717. Investigation of dynamic and precision characteristics of low frequency vibration measurement device

V. Volkovas, E. Uldinskas, M. Eidukevičiūtė
Technological Systems Diagnostics Institute, Kaunas University of Technology
Kaunas, Lithuania
E-mail: *tsdi@ktu.lt, ernestas.uldinskas@ktu.lt, marija.eidukeviciute@ktu.lt*

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Abstract. The measurement of low frequency vibration is important in evaluating vibration in constructions of wind power stations, buildings and also in television and retransmit antennas. The paper presents a new construction of low frequency vibration measurement device and methodology of parameters optimization based on BEM model. Experimental investigation of device prototype shows the adequacy of the theoretical model which can be used for synthesis of devices with desirable frequency parameters. The measurement precision of the device is evaluated by accomplishing measurement uncertainty analysis.

Keywords: low frequency vibration, measurement device, uncertainty, precision.

1. Introduction

Currently the number of unique construction objects in the world like and in Lithuania is increasing: multi-storey buildings, wind power-stations, many bridges, frame constructions and others. Some of the constructions after long exploitation are influenced by aging processes then environmental effects such as wind, earthquakes may induce low frequency vibrations in objects or constructions. In order to ensure their safety it is essential to observe their parameters by measuring low frequency vibrations.

Low frequency vibration measurements are used in such fields as: earthquake vibrations [1], wave measurements on sea ice [2], etc. The vibrations in various constructions are measured using vibration transducers of Endevco [3], Brüel&Kjær [4], Kistler [5] and of other known enterprises. Yet the analysis of the products shows that they have specific characteristics: they are based on specific physical principle of transforming mechanical energy into other type of energy; designed for specific frequency range and has nominal precision and purpose. Their constructional solutions and physical principles prevent using simple method to synthesize vibration transducers for specific low frequency vibration range with reliable measurement uncertainty and suitably extensive precision.

This paper suggests unified mechatronic system for low frequency vibration measurement and the construction of adequate device, which theoretical ground is dynamics of multi degree of freedom vibrating system (in simple case – of one degree). The dynamic properties of the device and possibilities of synthesis of its dynamics were examined, the components for measurement uncertainty model were analyzed and the uncertainty model itself provided.

2. Measurement scheme and construction of the device

The device for low frequency vibration measurement (Fig. 1) is constructed as mechatronic system, consisting of mechanical subsystem (console beam 2 with a seismic mass 4 at the end) and two inductive displacement sensors 3, which are placed in x and y directions of the corresponding orthogonal coordinate system. The measurement of the object's low frequency vibrations is supplemented with filter 5 and secondary (e. g. recorder or analyzer) equipment 6.

The device is fastened to the measured object 7. This is sufficiently simple constructional solution, which allows warrant frequency synthesis, is cheap and has possibilities for the wide application. They can be widely used in various industrial fields due to its simplicity, reliable work and good electric characteristics.

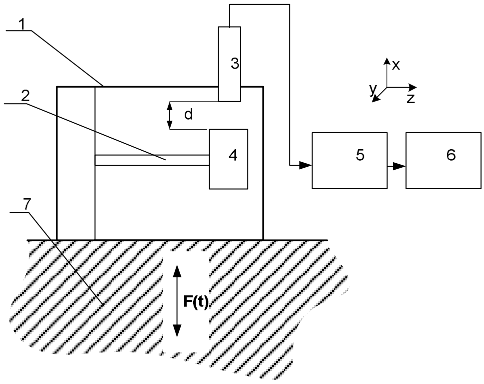

Fig. 1. The scheme of low frequency vibration measurement of the object: 1 – device's housing; 2 – steel rod, acting as spring; 3 – one of two noncontact displacement sensors; 4 – seismic mass; 5 – analog low frequency filter; 6 – signal recorder and analyzer; 7 – measured object; $F(t)$ – force induced of the object's vibrations; d – distance between seismic mass and displacement sensor

When force $F(t)$ induces low frequency vibration in the object 7, they are transferred into device's mechanical system. Inside it steel beam 2 is built-in, which acts as a spring with seismic mass 4, the latter begins to move according the law $A(x, y)$. Displacement sensors 3 are fastened perpendicularly to the plane yOz in two directions x and y. In that case we measure the displacement of seismic mass $x(t)$, which is the same range as low frequency vibrations $y(t)$ in direction y. The measured signal from displacement sensors 3 through analogue low frequency filter 5 is transmitted to signal recorder and analyzer 6. The paper investigates the independent vibration in mentioned directions and the following is only about one x direction of vibration.

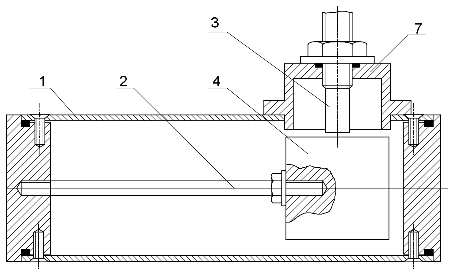

Fig. 2. The construction of low frequency vibration measurement device

The construction of the low frequency vibration measurement device is shown in Fig. 2. It consists of housing 1 in which on the beam 2 the seismic mass 4 is fastened. The beam 2 is fastened on the cover 1. The fastening elements 7 of the displacement sensor 3 are built-in the housing [4].

3. Theoretical model of the device

The transverse vibration of mechanical subsystem of the device is described using equation [7]:

$$EI\,\frac{\partial^4 x}{\partial u^4} + \rho F\,\frac{\partial^2 x}{\partial t^2} = 0; \quad u \in [0, L] \tag{1}$$

here $x(u, t)$ – transverse vibrations in u coordinate of L length (when $u = L$, the mass m is at the end of the beam); E – Young‘s modulus; I – inertia moment of the rod cross-section; ρ – the density of rod material; F – the area of rod cross-section.

The marginal conditions:

$$\begin{aligned}
&x\big|_{u=0} = 0;\\
&\left.\frac{\partial x}{\partial u}\right|_{u=0} = 0;\\
&\left.\frac{\partial^2 x}{\partial u^2}\right|_{u=0} = 0;\\
&EI\left.\frac{\partial^3 x}{\partial u^3}\right|_{u=L} = m\left.\frac{\partial^2 x}{\partial t^2}\right|_{u=L}.
\end{aligned} \tag{2}$$

The solution of the equation (1) using variables separation method gives mechanical subsystem vibration forms such as:

$$Q(u) = C_1 S(ku) + C_2 T(ku) + C_3 U(ku) + C_4 V(ku)\,, \tag{3}$$

here C_i – constants (i = 1, 2, 3, 4); S, T, U, V – Krylov functions; $k = \sqrt[4]{\dfrac{\rho F \omega^2}{EI}}$ – frequency parameter (ω – angular vibration frequency).

The equation (1) is embedded into marginal condition (2), then constants C_i determination equation system is obtained from which we get frequency. Its general form is:

$$\Delta \equiv \det(A) = 0, \tag{4}$$

here A – matrix, which consists of coefficients C_i. The equations (1) and (4) describe the dynamics of the investigated transducer mechanical subsystem. According this model the natural frequencies and vibration forms are calculated and also their change is analyzed with different meanings of parameters E, I, ρ, F and m.

The model parameters determine the precision and possibilities of the low frequency vibration measurement. To ensure them identification procedure is used. The frequency equation of the mathematical model of the investigated subsystem is denoted as $\Delta = 0$, its E, I, ρ, F and m, ensuring required low frequency vibration measurement, as unknown variables $p_1, p_2, \ldots p_5$, and the vector of natural frequencies $\boldsymbol{\Omega}$. Then frequency equation becomes:

$$\Delta(p_1, p_2, \ldots, p_5, \Omega_j) = 0;\; j = 1, 2, \ldots \tag{5}$$

The first component of the natural frequencies vector $\boldsymbol{\Omega}$ should be at least 7 times larger than the biggest measured frequency of low frequency vibration. Thus optimization task can be solved by searching optimal parameters p_i.

The analytical form (5) of the frequency equations narrow the range of possible mechanical subsystems constructions as sometimes it is impossible to compose frequency equations for complex mechanical systems.

For frequency equations composition also finite elements method can be used in which the task of natural frequencies evaluation is such [8]:

$$\det\left|\boldsymbol{K} - \Omega^2 \boldsymbol{M}\right| = 0 \tag{6}$$

here $\boldsymbol{K}$ – matrix of the system rigidity; $\boldsymbol{M}$ – matrix of the system masses.
Such access allows more extensive range of solvable task as it does not have limitations mentioned above, but it does possible using structural optimization [9] to design the device.

Suppose for investigative mechanical subsystem there is an information in the form of n resonant frequencies ω_j, and ω_j is 7 times larger than maximum measured frequency of low frequency vibration. It is known that resonant frequencies determined experimentally will not coincide with natural system frequencies due to energy loss in mechanical systems and measurement errors. Then equation (5) becomes:

$$\Delta(p_1, p_2, \ldots, p_n, \omega_j) = E_j;\ j = 1, 2, \ldots, n \tag{7}$$

here E_j – error due to mismatch between Ω_j and ω_j.

If n resonant quantities are measured, then equation (7) can be expressed by equation system of n unknown parameters p_i. Their identification procedure is determined by iterative procedure of digital equation solving, where $p_1^{(0)}$, p_2,$^{(0)}$…, $p_5^{(0)}$ are the initial meanings (evaluations) of the unknown parameters.

The right side of the equation (7) will be expanded using Taylor series according initial evaluation of parameters and lets limit ourselves only with linear members of the series. We will obtain such equation:

$$E_j = E_j^{(0)} + \left.\frac{\partial E_j}{\partial p_1}\right|_{(0)} dp_1 + \ldots + \left.\frac{\partial E_j}{\partial p_n}\right|_{(0)} dp_n \tag{8}$$

here index (0) denotes initial evaluation of the parameters. By assuming that residual error E_j is equal to zero, we get:

$$-E_j^{(0)} = \left.\frac{\partial E_j}{\partial p_1}\right|_{(0)} dp_1 + \ldots + \left.\frac{\partial E_j}{\partial p_n}\right|_{(0)} dp_n; \qquad i = 1, 2, \ldots, n$$

Or, by using least square method, total error E_s can be written in quadratic form:

$$E_s = \sum_{j=1}^{n} \left\{ e_j^{(0)} + \left.\frac{\partial e_j}{\partial p_1}\right|_{(0)} dp_1 + \ldots + \left.\frac{\partial e_j}{\partial p_n}\right|_{(0)} dp_n \right\}^2$$

By requiring that $E_s \to 0$, we get:

$$-\sum_{j=1}^{n} E_j^{(0)} \frac{\partial E_j}{\partial p_i}\bigg|_{(0)} = \sum_{j=1}^{n}\left(\frac{\partial E_j}{\partial p_1}\cdot\frac{\partial E_j}{\partial p_i}\bigg|_{(0)}\right)dp_1 + \ldots + \sum_{j=1}^{n}\left(\frac{\partial E_j}{\partial p_n}\cdot\frac{\partial E_j}{\partial p_i}\bigg|_{(0)}\right)dp_n;\ i = 1,2,\ldots,n \tag{9}$$

Thus we have linear equation system of n unknown parameters variation/changes (corrections) dp_1, $dp_2,\ldots,dp_n$. By determining these corrections, the initial evaluation of the parameters $p_i^{(o)}$ can be improved like this:

$$p_i^{(1)} = p_i^{(0)} + dp_i;\quad i = 1,2,\ldots,n \tag{10}$$

The iterative calculation is continued till the corrections of the parameters dp_i will become sufficiently small. The resultant meanings of the identification procedure p_i values are put into mathematical model of the system.

For practical engineering analysis the simplified model is used with lumped parameters of mechanical subsystem, i.e. mathematical model of the device analyses dynamical response $x(t)$ of the mass m to the kinematic excitation $X(t)$. Dynamic model of the device mechanical system is composed of the mass attached to the beam (as the spring in dynamical property) and dampener is provided in Fig. 3 [3]. The damping is assumed as viscous.

According the Hooke's law, if the amplitude of the seismic mass vibration is sufficiently small, then between displacement and vibrating system amplitude there is linear relationship. The mass m element is supported by beam (as the spring) of rigidity k, and the movement of the mass is damped by element, which has dampening coefficient c. The mass can move only in direction x respectively to the housing of vibration measurement device. During the action, the housing can achieve the acceleration $\frac{d^2X}{dt^2}$ and the output signal is proportional to the displacement $x(t)$ of the mass m.

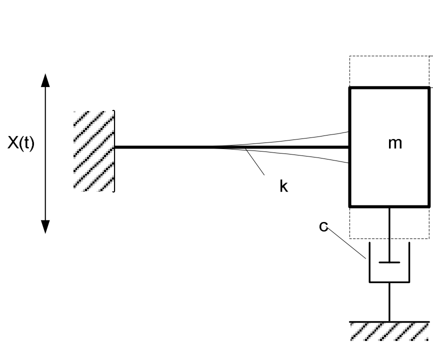

Fig. 3. Mechanical model of low frequency vibration measurement device

The model is analyzed as a system of one degree of freedom. According the Newton law we get:

$$ma = -kx - c\dot{x} \tag{11}$$

Here a – acceleration of the mass m and is expressed as:

$$a = \ddot{x} - \ddot{X} \tag{12}$$

Inserting the expression (12) into equation (11) we get approximate mathematical model of the simplified mechanical subsystem:

$$m\ddot{x}+c\dot{x}+kx=m\ddot{X} \tag{13}$$

5. The characteristics of the vibration measurement device

Assume that designed device should measure low frequency vibrations in two directions *x* and *y* in the frequency range of 0...15 Hz, it also must have small external dimensions and have adequate amplitude frequency characteristics. For this purpose design problem should be solved and appropriate creation and dynamic synthesis methodology should be created. The methodology of the transducer design is based on vibration transfer function and its analysis. In order for displacement measurement to correspond the required precision, the device should be technological, high reliability and good qualitative characteristics [6]. The latter requirements can be realized through these properties of the device:

1. Qualitative characteristics: sensitivity; precision; linearity of characteristics; reproducibility (repeatability); reaction speed; absence of hysteresis loop; small internal noise.
2. High reliability: long operating age; resistance to environmental influence (stability of characteristics on the influence of environment).
3. Technological parameters: overall dimensions and mass; simplicity of construction; low costs.

Some parameters of the transducer have much impact on measurement result, and the others small. The quality of device and measurement precision is much dependent on the main device‘s qualitative features. Thus it is important to determine the main criteria. For low frequency measurement systems few criteria can be distinguished to be considered:

- amplitude-frequency characteristic;
- natural frequency of the mechanical subsystem of the device;
- measurement range;
- sensitivity;
- linearity of characteristic;
- influence of external parameters.

Natural device frequency can be analyzed as one of criteria. By changing mechanical parameters of the device: rigidity of the beam (spring) and damping the measured low frequency range may be controlled. In our case the device should measure low frequency vibration in a range of 0...15 Hz. Fig. 4 shows the theoretical curves of device natural frequency with different damping coefficients. It can be noticed that by changing damping ratio also curve changes and appropriately changes measurement range. With small damping in the system the measurement range decreases from 15 Hz to 10 Hz. In order to increase measured frequency range system damping should be enlarged. Practical characteristics of the vibration measurement device are shown in Fig 5.

The theoretical results of parameters optimization of the vibration measurement device are presented in Fig. 6 and Fig. 7. From the last figure it is seen that created device measure vibration in frequency range of 0...15 Hz and amplitude-frequency characteristic is linear enough.

6. Analysis of device‘s measurement uncertainty components

The device‘s measurement range mostly depends on mechanical subsystem construction and parameters of inductive sensor. The main components are listed in the Fig. 8. The biggest influence on the measurement range has amplitude-frequency characteristic, which depends on one freedom degree vibration system elements: mass, rod (spring) and damping coefficient.

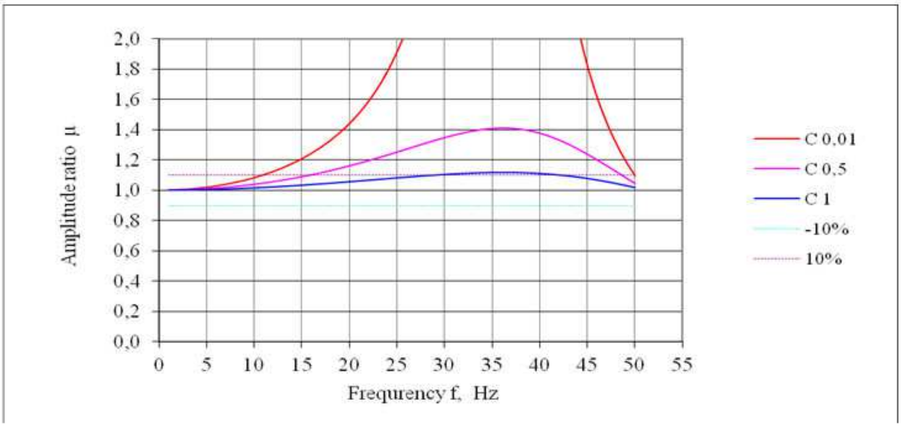

Fig. 4. Theoretical frequency characteristics of the amplitude ratio with different damping degrees

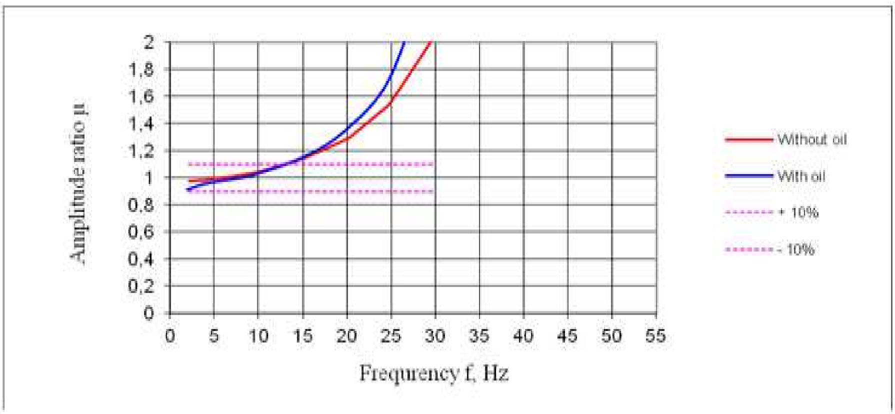

Fig. 5. Measured characteristics of amplitude ratio for mechanical system with damping and without it

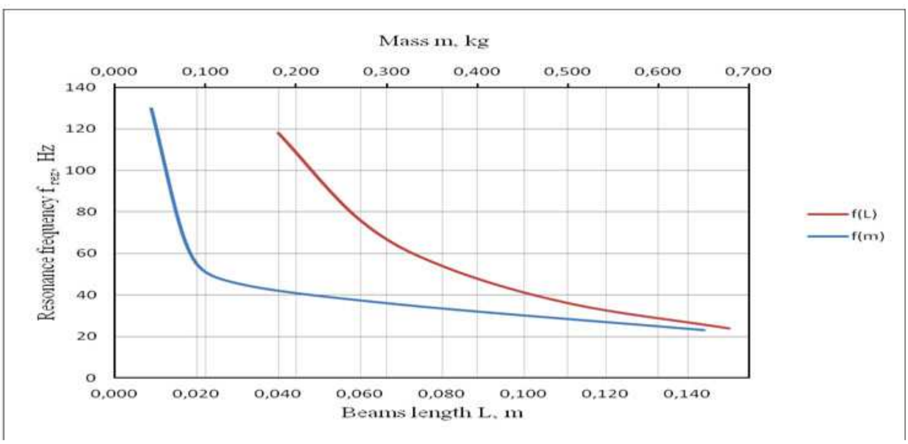

Fig. 6. Relationship between parameters (*m* and *L*) of the device and resonance frequency

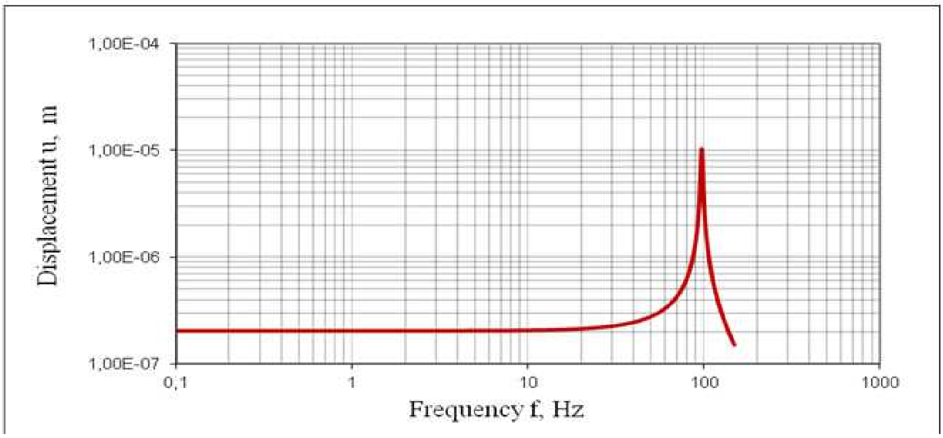

Fig. 7. Amplitude-frequency characteristic of device with optimal parameters (*m* and *L*) in frequency range of 0...15 Hz

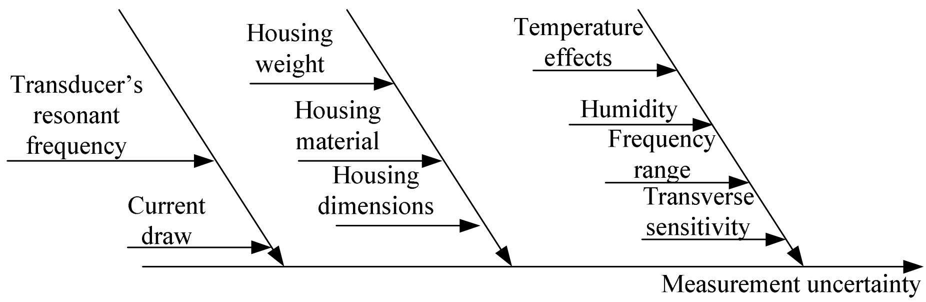

Fig. 8. Vibration measurement uncertainty component diagram

The classical uncertainty analysis theory suggests the use of formula (14) for measurement uncertainty expression [10]:

$$u_c(y) = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial f}{\partial x} \right)^2 u^2(x_i) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j)}. \quad (14)$$

Here $u_c(y)$ – total uncertainty value; $\frac{\partial f}{dx}$ – depicts influence factor of the input variable, $u(x_i)$ – uncertainty component of the input factor x_i and $u(x_i, x_j)$ depicts the correlation between the input factors x_i and x_j.

The measurement uncertainty can be used in transducer designing in two ways: by designing sensor by specifying its uncertainty; and the other way would be to design sensor, then calculate uncertainty. If necessary, then transducer has to be redesigned.

In this case the biggest input on the measurement result uncertainty is caused by displacement sensor which is built-in to the device. It's measurement uncertainty is already evaluated during calibration, thus the final value of the uncertainty depends on the environmental factors in which the device will be working and final dimensions of its housing and its material.

7. Conclusions

The device measuring low frequency vibrations (in the frequency range of 0…15 Hz) in two directions were constructed. The features which affect reliability and precision of the device were analyzed. The analysis showed that increasing damping allows to measure vibrations in the wider range of frequencies, but the further investigation has to be done.

Acknowledgments

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