730. The rings with molecular current as the model of the passive magnetic bearing

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Abstract. There will be presented a model of a passive magnetic bearing. The model uses the ring with molecular current as a source of external magnetic field (the unmovable magnet mounts in a case of machine). The next ring with molecular current moves in external field generated by unmovable ring (the magnet connects with the shaft of machine). The density of magnetic field is obtained from the Biota-Savarta law and the force is estimated from the Lorentz law. Moreover, there will be estimated the damping factor proportional to the speed of the movable magnet. The model can be used to design the radial passive magnetic bearings.

Keywords: passive magnetic bearing, molecular current, magnetic force.

Introduction

The passive magnetic bearings have got a lot of advantages. They eliminate friction between rotate elements of machine and remove cooling and lubrication system. The passive bearings don't power during a work and they are cheaper than active magnetic bearing. These bearings have got disadvantage. They don't assure coaxial position a rotor in an air gap and those are unable to design isolated system of magnetic suspension. The full system of magnetic levitation must one degree of freedom controlled by active magnetic suspension or other system of position stabilization.

The active magnetic bearing has got feedback loop between the position of rotor in the air gap and the magnetic force. The passive magnetic bearing hasn't got feedback loop. The repulsive or attractive magnetic forces in the passive magnetic bearing are resulted from magnetism phenomena [1].

The estimation of the property of the passive magnetic suspension as damping coefficient, natural frequency and stiffness coefficient is very difficult. The magnets generate the non-uniform magnetic field and the value and direction of magnetic field depend from point around the magnet. The classical approach doesn't give a good result. Only the finite element method makes possible evaluation of the passive magnetic bearing. Available different models are describing the passive magnet bearing [2] and [3].

The ring with the surface molecular current was presented by author as a mathematical model of the passive magnet bearing. There is derived damping and stiffness coefficient of the passive magnetic bearing. The resultant magnetic forces depend from molecular surface current and inductive surface current. The first part of paper is presented experimental result and the second part is mathematical model damping and stiffness forces. The experimental result formulates problem, which is solve in the second part.

The experimental result

The step response of the passive magnetic bearing was used to estimate the damping coefficient. The passive bearing was excited by the active magnetic bearing. There was changed a current in the winding of the active magnetic bearing. The step of current was generated a step of magnetic force which snatch away the rotor from nominal position [4]. The move of rotor was transform to the passive magnetic surface by the rigid shaft (Fig. 1).

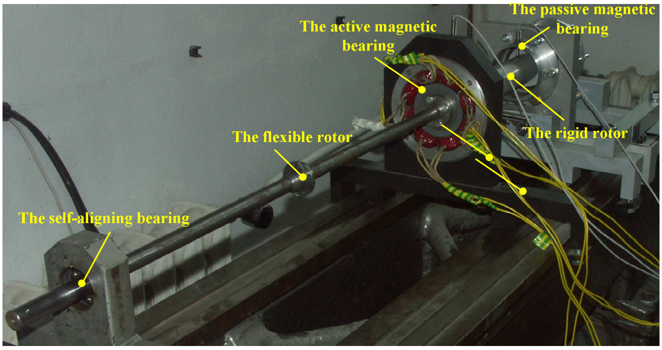

Fig. 1. The lab stand

There was used the negative slope to obtain the step response. When the current increased in the winding of electromagnet the rotor moved and glued to poles of magnetic core of electromagnet. The rotor was blocked. When the rotor was blocked by electromagnet the current in the winding decreased to zero and the rotor was sunk and the move of rotor was transformed to the passive magnetic bearing surface (Fig. 1). There was response of passive bearing for step move of rotor. The electromagnet of active magnetic bearing worked only as inductor. The feedback loop of the active magnetic bearing didn't work and the active bearing didn't damp the vibration of rotor [4].

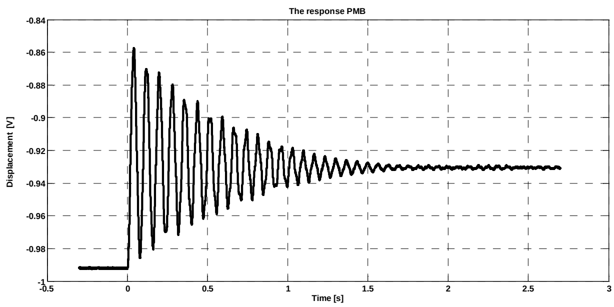

Fig. 2. The step response of the passive magnetic bearing

The laboratory stand is presented on the figure 1 and the step response of the passive magnetic bearing is shown on the figure 2. The move of rotor was measured by eddy current sensor in the surface of the passive magnetic bearing. Response of the passive magnetic bearing (Fig. 2) was recorded when the rotor was at the sink. On the picture is shown response with the small damping coefficient. There is a big overshot. The main problem is deriving a formula for the damping and stiffness coefficient of the passive magnetic bearing.

The molecular current in the passive magnetic bearing

The passive magnetic bearing is built from magnets or complex magnets. There are two groups of magnets. The first magnet generates the magnetic field and it doesn't move and it mounts in a case of machine. The second magnet connects with the rotor and it moves together with the rotor. The first magnet calls unmovable and the second magnet calls movable. The movable magnet moves as a needle of compass in the external field generates by unmovable magnet.

The source of magnetic field is the surface molecular current in an active wall of magnet. The surface molecular current $\vec{K}$ is equal [5] and [6]:

$$\vec{K} = \vec{M} \times \vec{n} \tag{1}$$

where: $\vec{M}$ – vector of magnetization, $\vec{n}$ – normal vector.

From equation (1) arises that the surface current subsist only in the wall of magnet that the normal is perpendicular to the vector of magnetization.

The amplitude of vector of magnetization is estimated from the curve of demagnetization of magnets:

$$B_r = \mu_0 M$$

where: B_r – a remanence of magnet, μ_0 – the magnetic permeability of vacuum, hence, for the left and right wall the surface current is equal (Fig. 3):

$$\vec{K}_l = \begin{bmatrix} \frac{B_r}{\mu_0}\sin\varphi & -\frac{B_r}{\mu_0}\cos\varphi & 0 \end{bmatrix}, \tag{2}$$

$$\vec{K}_r = \begin{bmatrix} -\frac{B_r}{\mu_0}\sin\varphi & \frac{B_r}{\mu_0}\cos\varphi & 0 \end{bmatrix}. \tag{3}$$

a) b)

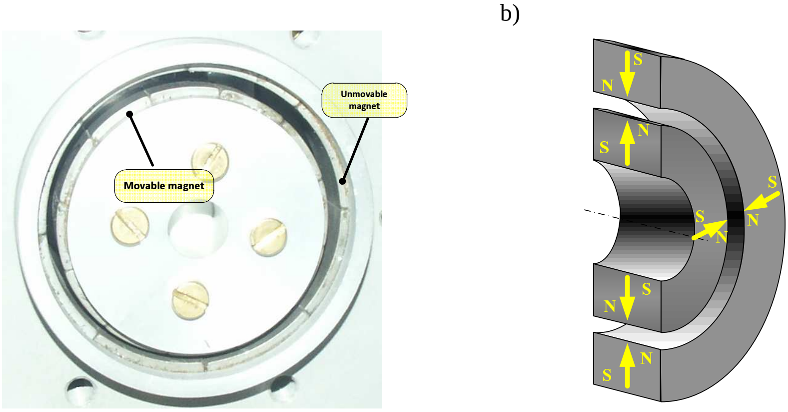

Fig. 3. The construction of the radial passive magnetic bearing

If the vector of magnetization changes direction the surface current is equal (Fig. 3):

$$\vec{K}_l = \begin{bmatrix} -\frac{B_r}{\mu_0}\sin\varphi & \frac{B_r}{\mu_0}\cos\varphi & 0 \end{bmatrix}, \tag{4}$$

$$\vec{K}_r = \begin{bmatrix} \frac{B_r}{\mu_0}\sin\varphi & -\frac{B_r}{\mu_0}\cos\varphi & 0 \end{bmatrix}. \tag{5}$$

The construction of radial passive magnetic bearing was presented on the figure 3. The unmovable magnet with radial orientation of vector magnetization has got pole N inside diameter of ring and pole S outside diameter of magnet. The movable magnet also has got radial orientation of vector of magnetization. It has got pole N outside diameter of magnet and pole S inside diameter of magnet. The magnetic polarity is different in magnets. The magnets have got self-same polarity on the opposite diameters. This is the differential radial passive magnetic bearing with repulsive magnetic forces. If the magnetic polarity of opposite diameter of magnets is S, it generates repulsive magnetic force too. The intersection of passive magnetic bearing with radial orientation of vector of magnetization is shown on the figure 3b. In figure 3a a radial passive magnetic bearing is presented. The ring shaped magnet was glued to ring made of aluminium (Fig. 3a).

The forces in the radial passive magnetic bearing

The forces in the passive magnetic bearing are equal:

$$\sum_i F_i = 0 \tag{6}$$

The equation (6) can be written as:

$$\sum_i F_i = F_m + F_b + F_c + F_g = 0 \tag{7}$$

where:
F_m – inertial forces proportional to acceleration of the rotor in the air gap,
F_b – damping forces proportional to speed of the rotor in the air gap,
F_c – spring forces proportional to the move the rotor in the air gap,
F_g – external forces (the gravity force or the forces generated during the work the machine.

In the passive magnetic bearing is two direction of move of rotor in the axes *Ox* and *Oy* (Fig. 4). The magnetic forces between two magnets can be obtained from Lorentz law [5]:

$$\vec{F} = \int \left(\vec{K} \times \vec{B}\right) da \tag{8}$$

where:
$\vec{K}$ – vector of surface current in the active walls of unmovable magnet,
$\vec{B}$ – vector of external magnetic flux density,
da – elementary surface of active walls of movable magnet.

The surface current $\vec{K}$ in the active walls is sum of the molecular surface current $\vec{K}_m$ and inductive surface current $\vec{K}_\varepsilon$:

$$\vec{K} = \vec{K}_m + \vec{K}_\varepsilon. \tag{9}$$

If (9) substitutes to (8), the force is equal [6]:

$$\vec{F} = \int \left(\left(\vec{K}_m + \vec{K}_\varepsilon\right) \times \vec{B}\right) da = \int \left(\vec{K}_m \times \vec{B}\right) da + \int \left(\vec{K}_\varepsilon \times \vec{B}\right) da \tag{10}$$

There are two components of magnetic force. The first depends from cross product between a molecular surface current and an external magnetic flux density. The second depends from cross product between an inductive surface current and an external magnetic flux density. The inductive surface current can be obtained from Ohm's law [6]:

$$\vec{K}_\varepsilon = h_m \sigma \left(\vec{v} \times \vec{B}\right), \tag{11}$$

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where:
$\vec{v}$ – the vector velocity of the movable magnet,
$\vec{B}$ – the vector of external magnetic flux density,
σ – conductivity of magnet,
h_m – height of magnet.

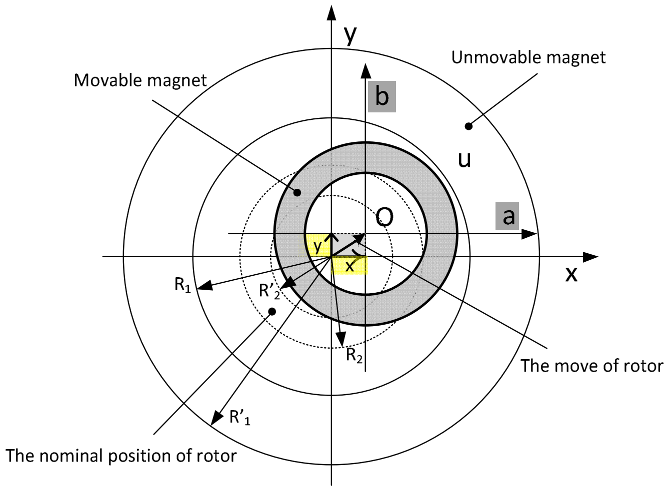

Fig. 4. The move of centre of movable magnet in the air gap of the passive magnetic bearing

In surface current (11) neglected influence of electric field and of other source of current [5]. The external magnetic flux density can be obtained from the Biota-Savarta law [5]. This solution provides to the elliptic integral. It can be solved only by numeric method. The expression for the magnetic flux density puts to the linearization around the point of work ($x = 0$ and $y = 0$ – Fig. 4), the magnetic flux density in is equal:

$$B_z(x, y, \varphi) = K_{B0}(\varphi) + K_{Bx}(\varphi)x + K_{By}(\varphi)y \tag{12}$$

where:
$K_{B0}(\varphi)$ – constant component of magnetic flux density,
$K_{Bx}(\varphi)$ – component x – displacement of magnetic flux density,
$K_{By}(\varphi)$ – component y – displacement of magnetic flux density,
x, y – displacement of movable magnet (rotor) in the air gap of bearing (Fig. 4).

The components of magnetic flux density are equal:

$$K_{B0}(\varphi) = \frac{\sqrt{2}}{32}\frac{\mu_0 M}{\pi R_1}\left(1 - \frac{R_1'^2}{R_1^2}\right)\left\{-R_1 + \frac{3\pi}{4R_1}R_2^2\left(\cos^2\varphi - \sin^2\varphi\right) + \frac{3\pi}{32R_1^3}R_2^4\left(\cos^4\varphi - \sin^4\varphi\right)\right\}$$

$$K_{Bx}(\varphi) = -\frac{3\sqrt{2}}{64}\frac{\mu_0 M}{R_1^2}\left(1 - \frac{R_1'^2}{R_1^2}\right)\left\{R_2\cos\varphi + \frac{1}{4R_1^2}R_2^3\cos^3\varphi\right\}$$

$$K_{By}(\varphi) = \frac{3\sqrt{2}}{64}\frac{\mu_0 M}{R_1^2}\left(1 - \frac{R_1'^2}{R_1^2}\right)\left\{R_2\sin\varphi + \frac{1}{4R_1^2}R_2^3\sin^3\varphi\right\}$$

where: M – vector of magnetization of unmovable magnet, R_1, R'_1 – inside and outside radius of unmovable magnet, μ_0 – magnetic permeability of vacuum. The unmovable magnet generates only component magnetic flux density in the axis Oz (Fig. 4).

The cross product for the left wall of magnet equal:

$$\vec{K}_{ml} \times \vec{B} = \frac{B_r}{\mu_0} B_z \cos\varphi \vec{i} + \frac{B_r}{\mu_0} B_z \sin\varphi \vec{j} \tag{13}$$

and right wall of magnet:

$$\vec{K}_{mr} \times \vec{B} = -\frac{B_r}{\mu_0} B_z \cos\varphi \vec{i} - \frac{B_r}{\mu_0} B_z \sin\varphi \vec{j} \tag{14}$$

where: $\vec{i}, \vec{j}, \vec{k}$ – the versos of axes Ox, Oy and Oz.

The magnetic force after integration expressions (13) and (14) is equal:

$$F_{mx} = K_{F_m} x \tag{15}$$

$$F_{my} = K_{F_m} y \tag{16}$$

where: K_{F_m} – constant of the passive magnetic bearing.

The constant of the passive magnetic bearing is equal:

$$K_{Fm} = \frac{3\sqrt{2}\pi}{64\mu_0} \frac{R_2}{R_1^2} \left(R_2^2 - R_2'^2\right) \left(1 - \frac{R_1'^2}{R_1^2}\right) \left(1 + \frac{R_2^2}{4R_1^2}\right) B_{r1} B_{r2} \tag{17}$$

where:
B_{r1}, B_{r2} – the remanence of unmovable and movable magnet,
R_2, R'_2 – inside and outside radius of movable magnet.

Next component of force (10) is depended from inductive surface current. The component of force is equal:

$$\vec{F}_{\varepsilon} = h_m \sigma \int \left(\left(\vec{v} + \vec{B} \right) \times \vec{B} \right) da \tag{18}$$

The double cross product is equal:

$$\left(\vec{v} + \vec{B} \right) \times \vec{B} = -B_z^2 v_x \vec{i} - B_z^2 v_y \vec{j} \tag{19}$$

The magnetic flux density B_z is proportional to displacement of rotor (equ. 12), so the equation (19) shows the nonlinear dependence between the displacement and speed of movable magnet. The double cross product after linearization in the point of work ($x = 0$, $y = 0$, $v_x = 0$ and $v_y = 0$) is equal:

$$\left(\vec{v} + \vec{B} \right) \times \vec{B} = -K_{B0}^2(\varphi) v_x \vec{i} - K_{B0}^2(\varphi) v_y \vec{j} \tag{20}$$

The force was generated by inductive surface current (11) and is equal to:

$$F_{\varepsilon_x} = -\sigma(R_2 - R_2')h_m K_{F_x} v_x \tag{21}$$

$$F_{\varepsilon_y} = -\sigma(R_2 - R_2')h_m K_{F_x} v_y \tag{22}$$

where: K_{F_x} – the constant of the inductive current:

$$K_{F_x} = \frac{\sqrt{2}}{16}\left(1 - \frac{R_1'^2}{R_1^2}\right)B_{r1}\left\{R_1 + \frac{9}{32}\frac{\pi^2 R_2^2}{R_1^3}\left[1 + \left(\frac{1}{2}\frac{R_2}{R_1}\right)^2 + 67\left(\frac{1}{8}\frac{R_2}{R_1}\right)^4\right]\right\} \tag{23}$$

The equation of motion of movable magnet

The move of rotor can be described by differential equations:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + cx = F_{z_x} \tag{24}$$

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + cy = F_{z_y} \tag{25}$$

The first component of equations (24, 25) describes the inertia force of mass of rotor m. The mass is reduced to the surface of the passive magnetic bearing. The next component of equation (24, 25) describes the damping force. It is proportional to the speed of rotor in the surface of the passive magnetic bearing. The damping factor of the passive magnetic bearing is equal:

$$b = \sigma(R_2 - R_2')h_m K_{F_{x_\varepsilon}} \tag{26}$$

The third component of equations (24, 25) describes the stiffness force. The force is proportional to the displacement of rotor in the surface of the passive magnetic bearing. The stiffness factor of the passive magnetic bearing is described by expression (17). The model of passive magnetic bearing is described by matrix of transfer function:

$$G(s) = \begin{bmatrix} G_{\frac{x}{f}}(s) & 0 \\ 0 & G_{\frac{y}{f}}(s) \end{bmatrix} = \begin{bmatrix} \dfrac{\frac{1}{m}}{s^2 + \frac{\sigma(R_2 - R_2')h_m K_{F_x}}{m}s + \frac{K_{F_m}}{m}} & 0 \\ 0 & \dfrac{\frac{1}{m}}{s^2 + \frac{\sigma(R_2 - R_2')h_m K_{F_x}}{m}s + \frac{K_{F_m}}{m}} \end{bmatrix} \tag{27}$$

In the picture 5 is shown the step response of the model of the passive magnetic bearing. The response is similar to response from Fig. 2. It was obtained for a dimensions and magnetic parameter as was used in the experiment from figure 2. The differences arise from gain and static characteristic of the eddy current sensor which measure displacement of rotor.

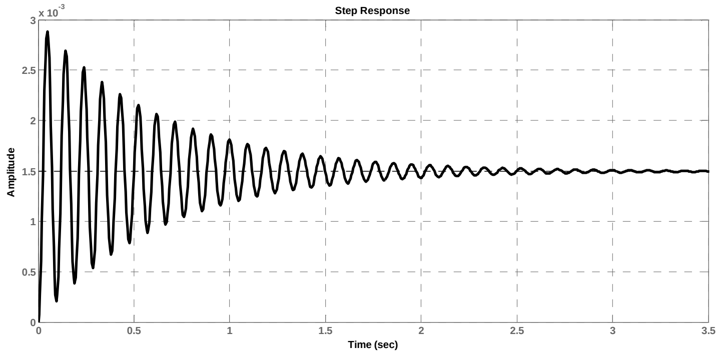

Fig. 5. The step response of the model (27) of the passive magnetic bearing

Summary

In the paper is presented reduction model of the radial passive magnetic bearing. The model of bearing was built as a matrix of transfer function. There was shown the damping factor and the stiffness factor of the passive magnetic bearing. The model assumes:

- that the movable magnet and unmovable magnet have got this same height of magnet h_m,
- the linearization of model in the point of work (it is equal $x = 0$, $y = 0$, $v_x = 0$ and $v_y = 0$).

The model is approximation of the real passive magnetic bearing. It can be used to estimate main parameters of the passive magnetic bearing. It will be useful to evaluate dynamics and static properties of the passive magnetic bearing.

This model comes down to coefficient (17) and (23). It doesn't ensure high precision of evaluation parameter of the passive magnetic bearing, only the finite elements method ensures full-up evaluation of bearing. The methodology of derivation of a model (27) can be used to obtain the model of the axial passive magnetic bearing and the passive magnetic bearing in the Halbah's array.

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