546. Influence of vertical vibration of support on the dynamic stability of subsea pipeline

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Abstract: The underwater suspended pipeline was investigated for the dynamic instability which is applied at the problem for the oscillation of the pipe-line part by inverted pendulum. The connection point of pendulum was received as vertical moving point by harmonic law. For definition of the dynamical equation it is used the analogy of the Mathieu equation. For solution it is used Ince-Strut diagram. As numerical example it was used the pipeline behavior at the project between Turkey and North Cyprus at the East Mediterranean Sea.

Keywords: Subsea pipeline**,** dynamic behavior, Ince-Strut diagram, pendulum.

Introduction

In this research, for defining the problem of the oscillation it is used the analogy with the offshore tension leg platforms [1, 2, 3, 4]. The vibration of sub sea cylindrical long body have many solutions given in the technical literature [2, 5, 8], but the dynamic equations of the motion at the structures have non-linear characteristics. Therefore, to find the direct solution of their equation it is impossible and the reseacher must apply the different numerical methods for stability investigation of the pipe-line. The main problem is to solve the stability of the inverted pendulum system which vibrates at the foundation of the system [6, 7]. In this research it is used the project of pipeline between Turkey and North Cyprus as key study [1] (see Fig.1).

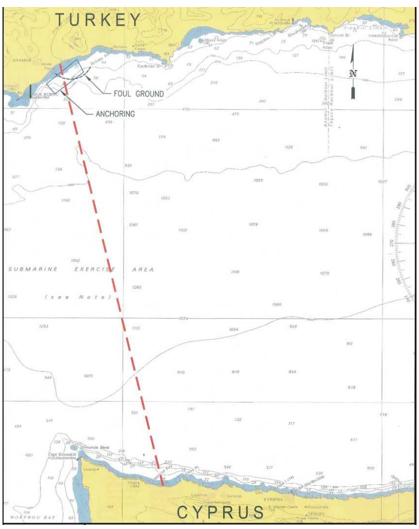


Fig. 1. Map of dislocation of pipeline between Turkey and Cyprus

Statement of the problem

The pipeline structure system consists of the pipe - line - 1, cable - 2, Y – type connection joint system - 3, ballast float - 4 and foundation plate - 5 (Fig. 2). Used in natural system foundation structure it is very rigid and fixed structure (Fig. 3) [5].

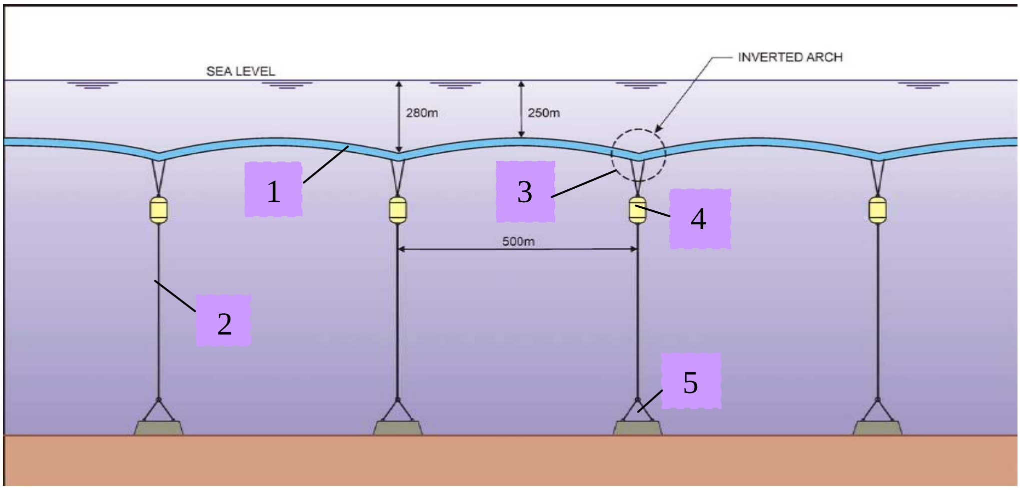


Fig. 2. Cyprus Peace Water Project Draft Sketch [5]

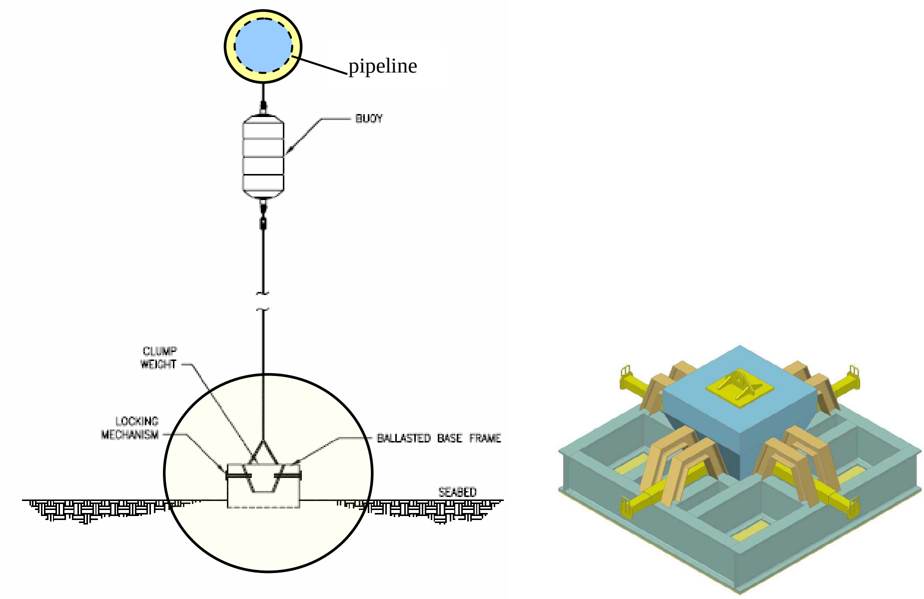
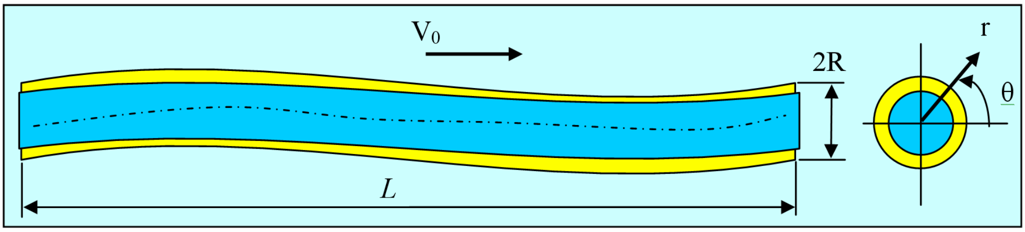
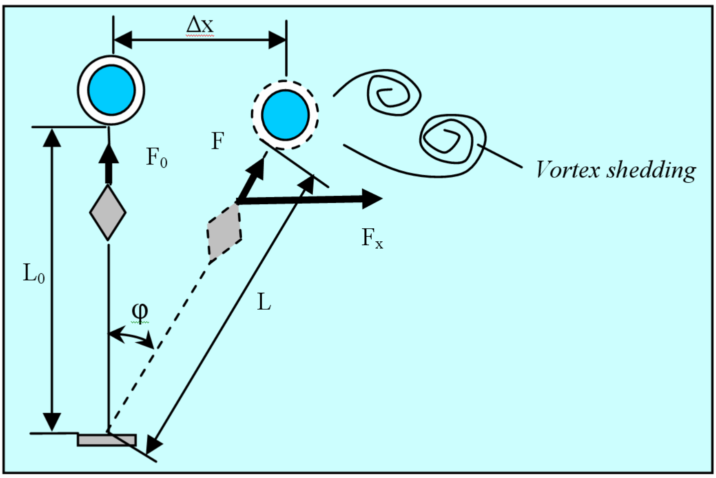


Fig. 3. Foundation structure [5]

The dynamical design scheme of the system is shown as inverted pendulum (Fig. 4a). Investigation from Figure 3, we have fixed support and at the foundation it is assumed that there is no displacement of this point then the system is unstable. If the support of the system has small motion then this system can be stabile (Fig. 4b).



a)



b)

Fig. 4. a) Vortex shedding of suspended pipeline horizontal sketch and b) vortex shedding of suspended pipeline vertical sketch

If the forces given in the Figure 4 loaded to the system according to the D'Alambert principle the system is in equilibrium and then we can write the dynamic equation system as

$$\begin{cases} m\ddot{x} = -N\sin\varphi \\ m\ddot{y} = N\cos\varphi - mg \end{cases} \tag{1}$$

The excepted *N* parameter of the system can be solved by multiplying the first equation with $\cos\varphi$ and the second equation with $\sin\varphi$ and then they are added as

$$m(\ddot{x}\cos\varphi + \ddot{y}\sin\varphi) = -mg\sin\varphi \tag{2}$$

Moreover, if *x* and *y* parameter can be written with cable length – *l*:

$$\begin{cases} y = l - l\cos\varphi - y_1 \\ x = l\sin\varphi \end{cases} \tag{3}$$

or

$$\begin{cases} \ddot{y} = l\left(\ddot{\varphi}\sin\varphi + \dot{\varphi}^2\cos\varphi\right) - y_0\omega^2\sin\omega t \\ \ddot{x} = l\left(\ddot{\varphi}\cos\varphi - \dot{\varphi}^2\sin\varphi\right) \end{cases} \tag{4}$$

If φ is a very small parameter, then $\cos\varphi = 1; \sin\varphi = \varphi; \dot{\varphi}^2 \approx 0$. After transformation we have:

$$\ddot{\varphi} + \left(\frac{g}{l} - \frac{y_0\omega^2}{l}\sin\omega t\right)\varphi = 0 \tag{5}$$

The equation (5) is known as the equation of Mathieu. In canonical form we can write this equation, like

$$\frac{d^2\varphi}{d\tau^2} + \left(a + 2q\cos 2\tau\right)\varphi = 0 \tag{6}$$

where $a = \dfrac{4g}{\omega^2 l}; \quad \tau = \dfrac{\omega t}{2}; \quad 2q = \dfrac{4y_0}{l}$

The Mathieu Equation has oscillating nature, depends on a and q constants and have two solutions as stabile and instable characters (Fig. 5).

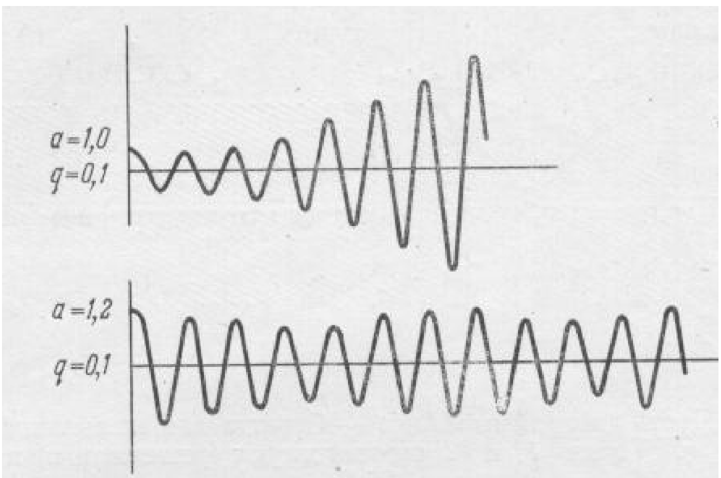


Fig. 5. Two solutions of Mathieu equation: a) instable; b) stable [17]

The domains of stability for the solution of the Mathieu equation are given in the Ince - Strutt diagram is given as (Fig. 6).

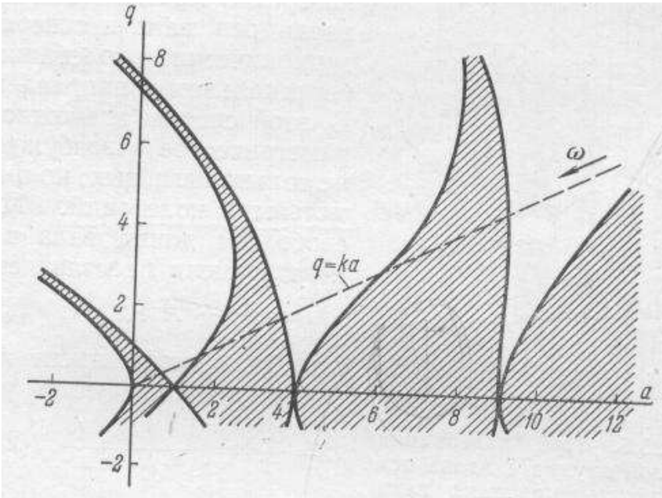


Fig. 6. Ince - Strutt diagram [6]

Every curve of the graph is given by Mathieu function. At first among four instable fields we can write exact equations, if we mark them as a_n^r and a_n^l (in this *r* index is right, and the *l* index is left hand side) as [13, 14].

Solution of the problem

There are many solutions of the Mathieu equation: *Whitaker, Watson (1963)* → $a=b$, $q=-8c$; *Stratton (1942)* → $a=b-c^2/2$, $4q=c^2$; *Yanke-Emde-Leush (1964)* → $a=4b$, $q=8c$; *National Bureau of Standards (1951)* → $a=b-c/2$, $q=c/4$ [3, 6, 7, 10, 13, and 14].

As an example in this research it is used the project of pipeline between Turkey and North Cyprus which is located at the narrowest section of the strait formed by Turkey and the North Cyprus and will provide water at a rate of 75 million m³ per year (2.38 m³/s). The pipeline will be a submerged floating structure and the sub sea section of the pipeline will consist of 1.6 m diameter HDPE (High Density Polyethylene) pipe approximately 78 km long. In the shore approaching sections of the route, the pipeline will be either resting on the seabed or be trenched and backfilled below seabed level. Between the 1000 m depth contours on both sides of Turkish and Cyprus, the pipeline will be suspended at a water depth of minimum 250 m. The pipeline will be spanning from vertical legs anchored to the sea bed in spans of approximately 400 – 500 meters length of each.

In this work we use following real data, which gives in present project [1]. The length of pipe for one section $l=500m$; radius $R=0.85m$ $(D=1.7m)$. The thickness of pipe $\delta=0.063m$. The Poisson ratio is $\nu=0.44$. The density of HDPE material of pipe $\rho=1.4x10^3 kg/m^3$. The density of sea water $\rho_0=1.03x10^3 kg/m^3$. The elasticity modulus of material $E=120000t/m^2$. The stiffness of pipe $EI=7500kN\cdot mm^2$. The initial tension of legs was as $F_0=600;\ 800;\ 1000kN$. The mass of pipe on the unit is $M=600N/m$.

During small amplitude, when $0<|q|<1$, the stability of pipeline may be if it has the condition $|a|<\frac{q^2}{2}$ or when $\omega>\frac{\sqrt{2gl}}{A}$. If we have cable length $l=1000m$, maximal displacement of foundation $y_0=1m$, then $q=0.002$ and $a=0.00002$ (Fig.7).

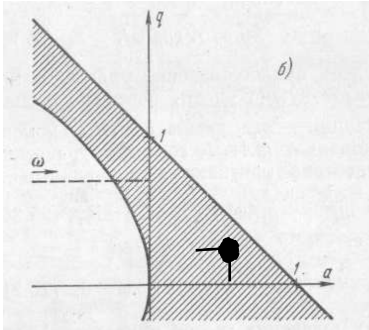


Fig. 7. Maximal displacement during small amplitude

From here we may calculate frequency and period of system:

$$\omega = \sqrt{\frac{4g}{al}} = \sqrt{\frac{4 \cdot 9.81}{0.00002 \cdot 1000}} = 44.3 rad/s \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{44.3} = 0.14s$$

Their systems are known as *high-frequency systems* [12]. There are experimental tests of stabilization of instable system with the help of displaceable foundation (Fig.9) [14, 15, 16]. One of this test was investigated by famous Russian scientist Kapitsa (Fig. 8) [17, 18].

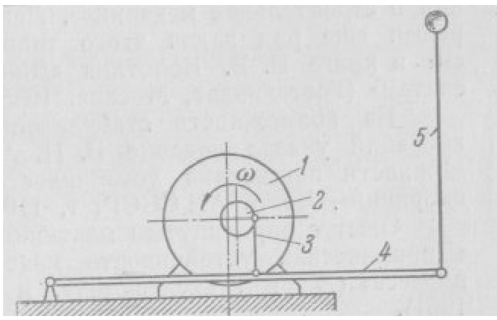


Fig. 8. The machine set up for the damping and stabilization of the oscillation [17]

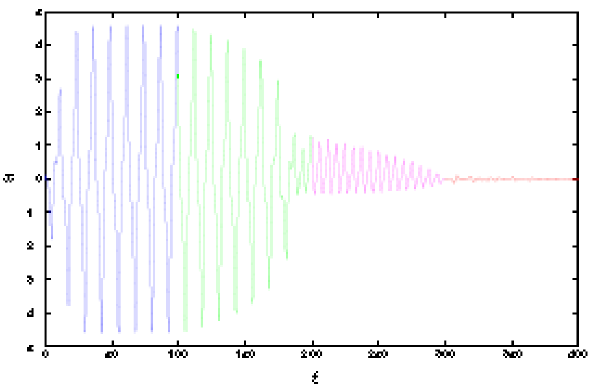


Fig. 9. Sketch of stabilization of system vibration [15]

Conclusion

- This problem is non-stationary and therefore the stability problem may be an example which can be analyzed by staticall methods;
- During symmetrical vibration modes it shows parametric resonance case;
- The Ince-Strutt diagram gives us a good possibility for defining coefficients *a* and *q* without solution of the Mathieu equation and can be defined by Mathieu functions with analytical methods;
- In order to avoid the unstable cases some engineering measures must be considered;
- Necessary is the connection of inverted pendulum make up as moving by harmonic law with the help of electrical motor.

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