553. Pipe damage detection method by combination of wavelet-based element and support vector regression

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(Received 23 July 2010; accepted 10 September 2010)

Abstract. This paper proposes a new method based on wavelet-based element and support vector regression (SVR) for pipe crack detection. The cracked pipe is modeled using B-spline wavelet on the interval (BSWI) element to obtain the precise frequencies database associated with different crack location and depth. Subsequently, the database is employed as training samples to construct the crack prediction model by means of SVR algorithm. The first three frequencies measured are inputted to the model to predict the location and severity of unknown crack. Both the numerical simulation and experimental study have verified the validity of the proposed method.

Keywords: damage detection method, wavelet-based element, support vector regression, pipe.

Introduction

The interest in the ability to monitor a structure and detect damage at the earliest possible stage is pervasive throughout the civil, mechanical, and aerospace engineering communities [1]. At present, the non-destructive testing methods such as ultrasonic methods, magnetic field methods, radiography, eddy-current methods and thermal field methods [2-6] are most currently used for damage identification. For higher efficiency and lower cost than non-destructive testing methods, the vibration-based damage detection has received considerable attention in the technical literature in recent year.

The existence of a crack in a structure results in a reduction of stiffness which in turn leads to a decrease in natural frequencies and changes in the mode shapes of vibration, which makes it possible to identify cracks in structures [7-9]. To detect structural crack fault, the model-based forward problem based on Finite Element Method (FEM) is studied for structural modal analysis when the crack is replaced by a rotational spring. Murigendrappa et al. [10] developed a natural frequency-based method for detecting the location of an unknown crack in a straight pipeline containing fluid under pressure. Naniwadekar et al. [11] presented a technique based on measurement of change in natural frequencies for detecting a crack with straight front in different orientation in a section of straight horizontal steel hollow pipe. Because wavelet-based FEM have the desirable advantages of multi-resolution properties and various basis functions for structural analysis, wavelet-based elements are applied to solve forward problem to obtain high precision crack detection database. Xiang and Zhong et al. [12] proposed a new crack detection method for detecting crack in a shaft by combination of wavelet-based element and genetic algorithm. Ye et al. [13] presented a new method to identify pipe crack location and size based on stress factor and finite element method of second generation wavelets. To solve inverse problem, one method to accomplish damage identification is to directly solve the optimization problem to yield a set of crack parameters, including location and depth of crack in structures. Evolutionary algorithms are powerful search algorithms and they have been applied to predict the crack location and severity by optimizing objective function [12]. Furthermore, the stochastic nature and parallel computational framework of Evolutionary algorithms allow to overcome the difficulties of conventional methods in converging to an absolute optimal solution. However, for the numerous train samples are needed, the computational efficiency of Evolutionary algorithms is slower than traditional optimization methods.

Support vector machine (SVM) is a relatively new computational learning method based on the statistical learning theory, which is famous and popular in machine learning community due to the excellence of generalization ability than the traditional method such as neural network. SVM have been successfully applied to face detection, verification, recognition and so on. Liu and Meng [14] introduced the SVM regression algorithm for the beam structure damage monitoring. Yang and Zhang et al. [15] used SVM for trend prediction of vibration signal of mechanical equipment and proved that SVM method had better prediction performance than BP neural network. For the above mentioned literatures, only numerical simulations were provided and traditional FEM was used.

In the present work, a method for pipe crack detection based on BSWI-based model and SVR is presented. Firstly, the cracked pipe is modeled using the BSWI element to obtain the precise frequencies database, and then the database is used as training samples to construct the crack prediction model using SVR algorithm. Both the numerical simulation and experimental study have verified the validity of the proposed method.

Support vector regression theory

Considering the case of linear function f, the form can be taken as [16]:

$$f(x) = < w, x > + b \quad \text{with} \quad w \in \chi, b \in R, \tag{1}$$

where $< w, x >$ denotes the dot product. To ensure this to minimize the norm, we can write this problem as a convex optimization problem:

$$\text{minimize} \ \frac{1}{2}\|w\|^2, \quad \text{subject to} \begin{cases} y_i - < w, x_i > - b \le \varepsilon \\ < w, x_i > + b - y_i \le \varepsilon \end{cases}. \tag{2}$$

To cope with otherwise infeasible constraints of the optimization problem, ξ_i and ξ_i^* are introduced, and the formulation can be written as:

$$\text{minimize} \ \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{l}(\xi_i + \xi_i^*), \text{ subject to} \begin{cases} y_i - < w, x_i > - b \le \varepsilon + \xi_i \\ < w, x_i > + b - y_i \le \varepsilon + \xi_i^* \\ \xi_i, \quad \xi_i^* \ge 0 \end{cases}, \tag{3}$$

where the constant $C > 0$ determines the trade-off between the flatness of f and the amount up to which deviations larger than ε are tolerated.

Applying the Lagrange technique, we have

$$\text{maximize} \begin{cases} -\frac{1}{2}\sum_{i=1}^{l}(\alpha_i - \alpha_i^*)(\alpha_j - \alpha_j^*) < x_i, x_j > \\ -\varepsilon\sum_{i=1}^{l}(\alpha_i + \alpha_i^*) + \sum_{i=1}^{l} y_i(\alpha_i - \alpha_i^*) \end{cases}, \text{ subject to } \sum_{i=1}^{l}(\alpha_i - \alpha_i^*) = 0, \alpha_i \quad \alpha_i^* \in [0, C] \tag{4}$$

where α_i and α_i^* are Lagrange parameters, and the support vector expansion can be written as follows:

$$f(x)=\sum_{i=1}^{l}(\alpha_i-\alpha_i^*)<x_i,x>+b\,. \tag{5}$$

For non-linear cases, the dot product is replaced by a kernel function $K(x_i,x)$, and the expansion for the non-linear can finally written as:

$$f(x)=\sum_{i=1}^{l}(\alpha_i-\alpha_i^*)K<x_i,x>+b \tag{6}$$

Pipe damage model based on BSWI element

Fig 1. shows the nodal layout of BSWI shaft element and the corresponding degrees of freedom (DOFs). Each node has one DOF, i.e., $w_i\,(i=2,\cdots r)$ $r=2j+m-4$, in which j is the scale and m is order of BSWI, except that two ends have two. i.e., $w_i,\theta_i\,(i=1,r+1)$.

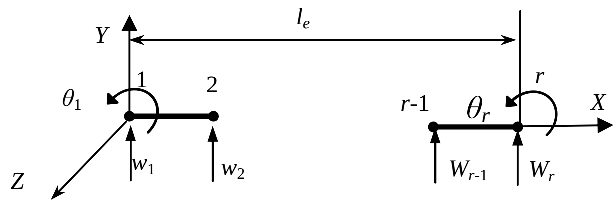


Fig. 1. The layout of elemental nodes and the corresponding DOFs

When the shaft vibrates freely, global potential energy [12] is:

$$\Pi_p=\int_0^{le}\frac{EI}{2}\left(\frac{d^2w}{dx^2}\right)^2dx-\int_0^{le}\frac{1}{2}\lambda\rho w^2dx \tag{7}$$

where l_e is the length of the element, w is the deflection function, E is the Young's modulus, I is the moment of inertia, λ is the eigenvalue of vibration, ρ is the material density and ω is the angular frequency of the shaft.

To obtain the finite element model, assume finite element approximation of deflection function w in the form:

$$w(\varepsilon,t)=\mathbf{\Phi}\mathbf{T}_b^e\mathbf{w}^e \tag{8}$$

where $\mathbf{\Phi}$ is the scaling function of BSWI mj, as shown in Ref. [17] and the corresponding DOFs in the local coordinate system are defined by:

$$\mathbf{w}^e=\{w_1\ \ \theta_1\ \ w_2\ \ \cdots\ \ w_r\ \ \theta_r\ \}^{\mathrm{T}} \tag{9}$$

where $\theta=\frac{1}{l_e}\frac{dw_1}{d\varepsilon}$ and $\theta=\frac{1}{l_e}\frac{dw_{r+1}}{d\varepsilon}$, the column vector and transformation matrix $\mathbf{T}_b^e$ is:

$$\mathbf{T}_b^e=([\mathbf{\Phi}(\varepsilon_1)\ \ \frac{1}{l_e}\frac{\mathrm{d}\mathbf{\Phi}(\varepsilon_1)}{d\varepsilon}\ \ \mathbf{\Phi}(\varepsilon_2)\cdots\mathbf{\Phi}(\varepsilon_r)\ \ \mathbf{\Phi}(\varepsilon_{r+1})\ \ \frac{1}{l_e}\frac{\mathrm{d}\mathbf{\Phi}(\varepsilon_{r+1})}{d\varepsilon}])^{-1} \tag{10}$$

Mapping Eq.(7) onto elemental standard solving domain $S = [0, 1]$ and submitting Eq.(8) into Eq.(7), according to the variation principle, with $\delta \prod_p = 0$, we can obtain elemental solving equations as:

$$(\mathbf{K}^{\mathbf{e}} - \lambda \mathbf{M}^{\mathbf{e}}) w^e = 0 \tag{11}$$

Where:

$$\mathbf{K}^{\mathbf{e}} = \frac{EI}{l_e^{\,3}} (\mathbf{T}_b^e)^{\mathrm{T}} \mathbf{\Gamma}^{2,2} \mathbf{T}_b^e$$

$$\mathbf{M}^{\mathbf{e}} = \rho l_e (\mathbf{T}_b^e)^{\mathrm{T}} \mathbf{\Gamma}^{0,0} \mathbf{T}_b^e$$

$$\mathbf{\Gamma}^{0,0} = \int_0^1 \mathbf{\Phi}^{\mathrm{T}} \mathbf{\Phi} \mathrm{d}\varepsilon$$

$$\mathbf{\Gamma}^{2,2} = \int_0^1 \frac{\mathrm{d}^2 \mathbf{\Phi}^{\mathrm{T}}}{\mathrm{d}\varepsilon^2} \frac{\mathrm{d}^2 \mathbf{\Phi}}{\mathrm{d}\varepsilon^2} \mathrm{d}\varepsilon$$

Therefore, the corresponding characteristic equation can be written as:

$$\left| \overline{\mathbf{K}} - \omega^2 \overline{\mathbf{M}} \right| = 0 \tag{12}$$

The BWSI shaft element was employed for damaged pipe discrete. Fig. 2. shows the cross-section of the cracked pipe. The part-through and the through to the thickness of pipe are shown in Fig. 2. (a) and Fig. 2. (b) respectively.

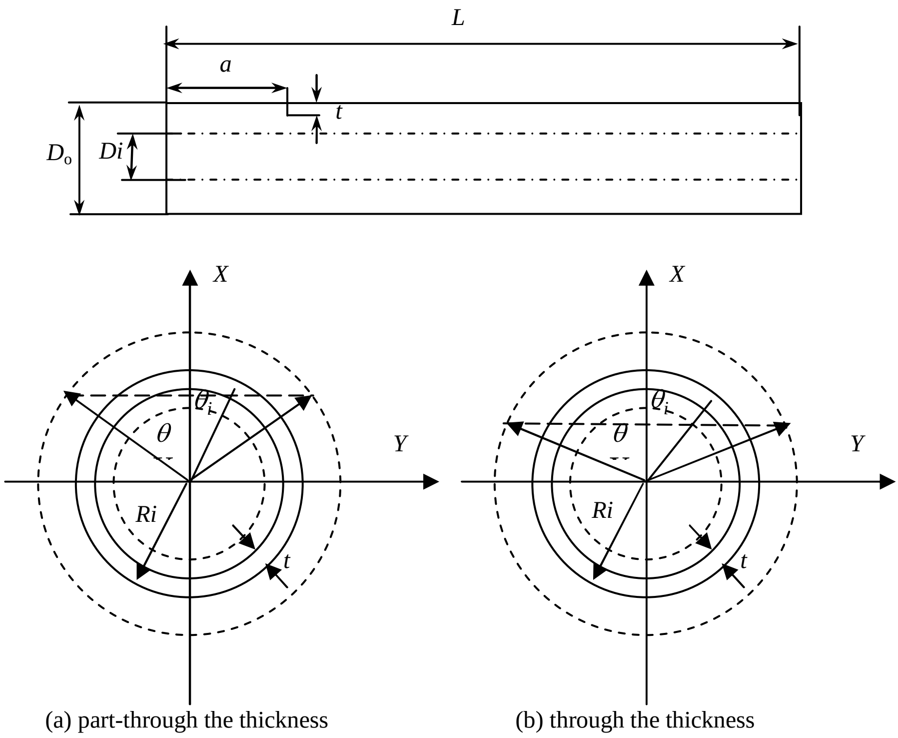


Fig. 2. The cross-section of the cracked pipe

The stiffness matrix of the damage is [13]:

$$\mathbf{K}_s = \begin{vmatrix} k_t & -k_t \\ -k_t & k_t \end{vmatrix} \tag{13}$$

where:

$$k_t = \frac{E}{2(1-v^2)t[\sum_{i=1}^{n}(R_i \int_0^{\theta_i} \frac{\partial^2 (k_i)}{\partial M^2} d\theta)]}$$

$$k_i = \sigma\sqrt{R}(\frac{\sqrt{2}}{\varepsilon})^{\frac{1}{2}} G(\theta)$$

$$\sigma = \frac{M}{\pi R^2 t}$$

$$\varepsilon^2 = (t/R)/\sqrt{12(1-\upsilon)^2}$$

$$G(\theta) = \sin\theta[1 + \frac{1}{2}\frac{\theta - \cot\theta(1-\theta\cot\theta)}{2\cot\theta + \sqrt{2}\cot(\frac{\pi-\theta}{\sqrt{2}})}]$$

E and v indicate the elastic modulus and Poisson's ratio respectively, R is the average value of inner and outer radii, R_1 and R_2 indicate the inner and outer radii of the pipe respectively. a and l is the cross-section size and location of the pipe with a transverse damage. t is the pipe wall thickness. M is the bending moment of damage at both ends. For part through-the-thickness damage, the value of n is 1400-1500, and for through-the-thickness damage, the value is 700-750. Then stiffness matrix $\mathbf{K}_S$ of the damage was assembled into the global stiffness matrix, the global mass matrix of the damaged pipe is equal to the undamaged one. From now on, the damaged pipe finite element model is constructed by using BSWI shaft element. The solution of the eigenvalue problem can then proceed as usual.

The relation between natural frequencies, damage location and size is:

$$f_r = F_i(\alpha, \beta), \quad r = 1,2,3 \tag{14}$$

Damage detection method

To detect the damage in a real structure, the damage location and size according to measured frequencies should be determined as follow:

$$(\alpha, \beta) = F_j^{-1}(f_j), j = 1,2,3 \tag{15}$$

The procedures for crack detection are:

(1) Solve the BSWI-based pipe damage model with various location and size to gain high precision damage detection database.

(2) Construct the detection database. The damage detection database are used as constructing the training sample set according to SVR algorithm, $\{X_i, H_i\}_{i=1}^{n} X_i = (\alpha, \beta), H_i = (f_1, f_2, f_3)$,

X_i is output, H_i is input. And then obtain the regression parameter using SVR algorithm to construct the detection data base.

(3) Obtain the time domain signal experimentally and get the first frequency using the frequency analysis fast Fourier transform (FFT).

(4) Detect the damage location and size using frequencies measured according to the detection database trained.

Numerical simulation

Our purpose in this study is to investigate the method proposed in this paper. Crack cases considered in this simulation are determined from previous studies paper by Murigendrappa and Maiti [10].

Example: A simply supported cracked mile steel pipe with length $L = 0.8$ *m*, outer diameter $D_o = 0.032$ *m*, inner diameter $D_i = 0.0195$ *m*, Young's modulus $E = 173.808$ Gpa, Poisson's ratio $\mu = 0.3$ and material density $\rho = 7860$ *kg/m*^{3}.

Table 1 lists the three natural frequencies ($\omega_2, \omega_3, \omega_4$) using 2 BSWI elements and those of experimental results given by Murigendrappa and Maiti [10]. Results in Table 1 indicate that the frequencies of contact pipe calculated by 2 BSWI elements are in good agreement with closed-form solutions. The four cases of different crack location and depth are also computed and the frequencies are close to the experimental ones.

The inverse problem is solved by employing the first three frequencies [10] as inputs to SVMdark tool, which is a windows implementation of SVM written by Martin Sewell [18]. The comparison of predicting crack location and depth using the present method and those by Murigendrappa and Maiti [10] are shown in the Table 2. The relative errors of both crack location and depth are no more than 1.6%.

Table 1. Comparison of 2 BSWI element and experimental solution

Method / Case	2 BSWI elements (Hz)			Experimental results (Hz)		
	f_2	f_3	f_4	f_2	f_3	f_4
uncracked	430.74 (430.50)	973.66 (973.13)	1730.95 (1730.00)	432.50	925.15	1602.50
1	415.04	935.57	1707.7	432.33	924.80	1602.40
2	391.88	899.65	1685.6	432.20	924.50	1602.00
3	407.84	919.19	1542.8	432.25	924.85	1600.80
4	399.84	902.83	1504.5	432.20	924.65	1599.50

Note: The value in () is closed-form solution of intact pipe.
Case 1: $\beta = 0.199$, $\alpha = 0.2032$; Case 2: $\beta = 0.199$, $\alpha = 0.3040$; Case 3: $\beta = 0.403$, $\alpha = 0.4064$; Case 4: $\beta = 0.403$, $\alpha = 0.5080$.

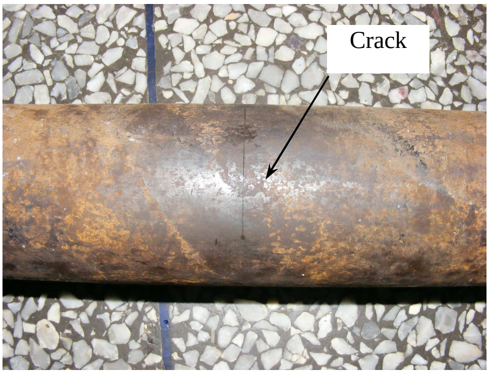
Table 2. Comparison of predicted and actual crack parameters

Parameters / Case	Actual		Predicted	
	β	α	β^* (Error %)	α^* (Error %)
1	0.199	0.2032	0.20(0.5)	0.20(1.6)
2	0.199	0.3040	0.20(0.5)	0.20(1.3)
3	0.403	0.4064	0.40(0.07)	0.41(0.08)
4	0.403	0.5080	0.39(0.3)	0.49(0.35)

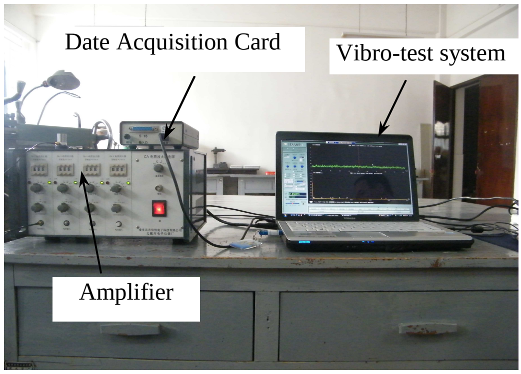
Experimental investigation

An experimental setup used for measuring the first three frequencies of the cracked pipe with the INV1601 vibration system and the accelerometer.

We tested four cracked cantilever pipes. The material of workpiece for experiment is 20# steel, and the geometries and the material properties are as follows: $L = 0.88\ m$, outer diameter $D_o = 0.073\ m$, inner diameter $D_i = 0.033\ m$, Young's modulus $E = 2.06 \times 10^{11}\ N/m^2$, material density $\rho = 7860\ kg/m^3$, Poisson's ratio $\mu = 0.3$. Figs.3 (a)-(c) demonstrate the cracked pipe, the vibration test system and the cracked cantilever pipe, respectively.



(a) The cracked pipe



(b) Vibration test system

(c) The cracked cantilever pipe

Fig. 3. The vibrate test system of cracked pipe

Table 3 shows the comparison of actual normalized crack location β and depth α and the predicted crack location β^* and depth α^*. For the given cases, the relative errors of β^* are less than 3.5% and the relative errors of α^* are less than 5.1%. Hence, the proposed method based on wavelet-based element and SVR is considered to be valid for actual application in detecting cracks in the pipe.

Table 3. Crack cases of pipe and detection results

Case \ parameters	β	α	Measured frequencies(Hz) f_1	f_2	f_3	β^* (Error/%)	α^* (Error/%)
1	0.316	0.137	72.07	460.74	1259.2	0.305(3.5)	0.144(5.1)
2	0.316	0.205	70.06	457.3	1223.4	0.305(3.5)	0.2(2.4)
3	0.659	0.137	73.77	447.68	1273.9	0.655(0.6)	0.144(5.1)
4	0.312	0.342	69.05	455.3	1251.3	0.309(0.9)	0.354(3.5)

Conclusions

A method has been proposed to detect the location and depth of the cracked pipe. The crack detection method is based on wavelet-based element and SVR algorithm. Firstly, the cracked pipe element model of BSWI is constructed to obtain computational frequencies under different crack location and depth, and then the frequencies database is used as training samples to construct the crack prediction model by means of SVR algorithm. The first three measured frequencies can be employed as inputs to the trained SVR model to predict the unknown crack location and depth on pipe. Both the numerical simulations and experiments were carried out and the performance of the presented method was verified. However, the cracks are so complex that much more work should be performed in order to detect the real rather than man-made damages in future research.

Acknowledgements

Authors are gratefully acknowledging the financial support by the projects of National Natural Science Foundation of China (No. 50805028), the Key Project of Chinese Ministry of Education (No. 210170), Guang Xi Key Laboratory of Manufacturing System & Advance Manufacturing Technology (No. 0900705_003). This work is also supported in part by the Excellent Talent Project of Guangxi University system, which is very much appreciated.

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