hz, 600-690 Hz, 795-885 Hz, 1215-1340 Hz, 2014-2080 Hz.

Frequency spectres for one of the investigated short frequency intervals of the input and output signals are shown in Fig. 4.



a)



b)

Fig. 4. Spectres of the input (a) and output (b) signals of the frequency interval 600-690 Hz

The modelling of these signals was made following [3-5].

Modelling of the spring input and output signals

We performed the modelling by creating a mathematical model in the form of the sum of damped interactive sin waves. The spring impulse response of both, the input and output, was modelled as a sum of formants – quasi-polynomials in the form:

$$h(t) = \sum_{i=1}^{m} f_i(t), \qquad (1)$$

where polynomials are composed from polynomials components having multipliers calculated from damped sinusoids of a particular frequency [5]. We restricted the polynomial till the quadratic order:

$$f_i(t) = e^{\lambda_i t}[a_{i1}\sin(2\pi\omega_i t + \varphi_{i1}) + + a_{i2}t \sin(2\pi\omega_i t + \varphi_{i2}) + a_{i3}t^2 \sin(2\pi\omega_i t + \varphi_{i3})], \tag{2}$$

where ω_i is the angular frequency of the searching formant, λ_i is its damping factor, $a_{i1} \dots a_{i3}$ are amplitudes of the searching formant components and $\varphi_{i1} \dots \varphi_{i3}$ are component phases.

The estimation of frequencies and damping factors was performed by using the Prony method, and for improving the initial estimated values the iterative non linear Levenberg optimisation was used following [3]. The algorithm of calculation of polynomial members parameters is described in detail in [3].

One of the particularities during modelling processes for the short frequency interval is the acceptation of the number of formants composing the model. We find that for a short frequency interval in our case is expedient to take one or two formants (m=1 or m=2) corresponding to frequencies close to those seen on the amplitude-frequency characteristic of the given frequency interval. The RSME evaluating the correctness of the model in such a case did not exceed 5% for all frequency ranges.

Calculated parameters of members of polynomials modelling input and output signals in the frequency interval 600-690 Hz are presented in Table 1.

TABLE 1: Parameters of model components for the frequency interval 600-690 Hz

Signal	Frequency, ω, Hz	Damp., λ, 1/s	Ampl., a, units	Phase, φ, rad
Input (before spring)	611	-67	654,6544	3,0749
			1,9419	-1,166
			0,0022	0,8889
	676	-76	974,2633	0,8257
			1,671	-0,814
			0,0013	-2,673
Output (after spring)	611	-69	5,9371	2,5187
			0,0032	2,9503
			0,0	2,0255
	673	-84	2,1644	0,5629
			0,0115	-2,214
			0,0	2,4062

Components representing three additives of the first polynomial, which models the input process, are shown in Fig. 5.

The input process for the frequency interval 600-690 Hz was modelled by using two polynomials – formants. The modelled and real oscillation processes at the spring input are shown in Fig. 6.

We found that the modelling of the oscillation process is more effective by using a small time interval signal taken at the beginning of the oscillation process, because all frequencies are expressed more clearly and can be singled out due to more powerful oscillations. During a long time interval some frequencies damp, the non-linearity of the system also results in changes of the process character, and the modelling of the long time interval process leads to a complicated model. Thus, for the modelling of the process of all researched frequency intervals we used the time interval of duration 0,0081-0,0435 s.

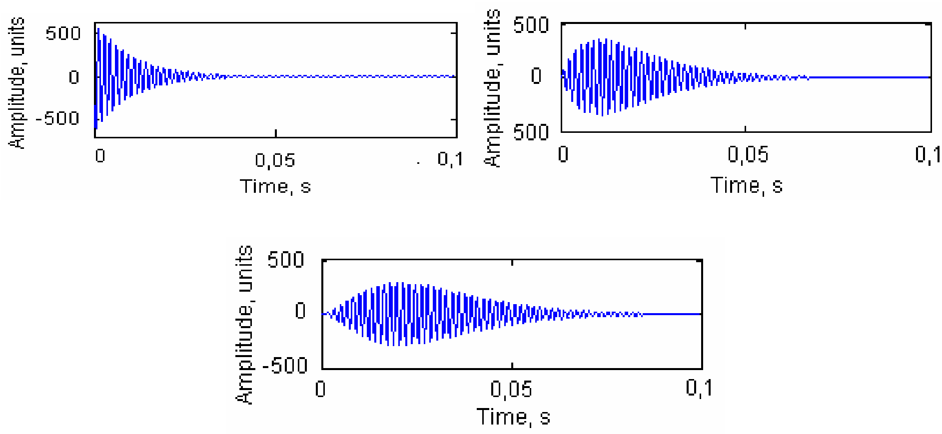


Fig. 5. Components of the first polynomial of the spring input process model for the frequency interval 600-690 Hz

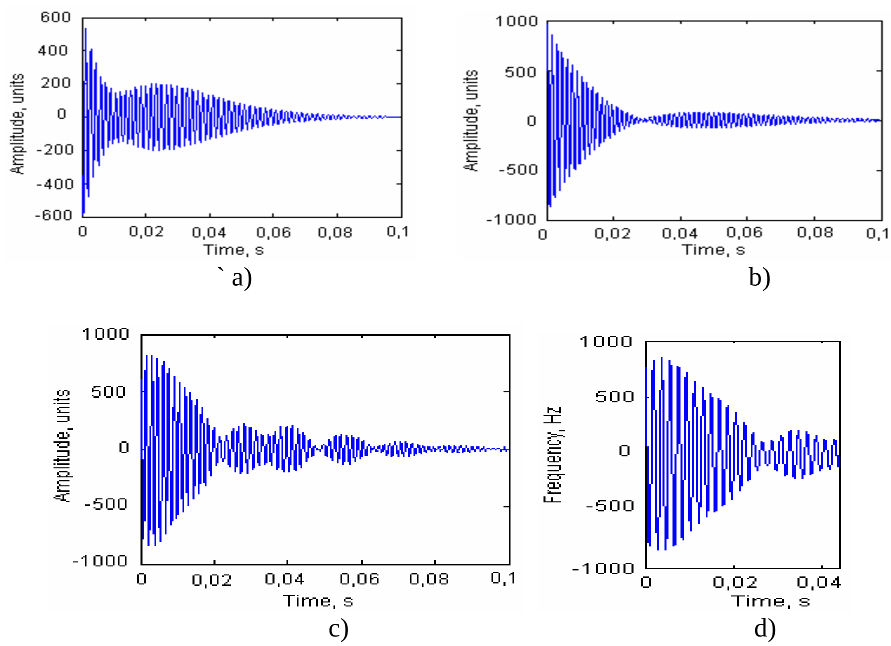


Fig. 6. The first (a), the second (b) formants, the model – the sum of these formants (c) of the oscillation process, and the actual oscillation process (d) for the frequency interval 600-690 Hz

Determination of the spring transfer function

We calculated the transfer function in the following way. The Fourier transform of each formant can be expressed as [5]:

$$h_i(f) = -\frac{1}{j}\frac{a_1 e^{j\varphi_{i1}}}{(\lambda_i - 2\pi j(f-\omega_i))} - \frac{a_1 e^{-j\varphi_{i1}}}{(\lambda_i - 2\pi j(f+\omega_i))} - \frac{1}{j}\frac{a_2 e^{j\varphi_{i2}}}{(\lambda_i - 2\pi j(f-\omega_i))^2} - \frac{a_2 e^{-j\varphi_{i2}}}{(\lambda_i - 2\pi j(f+\omega_i))^2} - \frac{1}{j}\frac{a_3 e^{j\varphi_{i3}}}{(\lambda_i - 2\pi j(f-\omega_i))^3} - \frac{a_3 e^{-j\varphi_{i3}}}{(\lambda_i - 2\pi j(f+\omega_i))^3}, \quad (3)$$

where i is the formant number.

The Fourier transform of the modelled signal will be the sum of Fourier transforms of all formants, composing the given signal, i.e., $\sum_{i=1}^{m} h_i(f)$.

The spring frequency response, *H(jw)*, is calculated as the ratio of the Fourier transforms of the output signal and the input signal

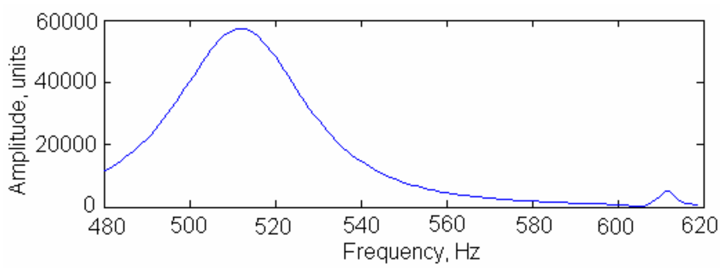
$$H(jw) = \frac{\sum_{i=1}^{m_{output}} h_{i\ output}(jw)}{\sum_{i=1}^{m_{input}} h_{i\ input}(jw)}, \quad (4)$$

where $w = 2\pi f$ and it is the angular frequency .

The range of frequencies, for which the Fourier function is determined, should be narrower to some extend than the range of modelling in order to avoid distortions related with the boundary divergences.

After dividing the Fourier function of the output signal by the Fourier function of the input signal, we will get the transfer function for the frequency range, for which these signals were determined. Amplitude-frequency responses of the spring input and output for the frequency intervals 480-620 Hz and 600-690 Hz are shown in Fig. 7.

The transfer function of the spring in the frequency interval 560-680 Hz, which is composed of the transfer functions calculated in two frequency intervals: 480-620 Hz and600-690 Hz – is shown in Fig. 8.



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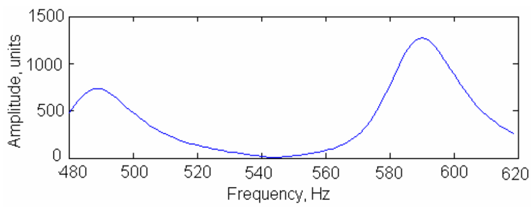

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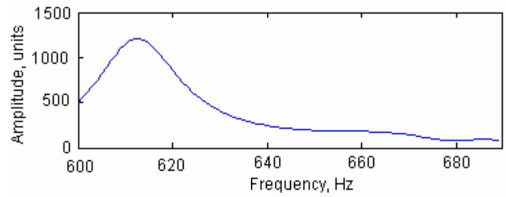
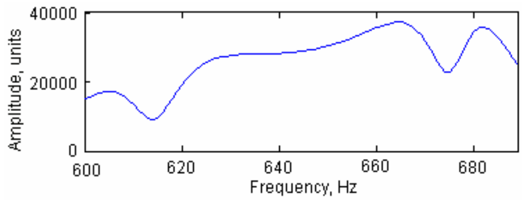


c)



d)

Fig. 7. Amplitude-frequency responses of the spring input (a, c) and output (b, d) for frequency intervals 480-620 Hz and 600-690 Hz

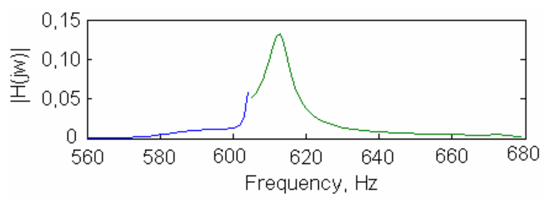


Fig. 8. The transfer function for frequency interval 560-680 Hz

It is seen (Fig. 8), that transfer functions of two frequency intervals calculated separately give a rather good correspondence in their touching point – at 605 Hz.

The transfer function of the spring in all frequency intervals related with the appearance of possible bearing failures frequencies is shown in Fig. 9.

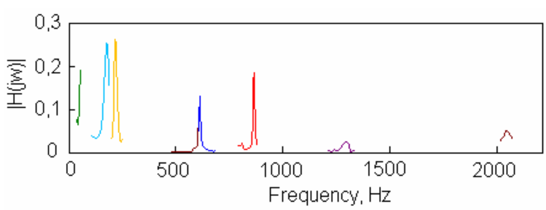


Fig. 9. The transfer function for all interesting frequency interval 0-2300 Hz

Conclusions

The spring transmits the input signal. The transfer function of the spring can be used to diagnose the bearing failure by analysing the change of the spectre of frame vibrations during the time and making decisions about reasons of frequencies appeared. However, the output signal of the spring in most investigated frequency intervals is approximately 20 times weaker than the input signal.

The transfer function is more expedient to create for small frequency ranges, in which frequencies caused by particular bearing failures can appear. The model constructed for such a case is more adequate and the error of the bias from the measured signal is less.

In order to determine the bearing failure frequency at the spring output (milk separator outer body) – to find the bearing failure frequency in the spectre of the body oscillations, it is necessary to fix and carefully analyse the oscillation level of the outer body with the right bearing and compare it with the current body oscillation level.

References

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