443. Analysis of calibration data of position of circular scale strokes

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Abstract. There is a wide variety of application of circular scales and rotary transducers used in micro-positioning devices for mechatronic measuring and control instruments. Their accuracy has direct influence on the accuracy of positioning or precision displacements of piezoelectric devices. There are a vast number of methods for accuracy calibration of the circular scales and rotary encoders. One of the modified methods (*constant angle setting in full circle with multiple turn*) has been proposed by the authors in this paper. Practical implementation of the method is described herein together with the analysis of the scale strokes biases (systematic errors) and random errors calculation. Values of scale strokes biases obtained by calibration of scale using the proposed method are compared with the reference values obtained by means of the multiangular prism (polygon) and photoelectric autocollimator.

Keywords: circular scale, measurement, accuracy, uncertainty.

1. Introduction

A test bench for testing and calibration of angle measuring geodetic instruments has been designed and implemented at the Institute of Geodesy VGTU [2]. The analysis of rotary disk positioning performed by angle encoder revealed unsatisfactory accuracy of the measurements [1]. To solve the problem circular scale/microscope means of measurement were used allowing to reach high accuracy (approx. 0.2″) at a comparatively small pitch of the scale strokes (down to 10′). To eliminate circular scale biases that are obviously present a special scale calibration method must be used.

Scale calibration using the same scale as reference is rather widely investigated since there is no need of other reference measure to use. This method is widely used and often segmented to sub-branches forming other calibration methods. These methods of circular scales calibration were created and developed by such famous scientists as H. Bruns, G. Schreiber, A. Perard, H. Wielde, H. J. Heuvelink, S. Yeliseyev, etc. [3, 7]. Most of the circular scale calibration methods described in the literature are fairly complicated, require several microscopes to be used and often do not provide unambiguous value of the systematic error of particular scale stroke. Therefore, a test bench with a special modified *constant angle setting in full circle with multiple turn* method has been suggested for calibration of the circular scale.

The suggested scale calibration method has the following *advantages*:

- bias of each scale stroke can be evaluated;
- only two microscopes can be used for calibration and only one for the further work;
- no need for precise mechanical positioning of the scale; the microscopes should be only capable to measure the particular stroke;
- calibration can be performed at any pitch of the circular scale;
- simplicity of the method and the error calculation process as the biases can be calculated by means of general mathematical computer programs or even manually;

Despite the advantages some *disadvantages* of the method are also present:

- random errors of calibration, accumulating during the calculation due to the sequential strokes biases determination. Therefore the biases of the last strokes may become fairly high. Nonetheless, in case of repeated (several times) calibration process the random errors should be eliminated.

The results of circular scale calibration on the test rig against the reference ones (provided by the multiangular prism/autocollimator) are described in this paper.

2. Calibration of the geodetic instruments

The main task of investigation was testing the possibilities of implementation of the proposed scale calibration method. The measurements on the test bench were performed using two photoelectric microscopes M1 and M2 (both fitted with CCD cameras) mounted at an approximate angle of 95° to each other [5]. To perform additional control of the results the multiangular prism (3) with 12 reflection surfaces and the autocollimator (1), also fitted with the CCD camera, were used. The positioning of the rotary table was performed by the worm gear drive with step motor (6) and controlled by the photoelectric angle encoder (4). The disc (2) with the circular scale was rotated at a pitch of 5° until full circle, and the readings from the microscopes ($\Delta M_{1.i}$ – for the first microscope and $\Delta M_{2.i}$ – for second microscope) at each position were performed. All measuring process was controlled by PC (7).

The measurements using the multiangular prism/autocollimator were performed at a pitch of 30° according to the number of flat surfaces of the multiangular prism. The measurement results are presented in Fig. 2.

After a full cycle of the measurements is performed, the real angle between the microscopes ($\varphi_r + \delta\varphi_p$) for each test can be calculated. The approximate angle between the microscopes (φ_r) is 95°, the deviations of angle will be determined by:

$$\delta\varphi_p = \frac{\sum_{i=1}^{n}\left(\Delta M_{2(i+1)} - \Delta M_{1i}\right)}{n}. \qquad (1)$$

where: $\delta\varphi_p$ – deviations of microscopes placement for each measurement series are listed in Table 1.

Table 1. Determined deviation ($\delta\varphi_p$) of microscopes placement angle

Test 1 (″)	Test 2 (″)	Test 3 (″)	Test 4 (″)	Test 5 (″)	Test 6 (″)	Test 7 (″)	Test 8 (″)
43.18	43.18	43.22	43.19	43.18	43.18	43.18	43.20

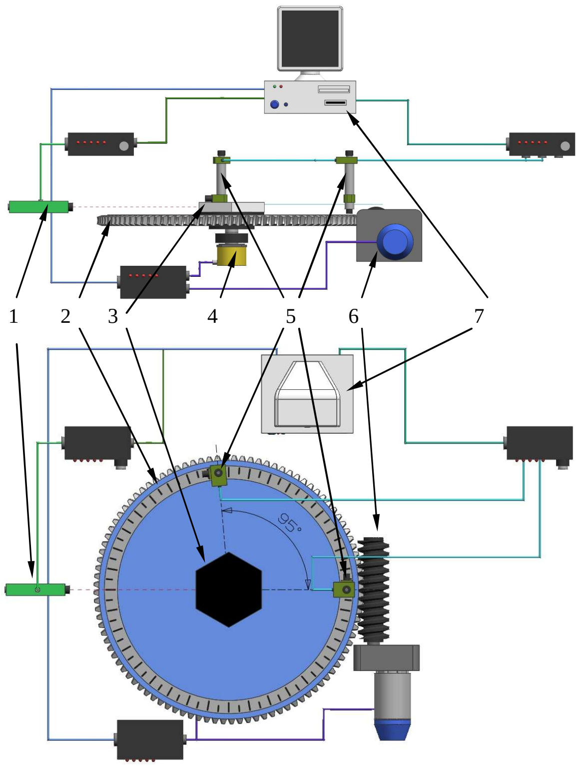

Fig. 1. Principal layout of the mechatronic measuring device:
1 – autocollimator, 2 – rotary table with the circular scale, 3 – multiangular prism, 4 – photoelectric rotary encoder, 5 – microscopes, 6 – worm gear drive with step-motor, 7 – control PC

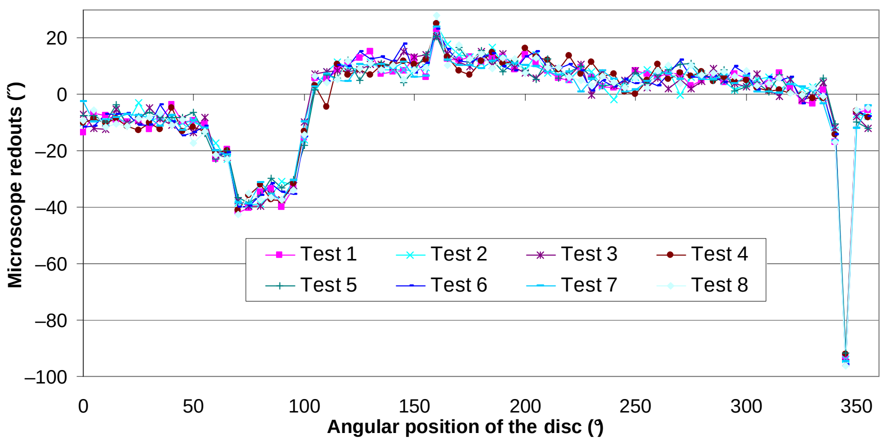

Fig. 2. Readouts of microscope 1

Each scale stroke bias at each test angle ($\delta\varphi_i$) regarding to the first (0°) stroke can be calculated using equation:

$$\delta\varphi_i = \delta\varphi_{i-1} + \left(\Delta M_{2i} - \Delta M_{1(i-1)}\right) - \delta\varphi_p . \qquad (2)$$

The results of the calculations of the scale strokes systematic errors are provided in Table 1.

The values of each scale stroke bias, using the calibration data, (with numerical compensation of the highest biases at 60°–100°, 160° and 340°–350°) are presented graphically in Fig. 3. and 4.

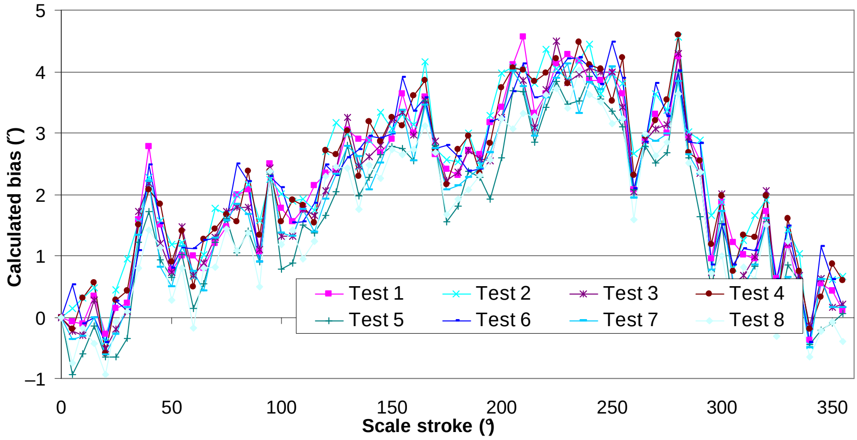

Fig. 3. Calculated bias of each scale stroke (with compensations of greatest biases)

According to Fig. 2. several noticeable scale strokes biases are present, the largest of which being at the strokes 350° and 60°–100°. Knowing the numerical values of those strokes biases (Fig. 4.), the precise rotary table angular position can be later determined by means of a single microscope.

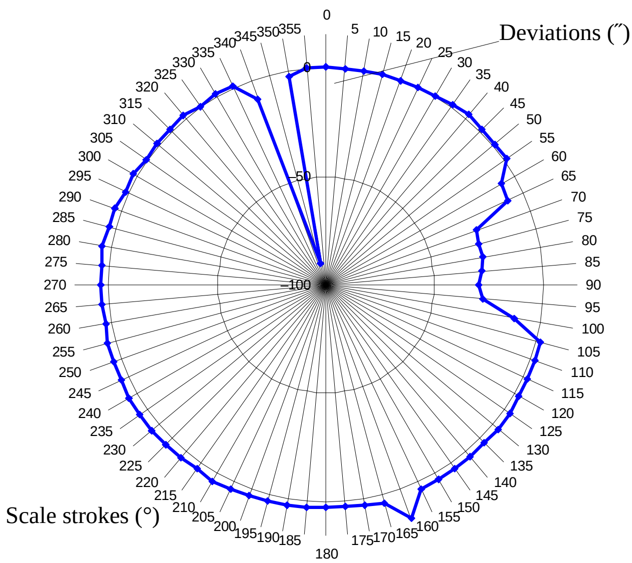

Fig. 4. Mean calculated bias of each scale stroke

Judging from the data presented in Fig 2 (the calculated scale strokes biases for each measurement series) an estimate of standard deviation of each bias measurement is relatively small (not exceeding $S_i = 0.423''$, including the estimate of standard deviation of the modernised microscopes measurements $S_m = 0.0125''$). Since the biases were calculated regarding the first stroke (bias is equal 0) the standard deviation estimate of each specific stroke should be determined as:

$$S_k = \sqrt{\sum_{i=1}^{k} S_i^2}\ , \tag{3}$$

where: k – number of the scale stroke.

Maximal standard deviation will be at the last calculated stroke:

$$S_{\max} = \sqrt{\sum_{i=1}^{n} S_i^2}\ . \tag{4}$$

where: n – total number of the calculated scale stroke.

In case of the described measurements (the angle of microscopes placement at 95°), the estimate of maximal standard deviation will be reached at a scale stroke marked as 90° and $S_{\max} = 2{,}621''$. It should be noted that despite a fairly high standard deviation of the last stroke bias, high amount of measurements should essentially eliminate any kind of a random error in scale bias determination. Since the values of biases need to be determined only periodically (during the rotary table scale calibration which should be performed quite seldom) it is possible to achieve appropriate calibration results.

After calculation of average biases of each scale stroke (at a pitch of 5°, Fig 4), the errors of each particular previously performed measurement (the errors of rotary table positioning) could be determined (Fig 5). It may be observed that the positioning of rotary table performed by the photoelectrical angle encoder with angular position determination by circular scale/microscope with scale calibration data used (Fig 5) show significant errors of photoelectrical angle encoder and are clearly influenced by eccentricity and other biases of the encoder [1].

Since during the scale calibration the multiangular prism/autocollimator was used (measurements performed every 30°) (Fig 1) and the measurements were taken simultaneously with those taken by the microscopes, the data received from both measurements could be compared, considering the multiangular prism/autocollimator data as the reference values [4, 7]. After that the rotary table position determination by the scale/microscope (with the scale calibration results used) could be practically analysed (Table 2 and Fig. 6).

Table 2. Error of angular positioning determined by microscope and multiangular prism/autocollimator

Angular position (°)	Positioning error determined by (″)															
	Test 1		Test 2		Test 3		Test 4		Test 5		Test 6		Test 7		Test 8	
	Microscope	Autocollimator	Microscope	Autocollimator	Microscope	Autocollimator	Microscope	Autocollimator	Microscope	Autocollimator	Microscope	Autocollimator	Microscope	Autocollimator	Microscope	Autocollimator
0	–13.31	–13.42	–9.91	–10.26	–7.07	–7.27	–10.03	–9.78	–7.49	–7.49	–11.43	–11.46	–2.32	–2.57	–9.18	–9.09
30	–12.69	–12.48	–10.17	–10.22	–5.05	–5.12	–10.33	–10.44	–5.84	–5.81	–11.07	–11.00	–8.33	–8.42	–8.39	–8.42
60	–15.79	–15.64	–9.85	–9.79	–14.05	–14.00	–13.12	–13.27	–15.15	–15.05	–14.69	–14.92	–12.44	–12.46	–13.08	–
90	–9.91	–10.09	–1.57	–1.44	–7.33	–7.40	–7.98	–8.04	–3.17	–3.40	–4.44	–4.63	–7.18	–6.96	–6.81	–6.80
120	8.80	9.24	6.87	6.97	6.03	6.05	4.41	4.36	10.50	10.37	7.50	7.60	2.85	3.04	9.51	9.40
150	10.20	10.23	5.31	5.36	10.06	9.87	7.32	7.22	6.00	5.95	4.73	4.81	3.07	3.12	6.13	6.37
180	9.27	9.47	8.63	8.52	13.03	12.99	9.31	9.52	13.74	13.87	10.76	10.72	6.99	6.73	12.59	12.39
210	3.97	4.54	3.59	3.65	5.83	5.80	8.10	8.26	8.86	8.70	5.63	5.46	4.26	4.55	6.28	5.99
240	–1.58	–1.59	–6.26	–6.13	–0.97	–1.13	3.19	3.13	–0.35	–0.41	–1.12	–1.24	1.55	1.36	0.50	0.23
270	3.84	3.99	–3.79	–3.72	2.43	2.40	4.54	4.70	8.12	7.99	8.59	8.62	2.67	2.56	1.19	1.20
300	4.08	4.01	1.89	1.85	0.93	0.90	3.18	3.06	1.37	1.24	5.95	6.15	4.82	4.38	7.32	7.66
330	–4.45	–4.45	1.51	1.60	–2.26	–1.94	–2.97	–2.82	0.76	1.11	0.96	0.79	0.71	0.52	1.19	1.22

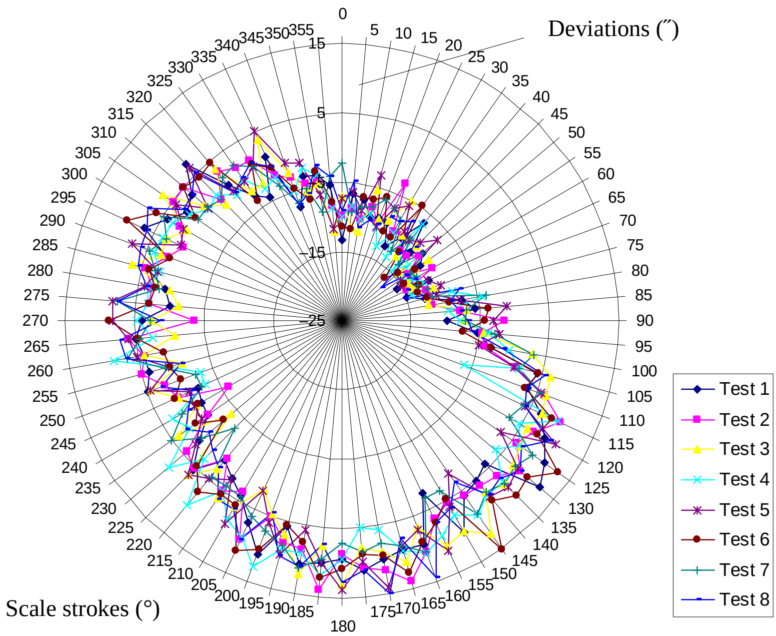

Fig. 5. Calculated rotary table positioning error

The graphical comparison (according to Table 2) of angular rotary disc position determined by polygon/autocollimator and circular scale/microscope is presented in Fig 6. It is obvious from the presented figures that rotary disc angular position determined by circular scale/microscope match the position determined by polygon/autocollimator fairly well.

Having an estimate of the standard deviation data of calibration of multiangular prism and the modernized autocollimator, a general evaluation of multiangular prism/autocollimator measurements' standard deviation can be calculated [4]:

$$S_{a/p} = \sqrt{S_a^2 + S_p^2}\,, \tag{5}$$

where: S_a – standard deviation of autocollimator measurements (S_a = 0.0405″), S_p – standard deviation of polygon calibration (S_p = 0.1″) [5].

According to the calculations it might be stated that general standard deviation of autocollimator/polygon measurements in our case does not exceed $S_{a/p}$ = 0.108″.

Having the standard deviation of the scale/ microscopes measurements (S_m = 0.0125″), a numerical comparison of calculated rotary table positioning errors data with the obtained by the multiangular prism/ autocollimator and the standard deviation of positioning at each scale stroke can be determined (Table 3).

Table 3. Standard deviation value of positioning angle determination by the microscope taking the multiangular prism/autocollimator readings as reference

Angular position (°)	0	30	60	90	120	150	180	210	240	270	300	330
Standard deviation (″)	0.197	0.0974	0.120	0.155	0.187	0.122	0.167	0.265	0.147	0.104	0.220	0.201

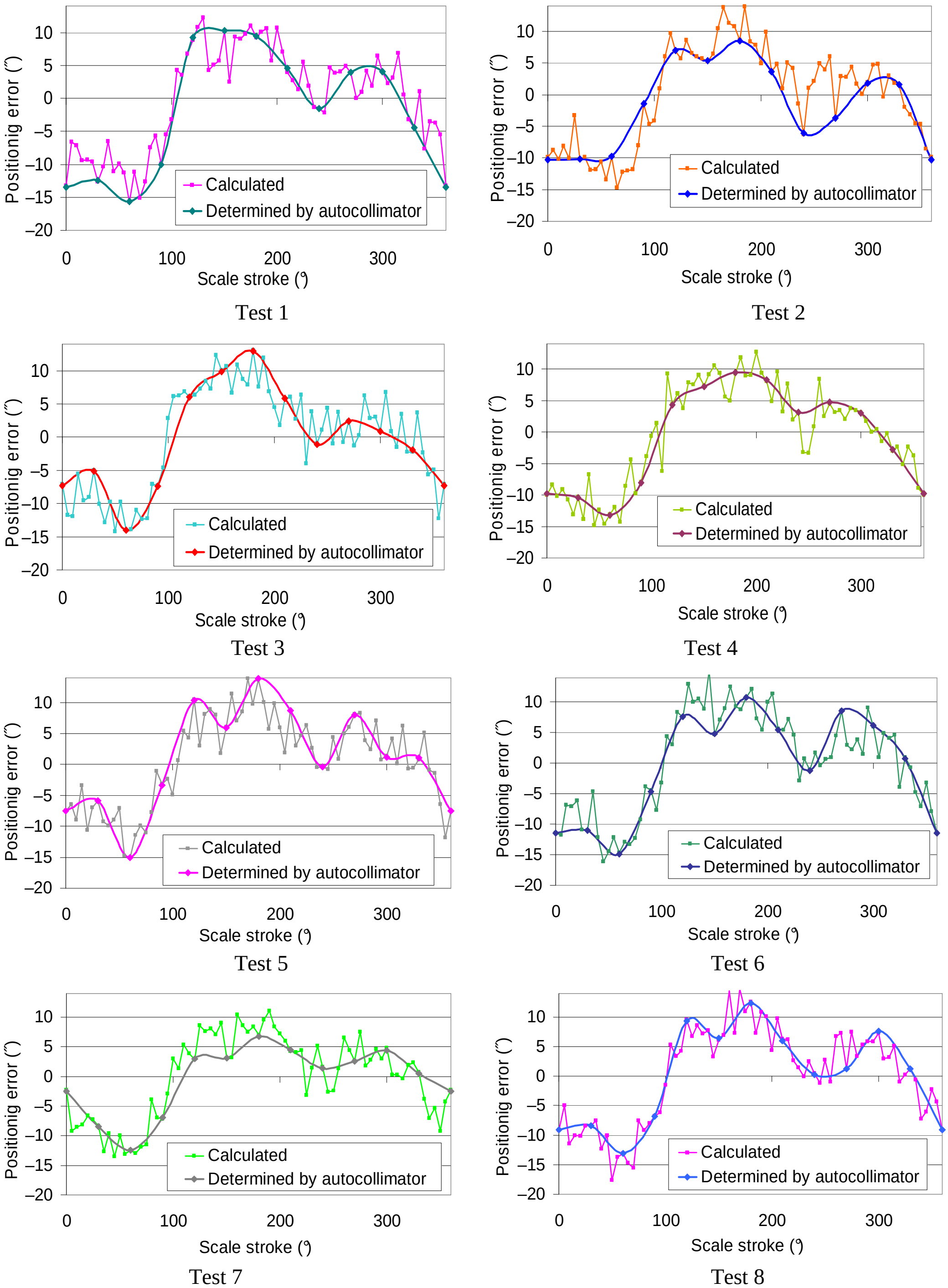

Fig. 6. Calculated rotary table positioning error compared to the one determined by multiangular prism/autocollimator

Using the calculated data, general (practical) standard deviation value and the uncertainty (at a confidence level of 0.95) of angle position determination can be calculated according to the proposed method, taking the multiangular prism/autocollimator measurements as the reference [6]:

$$S_p = 0.172'' ,$$

$$\varepsilon = 0.0295'' .$$

The random error (standard deviation and uncertainty) of the rotary disc position determination performed after the scale calibration by the applied method demonstrates good results. Theoretically estimated standard deviation of angular position determination (by proposed method) was fairly high (2.62″), practical comparison of the results to the ones determined by polygon/autocollimator indicates relatively high precision ($S_p = 0.172''$, $\varepsilon = 0.0295''$), suitable for implementation on the test rig for geodetic devices.

4. Conclusions

- A novel scale calibration method based on *constant angle setting in full circle with multiple turn* has been tested on the plane angle testing/calibration bench.
- The circular scale was calibrated using two photoelectric microscopes placed at an angle of 95° to each other with a pitch of 5° and additionally controlled by the multiangular prism/autocollimator measurements.
- The calibration indicates significant scale strokes biases of up to 85″, which can be numerically compensated for the subsequent measurements.
- Though due to the sequential nature of the scale calibration (considering the 0° as not having the bias), theoretical estimate of maximal standard deviation of the scale strokes bias is fairly high (S_t = 2.62″), practical comparison of the data to the reference one obtained by the multiangular prism/autocollimator demonstrates practical estimate of standard deviation low enough (S_p = 0.172″).

Acknowledgments

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