522. Optimal robust control of aeroelastic system vibrations

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Abstract. A method for global and robust stabilization of aeroelastic wing vibrations based on optimal feedback control concepts is described in the present paper using Lyapunov stability theory. The method consists in decomposing the system model into a stabilizable linear part and a nonlinear part that satisfies sector-bound inequality; then a control law is designed to guarantee the global stabilization of the system and a specified robustness degree of the closed-loop dynamics. The validation of the method on aeroelastic wing section demonstrates better control performances over existing methods. The main contribution of the proposed method is that it allows one to design a linear controller that globally stabilizes a highly nonlinear system up to a specified degree of robustness without assuming any stability condition about the linear part, or matching conditions about the nonlinear uncertainties, contrarily to existing methods about optimal robust control.

Keywords: aeroelastic vibrations, stabilization, optimal robust control, Lyapunov stability theory.

Nomenclature

V	freestream velocity
h	plunge displacement
α	angle of attack
ρ	air density
a	non-dimensional distance from midchord to elastic axis position
b	semichord of wing section
s	wing section span
k_h	plunge stiffness
c_h	plunge damping
c_α	pitch damping
m_{wing}	mass of wing section
m_W	total wing section plus mount mass
m_T	total mass of pitch-plunge system
I_{cam}	pitch cam moment of inertia
I_{cgwing}	wing section moment of inertia about the center of gravity
$C_{l\alpha}$	$\partial l/\partial\alpha$, where l is the lift force
$C_{m\alpha}$	$\partial m/\partial\alpha$, where m is the pitch moment
β	trailing-edge control surface deflection angle
$C_{l\beta}$	$\partial l/\partial\beta$
$C_{m\beta}$	$\partial m/\partial\beta$
$\mathbf{x}$, u	state vector, and control variable
$\mathbf{A}$	state matrix
$\mathbf{Q}$, $\mathbf{P}$	positive definite matrices
$\mathbf{K}$	control gain matrix

Introduction

It is well known that the structure of an airplane is not completely rigid as it seems to be, but rather elastic. It happens that the aerodynamic forces acting upon the body of the airplane induce deformations in its elastic structure, and that these structural deformations induce, in a feedback manner, changes in the aerodynamic forces. These additional aerodynamic forces cause increases in the structural deformations, which results in greater aerodynamic forces, and so on… The interactions between the structural deformations and the resulting aerodynamic forces may become smaller until a balance is attained, or may be amplified across-time. Indeed, when the aerodynamic and the structural deformations are in balance, harmonic oscillations occur at a certain speed of the airstream called the flutter boundary; beyond this critical flow speed, there is an increase in the vibrations, which requires control actions to damp out the oscillations. The vibration phenomena, be they induced in an aeroelastic system or not, are in general governed by highly nonlinear dynamics [1-8], which may in some conditions degenerate to chaos [9, 10].

Present paper deals with the stabilization of the vibrations in an aeroelastic aircraft wing whose section is depicted in fig. 1 [3]. Existing methods for stabilizing aeroelastic wing vibrations are based on linear control which relies on linearized models of the actual nonlinear dynamics or on feedback linearization that aims at finding a nonlinear control law that linearizes the closed-loop system. Indeed, Platanitis and Straganac [1], as well as Ko and co-workers [11], resort to feedback linearization to ascertain the stabilization of aerolastic wing vibrations for small scale freestream velocity with a control surface deflection angle ranging from -15 deg. to 15 deg. Demenkov and Goman [3] use a suboptimal linear control to suppress the vibrations of the same aeroelastic wing used by Platanitis and Straganac with the same control surface deflection angle value range. Their controller uses a method that maximizes the stability region of the linearized closed-loop system under actuator saturation constraints. They succeeded in effectively maximizing the stability region in that they could stabilize the aeroelastic wing vibrations for freestream velocities up to 30 m/s. Meanwhile, their method does not guarantee global stabilization, but local one, since the linearized model is only valid in a neighborhood of the equilibrium state. Similar linear and nonlinear methods were also used for controlling aeroelastic and smart wings of different characteristics as reported in [2, 6]. All the methods mentioned above deal with local stabilization of aeroelastic vibrations but not global stabilization, that is, they are not guaranteed to deal with vibrations that may have relatively larger amplitude or frequencies. Another limitation in existing methods is that they do not ensure robustness, i.e. they may be too sensitive to disturbances due to parameter uncertainties, measurement inaccuracies or atmospheric disturbances.

The main objective of the present paper is to propose a method that guarantees global stability and robustness of the control law using Lyapunov stability theory. Indeed, the aeroelastic wing dynamic model is decomposed into a linear part that involves the state vector and the control, and a nonlinear part involving only the state; the linear part is required to be stabilizable and the nonlinear part sector-bounded, as is actually the case. Then, a linear control gain is computed to ensure global stabilization taking into account the linear part and the sector bound of the nonlinear part. Since the control law uses sector boundedness of the nonlinearities, the method ascertains robustness according to a specified degree as will be explained in the paper. It is shown that this control law is optimal in the sense that it minimizes a certain quadratic functional when no saturation occurs. The main contribution of the approach described here, and consequently its distinction with respect to existing literature on robust control [12-17], is that the proposed method aims at determining a linear control that stabilizes a highly nonlinear system up to a specified degree of robustness without assuming the linear part to be stable or unstable, nor the nonlinear uncertainties to satisfy the matching conditions contrarily to existing methods on optimal robust control.

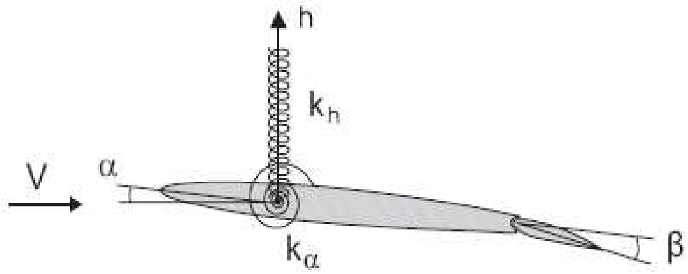

Fig. 1. Two-degree of freedom wing section

Problem statement

The equations of motion of an aeroelastic wing section with two degrees of freedom [3] (fig. 1) have been established in many references [1, 3, 6, 11]. Recently, Demenkov and Goman [3] showed that these equations can be expressed in the following form:

$$\mathbf{F}\ddot{\mathbf{y}} + \mathbf{G}\dot{\mathbf{y}} + \mathbf{C}(\alpha)\mathbf{y} = \bar{\mathbf{b}}u \tag{1}$$

where:

$$\mathbf{y} = \begin{bmatrix} h \\ \alpha \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} m_T & m_W x_\alpha b \\ m_W x_\alpha b & I_\alpha \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} c_h + \rho V b s C_{l_\alpha} & \rho V b^2 s C_{l_\alpha}(1/2 - a) \\ -\rho V b^2 s C_{m_\alpha \mathrm{eff}} & c_\alpha - \rho V b^3 s C_{m_\alpha \mathrm{eff}}(1/2 - a) \end{bmatrix},$$
$$\mathbf{C}(\alpha) = \begin{bmatrix} k_h & \rho V^2 b s C_{l_\alpha} \\ 0 & k_\alpha(\alpha) - \rho V^2 b^2 s C_{m_\alpha \mathrm{eff}} \end{bmatrix}, \quad \bar{\mathbf{b}} = \begin{bmatrix} -\rho V^2 b s C_{l_\beta} \\ \rho V^2 b^2 s C_{m_\beta \mathrm{eff}} \end{bmatrix}, \quad u = \beta \ (\mathrm{rad}). \tag{2}$$

with:

$$x_\alpha = -(0.0998 + a), \quad I_\alpha = I_{\mathrm{cam}} + I_{\mathrm{cgwing}} + m_{wing} r_{\mathrm{cg}}^2, \quad C_{m_\alpha \mathrm{eff}} = (1/2 + a)C_{l_\alpha} + 2C_{m_\alpha},$$
$$r_{\mathrm{cg}} = b x_\alpha, \quad C_{m_\beta \mathrm{eff}} = (1/2 + a)C_{l_\beta} + 2C_{m_\beta}, \quad k_\alpha(\alpha) = 12.77 + 53.47\alpha + 1003\alpha^2 \tag{3}$$

The numerical values in equations (3) are related to a specific wing chosen for the application. The overall data of the system are given in table 1.

Table 1. System parameters

Parameter	Value	Parameter	Value
ρ	1.225 kg/m^3	m_W	5.230 kg
a	-0.6719	m_T	15.57 kg
b	0.1905 m	I_{cam}	0.04697 kg.m^2
s	0.5945 m	I_{cgwing}	0.04342 kg.m^2
k_h	2844 N/m	$C_{l\alpha}$	6.757
c_h	27.43 kg/s	$C_{m\alpha}$	0
c_α	0.0360 kg.m^2/s	$C_{l\beta}$	3.774
m_{wing}	4.340 kg	$C_{m\beta}$	-0.6719

Setting $\mathbf{x} = \begin{bmatrix} h & \alpha & \dot{h} & \dot{\alpha} \end{bmatrix}^T$, equations (1) can be written as:

$$\dot{\mathbf{x}} = f(\mathbf{x}) + \mathbf{b}u, \tag{4}$$

where:

$$f(\mathbf{x}) = \begin{bmatrix} \mathbf{0}_2 & \mathbf{I}_2 \\ -\mathbf{F}^{-1}\mathbf{C}(\alpha) & -\mathbf{F}^{-1}\mathbf{G} \end{bmatrix}\mathbf{x}, \quad \mathbf{0}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \mathbf{F}^{-1}\overline{\mathbf{b}} \end{bmatrix}. \tag{5}$$

The problem to be solved consists in driving robustly the state vector $\mathbf{x}$ to zero, that is, cancelling the vibrations induced in the plunge displacement and in the angle of attack whenever this phenomenon occurs. Furthermore, the amplitude of the control surface deflection is known to be bounded as $|u| \le u_{\max}$. Therefore, the problem to be solved requires dealing with bound constrained robust control.

Proposed Method

Before presenting the robust control method that will be described in this paper, it is necessary to give some definitions and a theorem.

Basic Definitions and Theorems

Let us start this section with a definition:

Definition 1. *A matrix* $\mathbf{M}$ *is said to be* ***Hurwitz*** *if any of its eigenvalues has negative real part.*

Let us consider a nonlinear system described by the following ordinary differential equation:

$$\dot{\mathbf{x}} = \hat{\mathbf{A}}\mathbf{x} + \varphi(\mathbf{x}), \tag{6}$$

where $\mathbf{x} \in R^n$ is the state of the system, $\hat{\mathbf{A}}$ is a Hurwitz matrix with *n* rows and columns, $\varphi : R^n \to R^n$ is a nonlinear function satisfying a sector-bound inequality for any $\mathbf{x}$, that is:

$$\|\varphi(\mathbf{x})\| \le \gamma \|\mathbf{x}\|, \tag{7}$$

where $\gamma > 0$ is the bounding parameter.

Definition 2. *The equilibrium state* $\mathbf{x}_* = 0$ *is said to be* ***globally asymptotically stable*** *if the following two properties hold:*

1. *Stability:* $\forall \varepsilon > 0, \exists \delta > 0$ *such that for each solution s(t) of equation (6),* $\|s(0)\| < \delta \Rightarrow \|s(t)\| < \varepsilon \;\; \forall t \ge 0$ *;*
2. *Global attraction: Any solution s(t) of equation (6) vanishes in long term, that is:* $\lim_{t \to +\infty} s(t) = 0$.

Theorem 1 (Lyapunov asymptotic stability theorem). *Let $\mathbf{x}_* = 0$ be an equilibrium point of a system described by the equation $\dot{x} = g(\mathbf{x})$. Then that system is **stable** at $\mathbf{x}_*$ if there exists a continuously differentiable function $V(\mathbf{x})$ defined in a neighborhood $D \subseteq R^n$ of $\mathbf{x}_*$ that satisfies the following conditions:*

1. *$V(\mathbf{x}) > 0$ and $\dot{V}(\mathbf{x}) = V_x(\mathbf{x}))^T g(\mathbf{x}) \leq 0$ for any $\mathbf{x} \in D - \{\mathbf{x}_*\}$; where $(V_x(\mathbf{x}))^T$ denotes the transpose of the gradient of function V at $\mathbf{x}$;*
2. *$V(\mathbf{x}_*) = 0$ and $\dot{V}(\mathbf{x}_*) = 0$.*

*The system is **asymptotically stable** at $\mathbf{x}_*$ if:*

1. *$V(\mathbf{x}) > 0$ and $\dot{V}(\mathbf{x}) < 0$ for any $\mathbf{x} \in D - \{\mathbf{x}_*\}$;*
2. *$V(\mathbf{x}_*) = 0$ and $\dot{V}(\mathbf{x}_*) = 0$.*

One may notice that $\dot{V}(\mathbf{x}) < 0$ for any $\mathbf{x} \in D - \{\mathbf{x}_*\}$ in case of asymptotic stability, whereas $\dot{V}(\mathbf{x}) \leq 0$ for any $\mathbf{x} \in D - \{\mathbf{x}_*\}$ in case of stability.

Definition 3. *The system (6) is **robustly stable with degree** γ if the equilibrium state $\mathbf{x}_* = 0$ is asymptotically stable for any nonlinear term $\varphi(\mathbf{x})$ that satisfies equation (7).*

Definition 4. *Given two matrices $\mathbf{A}$ and $\mathbf{B}$, the pair $(\mathbf{A}, \mathbf{B})$ is said to be **stabilizable** if there exists a matrix $\mathbf{K}$ such that matrix $\mathbf{A} - \mathbf{BK}$ is Hurwitz.*

Solution to the Stated Problem

Based on the definitions and theorem given above, a robust control method that is optimal (in some sense that will be explained later) can be proposed in the sequel.

Since function f in equation (4) is nonlinear and $f(0) = 0$ (but not equal to zero for all $\mathbf{x} \neq 0$), it can be decomposed into the sum of two functions, one being linear and the other nonlinear in the following sense:

$$f(\mathbf{x}) = \mathbf{A}\mathbf{x} + \varphi(\mathbf{x}), \tag{8}$$

where $\mathbf{A}$ is a matrix with $\mathbf{A} \neq 0$, $\mathbf{A}$ may be, for instance, the Jacobian matrix of function f at $\mathbf{x} = 0$, and φ a nonlinear function that is defined as $\varphi(\mathbf{x}) = f(\mathbf{x}) - \mathbf{A}\mathbf{x}$ for all $\mathbf{x}$. Therefore, equation (4) may be written as:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u + \varphi(\mathbf{x}). \tag{9}$$

The following assumptions are supposed to be satisfied with equation (9):

H$_1$: The pair $(\mathbf{A}, \mathbf{b})$ is stabilizable;
H$_2$: $\exists \gamma > 0$ such that $\forall \mathbf{x}, \|\varphi(\mathbf{x})\| \leq \gamma \|\mathbf{x}\|$.

Since the pair $(\mathbf{A}, \mathbf{b})$ is stabilizable, there exists a linear feedback control law:

$$u(\mathbf{x}) = -\mathbf{K}\mathbf{x} \tag{10}$$

such that the closed-loop state matrix $\hat{\mathbf{A}} = \mathbf{A} - \mathbf{b}\mathbf{K}$ is Hurwitz. Using this control law, the closed-loop system may be written as:

$$\dot{\mathbf{x}} = \hat{\mathbf{A}}\mathbf{x} + \varphi(\mathbf{x}). \tag{11}$$

To stabilize the system described by equation (9) with the control law in equation (10), we use Theorem 1 to find the appropriate matrix $\mathbf{K}$ as described hereafter.

Consider the following quadratic Lyapunov function:

$$V(\mathbf{x}) = \mathbf{x}^T\mathbf{P}\mathbf{x}, \tag{12}$$

where $\mathbf{P}$ is positive definite matrix, and assume matrix $\mathbf{K}$ to be put on the form:

$$\mathbf{K} = \eta\mathbf{b}^T\mathbf{P} \tag{13}$$

with $\eta > 0$, fixed by the control designer as the control effort penalizing weight (for instance, $\eta = 1$). Then, for $\hat{\mathbf{A}} = \mathbf{A} - \mathbf{b}\mathbf{K}$ to be Hurwitz, there shall exist two positive definite matrices $\mathbf{P}$ and $\mathbf{Q}$ such that:

$$(\mathbf{A} - \mathbf{b}\mathbf{K})^T\mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{b}\mathbf{K}) = -\mathbf{Q} \tag{14}$$

which can be written as (when $\mathbf{K}$ is replaced in (14) by its expression):

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} - 2\eta\mathbf{P}\mathbf{b}\mathbf{b}^T\mathbf{P} + \mathbf{Q} = 0. \tag{15}$$

On the other hand, we would like the aeroelastic system to be stabilized when the control law in equation (10) is used, that is we would like the system described by equation (11) to be stable. Therefore, using Theorem 1, we express that the time-derivative of the Lyapunov function *V* shall be negative for all $\mathbf{x} \neq 0$ along the solutions of equation (11):

$$\dot{V}(\mathbf{x}) = \dot{\mathbf{x}}^T P\mathbf{x} + \mathbf{x}^T P\dot{\mathbf{x}} = (\hat{\mathbf{A}}\mathbf{x} + \varphi(\mathbf{x}))^T\mathbf{P}\mathbf{x} + \mathbf{x}^T\mathbf{P}(\hat{\mathbf{A}}\mathbf{x} + \varphi(\mathbf{x})) < 0 \tag{16}$$

However, denoting by $\sigma_{\min}(\mathbf{Q})$ the lowest singular value of $\mathbf{Q}$, and by $\sigma_{\max}(\mathbf{P})$ the largest singular value of $\mathbf{P}$, it comes:

$$\begin{aligned}(\hat{\mathbf{A}}\mathbf{x}+\varphi(\mathbf{x}))^T\mathbf{P}\mathbf{x}+\mathbf{x}^T\mathbf{P}(\hat{\mathbf{A}}\mathbf{x}+\varphi(\mathbf{x})) &= \mathbf{x}^T(\hat{\mathbf{A}}^T\mathbf{P}+\mathbf{P}\hat{\mathbf{A}})\mathbf{x}+2\varphi^T(\mathbf{x})\mathbf{P}\mathbf{x}\\ &\le -\mathbf{x}^T\mathbf{Q}\mathbf{x}+2\left\|\varphi(\mathbf{x})\right\|.\left\|\mathbf{P}\mathbf{x}\right\|\\ &\le -\mathbf{x}^T\mathbf{Q}\mathbf{x}+2\gamma\left\|\mathbf{x}\right\|.\sigma_{\max}(\mathbf{P})\left\|\mathbf{x}\right\| \\ &\le -\mathbf{x}^T(\mathbf{Q}-2\gamma.\sigma_{\max}(\mathbf{P})\mathbf{I})\mathbf{x}\ ,\\ (\hat{\mathbf{A}}\mathbf{x}+\varphi(\mathbf{x}))^T\mathbf{P}\mathbf{x}+\mathbf{x}^T\mathbf{P}(\hat{\mathbf{A}}\mathbf{x}+\varphi(\mathbf{x})) &\le -\mathbf{x}^T(\sigma_{\min}(\mathbf{Q})-2\gamma.\sigma_{\max}(\mathbf{P}))\mathbf{x}\ .\end{aligned} \tag{17}$$

Consequently, $\dot{V}(\mathbf{x})<0$ if:

$$\gamma<\frac{\sigma_{\min}(\mathbf{Q})}{2\sigma_{\max}(\mathbf{P})}. \tag{18}$$

We can now state the following theorem as the results of the reasoning above:

Theorem 2. *In case of no control saturation, the system (9) is robustly stabilized by control law* $u(\mathbf{x})=-\mathbf{K}\mathbf{x}=-\eta\mathbf{b}^T\mathbf{P}\mathbf{x}$ *under the assumption* $\mathbf{H_2}$ *if there exist two positive definite matrices* $\mathbf{P}$ *and* $\mathbf{Q}$ *such that the following two conditions are satisfied:*

(i) $\mathbf{A}^T\mathbf{P}+\mathbf{P}\mathbf{A}-2\eta\mathbf{P}\mathbf{b}\mathbf{b}^T\mathbf{P}+\mathbf{Q}=0$,

(ii) $\gamma<\dfrac{\sigma_{\min}(\mathbf{Q})}{2\sigma_{\max}(\mathbf{P})}$.

Theorem 2 provides a way to determine a robust control signal for stabilizing the vibrations of the aeroelastic wing. This is accomplished mainly by finding matrices $\mathbf{P}$ and $\mathbf{Q}$ that satisfy the joint equality (i) and inequality (ii) of Theorem 2, then to compute the controller output as given in the following equation due to actuator saturation:

$$\hat{u}(\mathbf{x})=\mathrm{sign}(u(\mathbf{x})).\min\{u_{\max},u(\mathbf{x})\}, \tag{19}$$

where $u(\mathbf{x})=-\eta\mathbf{b}^T\mathbf{P}\mathbf{x}$.

The resolution of the equality (i) and inequality (ii) above may be performed straightforwardly by solving rather the following optimization problem (instead of proceeding by trial-and-error):

$$\begin{aligned}&\underset{\mathbf{Q}}{\mathrm{Max}}\left\{\frac{\sigma_{\min}(\mathbf{Q})}{\sigma_{\max}(\mathbf{P})}\right\}\\ &\text{subject to: } \mathbf{A}^T\mathbf{P}+\mathbf{P}\mathbf{A}-2\eta\mathbf{P}\mathbf{b}\mathbf{b}^T\mathbf{P}+\mathbf{Q}=0\ ,\\ &\qquad\qquad \mathbf{Q} \text{ and } \mathbf{P} \text{ being positive-definite}\ .\end{aligned} \tag{20}$$

This optimization problem can be easily solved using, for instance, the Dynamic Canonical Descent method described in detail in [18].

Optimality of the Robust Control Solution

Consider the following cost functional:

$$J(u) = \frac{1}{2}\int_0^{+\infty} (\mathbf{x}^T \mathbf{Q}\mathbf{x} + \eta^{-1}u^2)dt\,, \tag{21}$$

where η and $\mathbf{Q}$ are the same as used above.

It is known [17] that the control that minimizes the cost functional $J(u)$ above for the system described by the equation $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u$, is given by equations (10) and (13) where matrix $\mathbf{P}$ satisfies equality constraint (i) in Theorem 2. This fact gives rise to the following theorem:

Theorem 3. *In the absence of control saturation, the robust control law* $u(\mathbf{x}) = -\mathbf{K}\mathbf{x} = -\eta\mathbf{b}^T\mathbf{P}\mathbf{x}$ *given in Theorem 2 is optimal in the sense that it minimizes the cost functional in equation (21) for the system with model:* $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u$ *(which is the linear part of the model in equation (9)).*

Simulation

Four examples will be dealt with to validate the proposed method. The bound of the control in each example is $u_{\max} = 15\,\text{deg.} = 0.261\,\text{rad.}$, therefore, $-0.261 \le u \le 0.261\,(\text{rad.})$. In all the examples, matrix $\mathbf{A}$ is computed as the Jacobian matrix of function f at the equilibrium state $\mathbf{x}_* = [0,\ 0,\ 0,\ 0]^T$; hence, one may notice that assumption $\mathbf{H_1}$ holds in all the examples. We choose the control penalizing weight as: $\eta = 10$.

The initial conditions on the state from which the simulation is done will be denoted by $\mathbf{x_0}$, and their values in examples 1 to 3 are similar to those used in most of the existing publications on the subject [1, 3, 6, 11]. However, the initial conditions in example 4 are more aggressive in the sense that they are farther from the equilibrium state than the ones used by other researchers.

Existing work on the stabilization of the specific aeroelastic wing model described in this paper can only deal with the cases when the freestream velocity is less than or equal to $30\ m/s$; meanwhile, we show in examples 3 and 4 that the method that is proposed in the present paper enables us to stabilize vibrations even when the freestream velocity is far beyond $30\ m/s$.

Example 1: $V = 23.15\ m/s,\ \mathbf{x_0} = [0.01,\ 11.5\pi/180,\ \text{-}0.05,\ \text{-}0.05\pi/180]^T$.

For this value of the freestream velocity *V*, we have:

$\gamma = 0.34$; Q: diagonal matrix with diagonal elements: $Q_{11} = 10.01, Q_{22} = 1, Q_{33} = 2, Q_{44} = 1$;

$$P = 0.01 \times \begin{pmatrix} 93.8016 & 6.9818 & 0.3861 & 0.5965 \\ 6.9818 & 4.1126 & -0.3163 & 0.0771 \\ 0.3861 & -0.3163 & 0.3237 & -0.0182 \\ 0.5965 & 0.0771 & -0.0182 & 0.0245 \end{pmatrix}; \quad \frac{\sigma_{\min}(Q)}{2\sigma_{\max}(P)} = 0.53 > \gamma = 0.34.$$

The simulation results for this case are depicted in fig. 2. The uppermost charts illustrate the plunge displacement and its derivative with respect to time, the middle charts display the angle of attack and its derivative, and the lower charts illustrate the control (in fact, the trailing-edge control surface deflection angle). Stabilization occurs in less than 1 second, which is similar to the results reported in [3], but slightly faster than in [1]. Besides, the amplitudes of the oscillations are globally less than those reported in publications [1, 3, 6, 11] in the same simulation conditions. Saturation of the control occurs at startup and lasts during less than 0.1 second, which is much less than in other work [1] where the control is sometimes of bang-bang type during the full stabilization process.

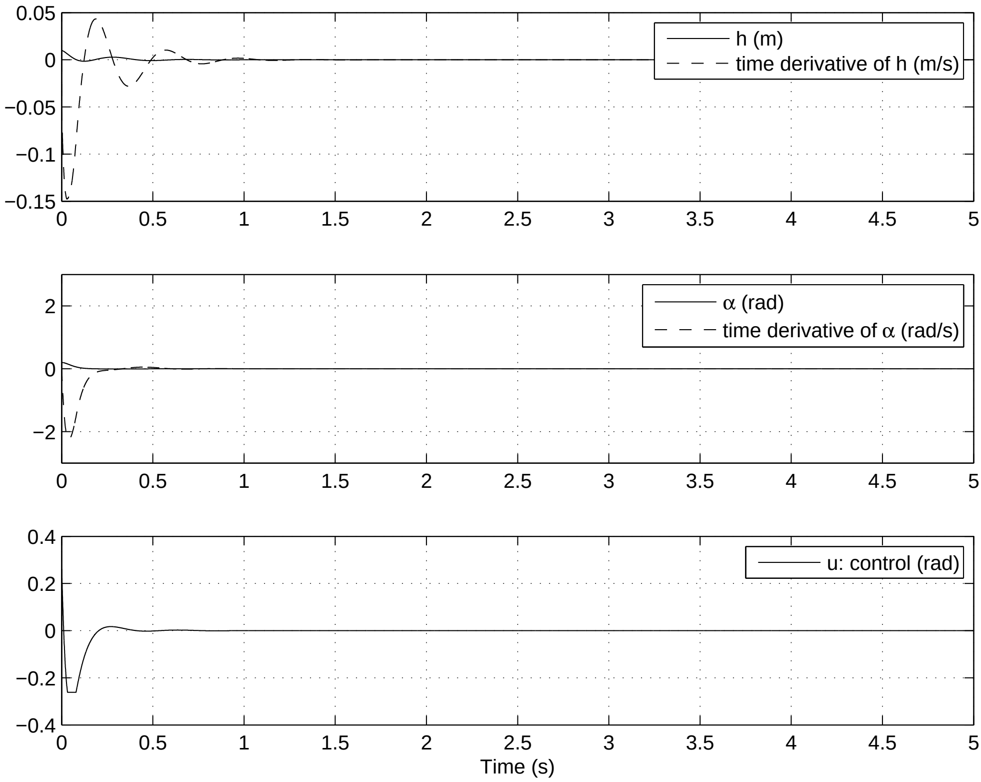

Fig. 2. Wing vibration stabilization for freestream velocity $V = 23.15\, m/s$, and initial conditions:

$$\mathbf{x_0} = [0.01,\ 11.5\pi/180,\ \text{-}0.05,\ \text{-}0.05\pi/180]^T$$

Example 2: $V = 30\, m/s,\ \mathbf{x_0} = [0.01,\ 11.5\pi/180,\ \text{-}0.05,\ \text{-}0.05\pi/180]^T$ **.**

For this value of the freestream velocity *V*, we have:

$\gamma = 0.98$; Q : diagonal matrix with diagonal elements: $Q_{11} = 20.0, Q_{22} = 1, Q_{33} = 1, Q_{44} = 1$;

$$P = 0.01 \times \begin{pmatrix} 41.7255 & 5.9746 & 0.1787 & 0.2740 \\ 5.9746 & 4.3026 & -0.2034 & 0.0655 \\ 0.1787 & -0.2034 & 0.1463 & -0.0097 \\ 0.2740 & 0.0655 & -0.0097 & 0.0159 \end{pmatrix}; \quad \frac{\sigma_{\min}(Q)}{2\sigma_{\max}(P)} = 1.17 > \gamma = 0.98.$$

The simulation results of this case are depicted in fig. 3. These results are qualitatively similar to those of the previous example, with the exception that no saturation occurs in the control and that the maximum amplitude of the plunge displacement time derivative is 0.145 m/s, that is, higher than in previous example where it was equal to 0.049 m/s, but anyway much less than in the results obtained by [3] where the maximum plunge displacement rate was 0.226 m/s.

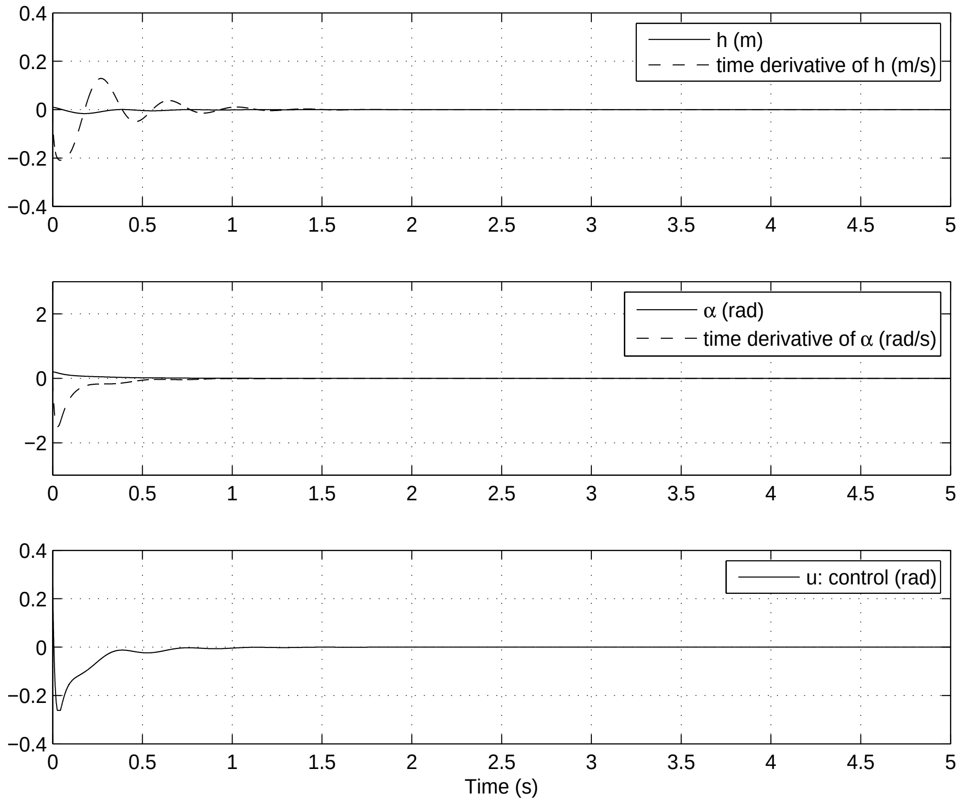

Fig. 3. Wing vibration stabilization for freestream velocity $V = 30\ m/s$, and initial conditions:

$$\mathbf{x_0} = [0.01,\ 11.5\pi/180,\ -0.05,\ -0.05\pi/180]^T$$

Example 3: $V = 45\ m/s$, $\mathbf{x_0} = [0.01,\ 11.5\pi/180,\ -0.05,\ -0.05\pi/180]^T$.

For this value of the freestream velocity, we have:

$\gamma = 1.18$; Q: diagonal matrix with diagonal elements: $Q_{11} = 20.0, Q_{22} = 1, Q_{33} = 2, Q_{44} = 1$;

$$P = 0.01 \times \begin{pmatrix} 31.8928 & 11.5465 & -0.2786 & 0.1296 \\ -0.2786 & 7.5311 & -0.2981 & 0.0713 \\ 0.1787 & -0.2981 & 0.1255 & -0.0113 \\ 0.1296 & 0.0713 & -0.0113 & 0.0070 \end{pmatrix}; \quad \frac{\sigma_{\min}(Q)}{2\sigma_{\max}(P)} = 1.37 > \gamma = 1.18.$$

The simulation results of this case are depicted in fig. 4. In this example, the freestream velocity is 45 m/s, which is higher than in any other case considered in existing work [1, 3, 6, 11] where former methods enable, as shown experimentally in [3], to stabilize the vibrations in case when the velocity is less than 30 m/s. Hence, this example aims at showing clearly that the method that is proposed in the present paper is much more efficient than other methods. It is noticeable with the charts in this figure that the oscillations are stabilized in much less than 1 second, but with higher amplitudes of the plunge displacement and angle of attack rates at startup. There is also a saturation of the control at startup, but this occurs during less than 0.1 second.

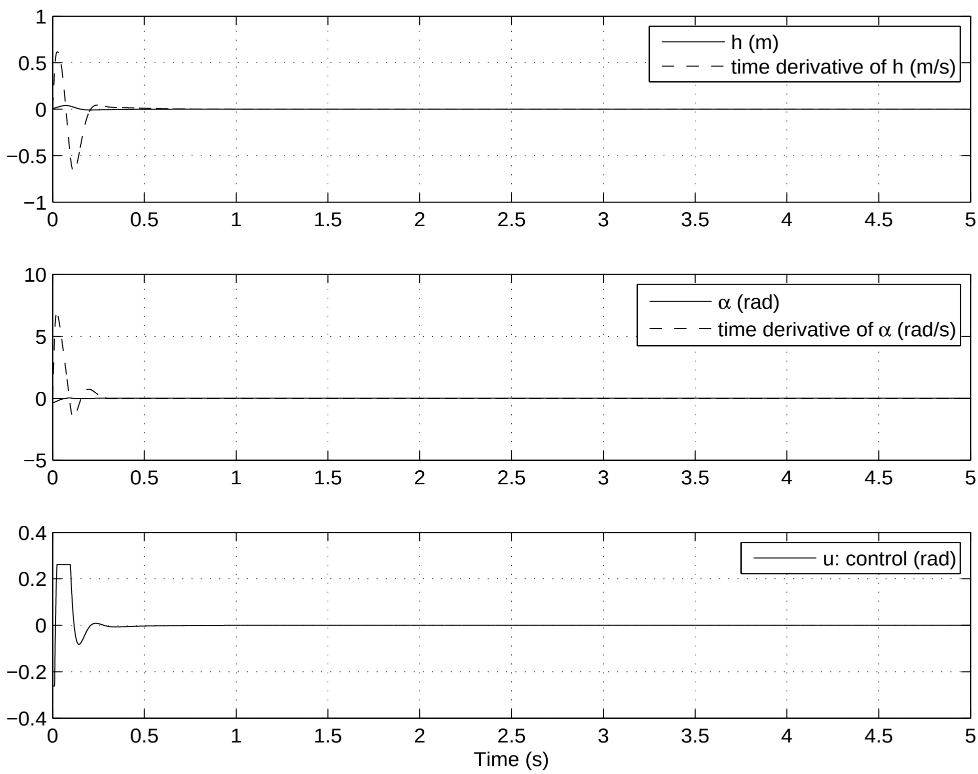

Fig. 4. Wing vibration stabilization for freestream velocity $V = 45\, m/s$, and initial conditions: $\mathbf{x_0} = [0.01,\ 11.5\pi/180,\ -0.05,\ -0.05\pi/180]^T$

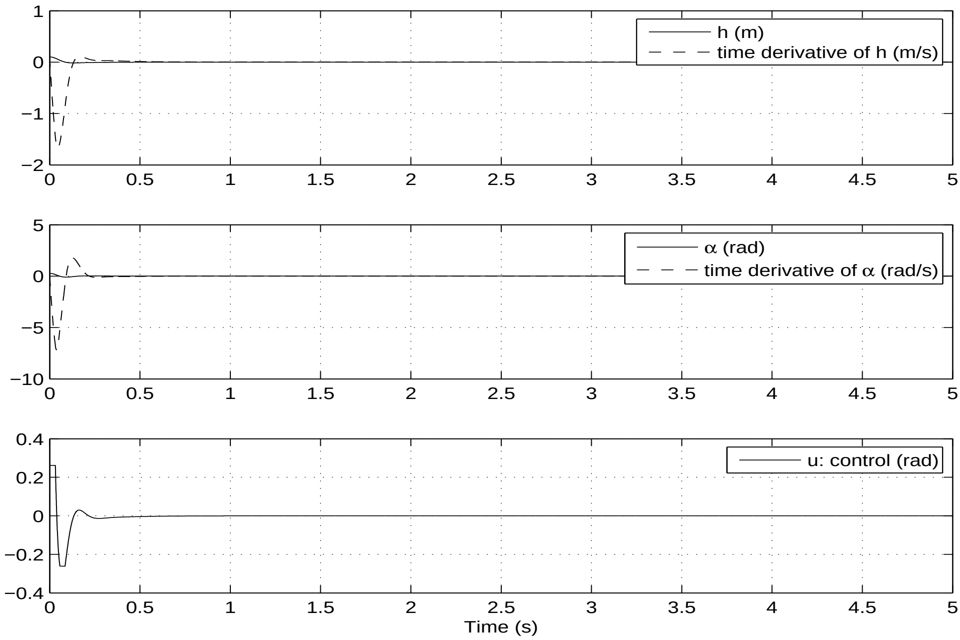

Fig. 5. Wing vibration stabilization for freestream velocity $V = 45\, m/s$, and initial conditions:

$$\mathbf{x_0} = [0.1,\ 15.5\pi/180,\ 0.08,\ \text{-}0.07\pi/180]^T$$

Example 4: $V = 45\, m/s$, $\mathbf{x_0} = [0.1,\ 15.5\pi/180,\ 0.08,\ \text{-}0.07\pi/180]^T$ **.**

In this example, the value of the freestream velocity is the same as the one used in the previous example, therefore, the parameters that were computed in the previous example are identical to those of the present example; hence, they will not be repeated here. However, to demonstrate the efficiency of the proposed method, we choose the initial conditions farther from the equilibrium state than those already used.

The simulation results of this case are depicted in fig. 5. The results show that even in this case where the initial state is very far away from the equilibrium state, the oscillations are stabilized in much less than 1 second as well, and that oscillations occur at startup but during a short time-interval. Qualitative results are similar to the former cases above, noting that the amplitudes of the oscillations at startup are much higher.

Conclusion

A control method that enables global, robust and optimal stabilization of aeroelastic system vibrations has been described in the present paper. The proposed method is described as follows: the nonlinear model of the system is decomposed into a stabilizable linear part that involves the state vector and the control, and a sector-bounded nonlinear part involving only the state; then, a linear control gain is computed taking into account the linear part and the sector-bound of the nonlinear part to ensure global and robust stabilization using Lyapunov stability theory. It is shown that the control law is optimal in the sense that, in fact, it is the solution of an optimal linear quadratic regulation (LQR) problem when no saturation occurs. The validation of the method on the aeroelastic wing system demonstrates excellent performances of the proposed method. Indeed, the simulation results demonstrate that the stabilization of the vibrations is faster, has fewer oscillations in the response and less saturations in the control than reported in former work for the same inputs of the simulation procedure.

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