749. Damage detection in time-varying beam structures based on wavelet analysis

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Abstract. Existing system identification methods are mostly dealing with local changes that are time-invariant. However, the process of local change in a real structure caused by damage is mostly time-varying, which can be modeled with a change in the physical parameters such as structural stiffness and damping. An identification algorithm for a time-varying beam system is proposed based on wavelet analysis. The response signal is firstly decomposed using the Daubechies wavelet scaling function. The governing differential equation of a structure is then transformed into a set of linear equations based on the orthogonality property of the scaling functions in a wavelet space. Finally, the proposed algorithm is illustrated with studies on a cantilever beam structure. The precision of identification with respect to different wavelet scales is investigated and discussed. Numerical results demonstrate that the proposed method can identify smoothly, periodically and abruptly time-varying physical parameters with excellent accuracy.

Keywords: time-varying, system identification, wavelet-Galerkin, Daubechies, scaling function, orthogonality, cantilever beam.

1. Introduction

Many methods have been developed for the system identification of linear time-invariant (LTI) system [1-6]. However, the structures are more or less time-varying during the damage process when they are still under service. System parameters and dynamic characteristics of the structure also vary during the process. The LTI model cannot fully describe the time-varying dynamic characteristics of the structure with the occurrence of local damages. The associated system identification methods cannot track the variation of time-varying parameters accurately. Therefore the study on linear time-varying (LTV) systems is of great significance.

The identification of LTV systems for damaged structure has received increasing attention recently. Some commonly used algorithms include the Hilbert transform method [7-10], the state-space method [11-14], the least-squares estimation with adaptive tracking method [15-18] and the wavelet-based method [19-20]. Each of these methods has its own merits and disadvantages. The development of an exact and efficient identification approach for the time-varying system remains to be a challenging problem.

Compared with the Fourier analysis, the wavelet transform makes use of adjustable window location and size to decompose a signal and it can capture more time-localized information. A discrete wavelet analysis approach was developed later by Daubechies [21]. The wavelet methods have been applied to signal analysis with enhanced performance. With the benefits of the orthogonality properties, compact support and enriched representation of a function at different levels of resolution, the compactly supported wavelets have also been applied to the numerical solution of partial differential equations based on the Wavelet-Galerkin method [22-23].

Wavelet-based system identification approaches have been proposed in the last decade. Staszewski [24] proposed three methods to identify modal damping ratios based on the continuous wavelet transform. Ruzzene et al. [25] showed that the application of the wavelet analysis to the free responses of a multi-degrees-of-freedom system represents a good improvement for the identification of instantaneous frequencies compared with the Hilbert transform. Newland [26] used harmonic wavelet to estimate the frequency of a transient signal. Approaches to identify natural frequencies and viscous damping ratios as well as mode shapes have been developed by Argoul et al. [27-29]. The wavelet approach for system identification was based on the comparison of the natural frequencies and damping ratios obtained from analyzing two segments of data recorded before and after the damage [30]. The sensitivities of wavelet coefficient, wavelet packets and the impulse response function with respect to a local damage have also been derived for the identification of a damage structure (31-33). The identification algorithms for non-linear systems have also been developed [34]. However, all the above-mentioned approaches are only valid for the identification of LTI systems.

The application of wavelet analysis to the identification of linear time-varying systems can be found in Ghanem and Romeo [19]. Single and two-degrees-of-freedom systems are studied to demonstrate the effectiveness in tracking three different time-varying parameters. A new identification algorithm is presented in this paper to estimate the physical time-varying parameters of a beam system based on the wavelet analysis. The response signal is firstly decomposed using the Daubechies wavelet scaling function. The governing differential equations are then projected into a subspace spanned by a finite number of wavelets. With the orthogonality property of the scaling functions, the differential equations are simplified into a set of linear algebraic equations. All the physical parameters can then be identified by solving the set of simultaneous equations. Section 2 introduces briefly the basic properties of the Daubechies wavelet. Section 3 develops the identification algorithm for estimating the time-varying physical parameters of a beam structure. Section 4 presents numerical examples to illustrate the procedure of the proposed identification algorithm and demonstrate its effectiveness. Sections 5 and 6 provide the discussion and conclusions respectively.

2. Basic introduction to Daubechies wavelet

The Daubechies wavelet is briefly introduced before the presentation of the proposed method. Wavelet is a family of orthonormal functions characterized by the translation and dilation (scale) of the basis wavelet function $\psi(\bullet)$. This family of functions, denoted by $\psi_{j,k}(x)$, can be written as:

$$\psi_{j,k}(x) = 2^{\frac{j}{2}}\psi\left(2^j x - k\right) \tag{1}$$

where 2^j is the scale parameter and k is the translation parameter.

The wavelet $\psi(x)$ and the associated scaling functions $\phi(x)$ satisfy the following recursive relations as:

$$\begin{cases} \psi(x) = \sum_k (-1)^k \alpha^{1-k}\phi(2x-k) \\ \phi(x) = \sum_k \alpha^k \phi(2x-k) \end{cases} \tag{2}$$

in which a finite number of the filter coefficients α^k and α^{1-k} are nonzero.

The translation of the scaling function $\phi(x)$ is orthonormal for any k and m parameters with

$$\int \varphi(x-k)\varphi(x-m)\,dx = \delta_{k,m}. \tag{3}$$

The functions derived from the translation and dilation of the wavelet function $\psi_{j,k}(x)$ form a space of complete and orthogonal basis for the square integrable functions $L^2(R)$. Any $L^2(R)$ function $f(x)$ may be approximated at resolution level j by:

$$P_j f(x) = \sum_k \alpha_{j,k}\phi_{j,k}(x) \tag{4}$$

where $\alpha_{j,k}$ is a vector for resolution level j and translation k. $P_j f(x)$ represents the projection of the function $f(x)$ onto the space of scaling functions at resolution level j with:

$$\phi_{j,k}(x) = 2^{\frac{j}{2}}\phi\left(2^j x - k\right) \tag{5}$$

where $\phi_{j,k}(x)$ is a scaling function basis for the scale j approximation of $L^2(R)$. The set of approximations $P_j f(x)$ constitutes a multi-resolution representation of the function $f(x)$.

3. Identification technique for a beam structure

The identification equation for a beam structure is proposed in this section. A time-varying planar beam structure is modeled with finite element method. The equation of motion of the n degrees-of-freedom system can be expressed as:

$$M(t)\ddot{x}(t) + C(t)\dot{x}(t) + K(t)x(t) = f(t) \tag{6}$$

in which $M(t)$, $C(t)$, $K(t)$ are $n \times n$ time-dependant mass, damping and stiffness matrices of the system respectively. $f(t)$ is a $n \times 1$ excitation vector, and $x(t)$, $\dot{x}(t)$, $\ddot{x}(t)$ are $n \times 1$ displacement, velocity and acceleration vectors respectively. The system matrices can be expressed as:

$$\begin{aligned} M(t) &= \sum_e M^e(t) = \sum_e \rho_e A_e(t) M_C^e \\ K(t) &= \sum_e K^e(t) = \sum_e E_e(t) I_e(t) K_C^e \\ C(t) &= \sum_e \left(\beta_M M^e(t) + \beta_K K^e(t)\right) \end{aligned} \tag{7}$$

where ρ_e is the mass density, $A_e(t)$, $I_e(t)$ and $E_e(t)$ are the time-dependent cross-sectional area, moment of inertia of the beam cross-section and modulus of elasticity of material in each element respectively. $M^e(t)$ and $K^e(t)$ are respectively the time-varying mass and stiffness matrices of each element. $C(t)$ is the viscous proportional damping matrix with proportional coefficients β_M and β_K. The consistent matrices M_C^e and K_C^e of a planar beam element can be written as:

$$M_C^e = \frac{L}{420}\left[\begin{array}{cc:cc} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ \hdashline 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{array}\right] = \begin{bmatrix} M_{11}^e & M_{12}^e \\ M_{21}^e & M_{22}^e \end{bmatrix} \tag{8a}$$

$$K_C^e = \frac{1}{L^3}\left[\begin{array}{cc:cc} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ \hdashline -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{array}\right] = \begin{bmatrix} K_{11}^e & K_{12}^e \\ K_{21}^e & K_{22}^e \end{bmatrix} \tag{8b}$$

in which L is the length of each element. Only the flexural rigidity $EI_e(t)$ of each element is taken as the unknown time-varying parameter in this study. Substituting Equations (7) and (8) into Equation (6), the free vibration equation of the beam structure can be re-arranged as:

$$M[\ddot{x}(t) + \beta_M \dot{x}(t)] + K_C[x_C(t) + \beta_K \dot{x}_C(t)](EI)_S(t) = 0 \tag{9}$$

in which:

$$(EI)_s(t) = (EI_1(t) \quad EI_2(t) \quad \cdots \quad EI_{NE}(t))^T \tag{10a}$$

$$K_C = \sum_e K_C^e = \begin{bmatrix} K_{11}^1 & K_{12}^1 & & & & & & 0 \\ K_{21}^1 & K_{22}^1 & K_{11}^2 & K_{12}^2 & & & & \\ & & K_{21}^2 & K_{22}^2 & K_{11}^3 & K_{12}^3 & & \\ & 0 & & & \vdots & \vdots & \vdots & \vdots \end{bmatrix}_{n\times(2n-4)} \tag{10b}$$

where NE is a number of elements, and $x_C(t)$ and $\dot{x}_C(t)$ are the displacement and velocity matrices respectively denoted by:

$$x_C(t) = \\ = \begin{bmatrix} x_1(t) & x_2(t) & x_3(t) & x_4(t) & & & & & & & 0 \\ & & x_3(t) & x_4(t) & x_5(t) & x_6(t) & & & & & \\ & & & & x_5(t) & x_6(t) & x_7(t) & x_8(t) & & & \\ & & 0 & & & & & & \vdots & \vdots & \vdots & \vdots \end{bmatrix}^T_{NE\times(2n-4)} \tag{11a}$$

$$\dot{x}_C(t) = \\ = \begin{bmatrix} \dot{x}_1(t) & \dot{x}_2(t) & \dot{x}_3(t) & \dot{x}_4(t) & & & & & & & 0 \\ & & \dot{x}_3(t) & \dot{x}_4(t) & \dot{x}_5(t) & \dot{x}_6(t) & & & & & \\ & & & & \dot{x}_5(t) & \dot{x}_6(t) & \dot{x}_7(t) & \dot{x}_8(t) & & & \\ & & 0 & & & & & & \vdots & \vdots & \vdots & \vdots \end{bmatrix}^T_{NE\times(2n-4)} \tag{11b}$$

with the subscripts 1, 2, 3, … etc. denoting the degree-of-freedom of the system. With the use of compactly supported Daubechies wavelets, any square integrable function $f(x)$ can be approximated at resolution level j as in Equations (4) and (5). The response $x(t)$ can then be rewritten with wavelet presentation as:

$$x(t) = \sum_k \tilde{\alpha}_k 2^{\frac{j}{2}} \phi(2^j t - k) \tag{12}$$

where $\tilde{\alpha}_k$ is a coefficient vector which can be computed with the periodic assumption [22]. The time derivatives of $x(t)$ can be expressed in terms of the corresponding time derivatives of the wavelet function as:

$$\dot{x}(t) = 2^j \sum_k \tilde{\alpha}_k 2^{\frac{j}{2}} \dot{\phi}\left(2^j t - k\right), \quad \ddot{x}(t) = 2^{2j} \sum_k \tilde{\alpha}_k 2^{\frac{j}{2}} \ddot{\phi}\left(2^j t - k\right) \tag{13}$$

Substitute Equations (12) and (13) into Equation (9) and let $y = 2^j t$ and $\alpha_k = 2^{\frac{j}{2}} \tilde{\alpha}_k$. Taking the inner product of both sides of Equation (9) with $\phi(y-l)$, and with the orthogonality property of the translation of the scaling functions, Equation (9) is projected from the physical space into the wavelet space and is written as:

$$M_l\left(2^{2j} \sum_k \alpha_k \Gamma_{l-k}^{(2)} + 2^j \beta_M \sum_k \alpha_k \Gamma_{l-k}^{(1)}\right) + K_C\left(\alpha_{C,l} + 2^j \beta_K \sum_k \alpha_{C,k} \Gamma_{l-k}^{(1)}\right)(EI)_{s,l} = 0 \tag{14}$$

for the l-th time instance with $k = \left[l - 2N + 2, l - 2N + 3, \cdots, l + 2N - 3, l + 2N - 2\right]$, $l = 0,1,2,\cdots, y - 1$,

$$\Gamma_{l-k}^{(1)} = \int \dot{\phi}(y-k)\phi(y-l)dy; \quad \Gamma_{l-k}^{(2)} = \int \ddot{\phi}(y-k)\phi(y-l)dy \tag{15}$$

in which $\Gamma_{l-k}^{(1)}$ and $\Gamma_{l-k}^{(2)}$ are the connection coefficients described by Latto et al. [35], and N denotes the number of vanishing moments for the particular class of Daubechies wavelets. It is noted that Equation (14) is a set of n simultaneous equations for time instance t. Assuming the mass of the system is known and time-invariant, Equation (14) can be rewritten in matrix form for all the time instances as:

$$T\theta = B \tag{16}$$

For the generic l-th time instance in Equation (14), matrices T and B are given by:

$$\begin{aligned} T_l &= K_C\left(\alpha_{C,l} + 2^j \beta_K \sum_k \alpha_{C,k} \Gamma_{l-k}^{(1)}\right) \\ \theta_l &= (EI)_{S,l} \\ B_l &= -M_l\left(2^{2j} \sum_k \alpha_k \Gamma_{l-k}^{(2)} + 2^j \beta_M \sum_k \alpha_k \Gamma_{l-k}^{(1)}\right) \end{aligned} \tag{17}$$

The set of simultaneous Equation (16) is the identification equation for the unknown parameters. For the l-th time instance, Equation (16) gives the information on the system within the time interval from l-2N+2 to l+2N+2 time instances. Thus, the evolution of the parameters over the duration associated with each time interval can be estimated by means of the above algorithm. For a linear time-invariant system, Equation (16) can be used in different combinations to estimate the unknown parameters, which are assumed constant during the whole duration of analysis.

This identification algorithm can also be used to identify any linear time-varying system with an assumption that the time-varying physical parameters remain constant within the time interval. Sequential arrays of Equation (16) can be obtained for the sequence of time instances to constitute an over-determined set of equation for a least-squares solution with the estimated parameters assumed to be time-invariant over the time interval under consideration.

4. Simulation results

Four examples are investigated to identify the time-varying parameters for a cantilever beam as shown in Fig. 1. The geometrical and physical parameters are: length of beam $L = 1.0\text{m}$, height $H = 0.005\text{m}$, breadth $W = 0.01\text{m}$, mass density $\rho = 7800\text{kg/m}^3$. The proportional damping coefficients are $\beta_M = \beta_K = 0.001$. The beam section has an original flexural rigidity of $EI = 20.834\text{Nm}^2$. The first eight modal frequencies of the beam are: 4.09, 25.632, 71.787, 140.77, 233.07, 349.16, 489.89 and 656.32 Hz respectively. The cantilever beam is divided into ten equal length elements. Only the flexural rigidity of each element is assumed as time-varying parameters. A support excitation in the vertical direction is simulated with an initial velocity and acceleration at the supporting node as $\dot{x}_0 = 1.0\text{m/s}$, $\ddot{x}_0 = 1.0\text{m/s}^2$. The response of the beam structure is obtained from numerical solution of the equation of motion using the Newmark-beta method.

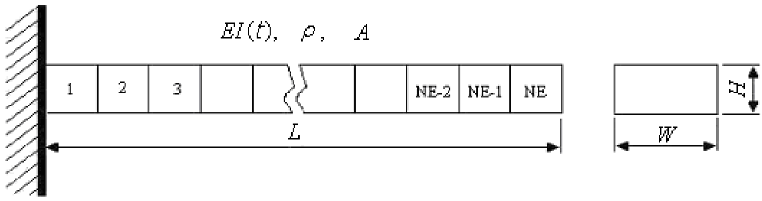

Fig. 1. Model of cantilever beam

Example One: Smooth change of flexural rigidity in a single element.

Suppose the flexural rigidity of the 1-st element changes with time as $EI_1(t) = EI(1 - 0.2t^2)\text{Nm}^2$ and the other elements retain their original flexural stiffness values. Two seconds of free vibration response from all the degrees-of-freedom of the system are used in the identification. The number of sampled data varies depending on the resolution. If $j = 9$ and $t = 2\text{s}$, then $y = 2^j t = 1024$, and there are 1024 time instances for the analysis indicating a sampling rate of 512 Hz. The dB3 Daubechies wavelet with three vanishing moments is used, i.e. $N = 3$. For each of the time instance, we have 22 numbers of equations in Equation (16), which is more than the 10 number of unknown flexural rigidity. The identification result on $EI_1(t)$ for $j = 9$ is represented by the dotted line in Fig. 2 alongside with the true values for comparison. The estimated results have a good tracking behavior on the smooth change of the 1-st elemental flexural rigidity.

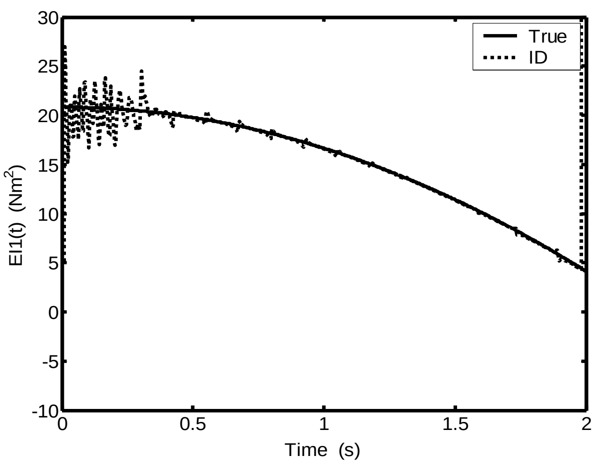

Fig. 2. Comparison of identified result and true value for Example 1

Example Two: Periodical change of flexural rigidity in a single element.

A periodical change in the 1st elemental flexural rigidity of the beam structure is modeled as $EI_1(t) = EI[1-0.6\cos(8\pi t)]\,\mathrm{Nm}^2$. Other elemental flexural rigidities are assumed time-invariant at their original values. The unknown flexural rigidities of the beam elements are identified in the same way as for the preceding example from two seconds of measured data. The identified value of $EI_1(t)$ obtained for j = 9 is compared with the true value in Fig. 3. Results indicate that the proposed identification algorithm can track the periodical change of flexural rigidity of the beam structure during the whole time duration. Fig. 4 shows the error of identification, defined as $\left\|EI^{True} - EI^{ID}\right\| / \left\|EI^{True}\right\| \times 100\%$, in the estimated flexural rigidity of the 1-st element when using three different wavelet resolutions of j = 7, 8 and 9 to analyze the response data. Higher resolution is noted to give higher identification accuracy.

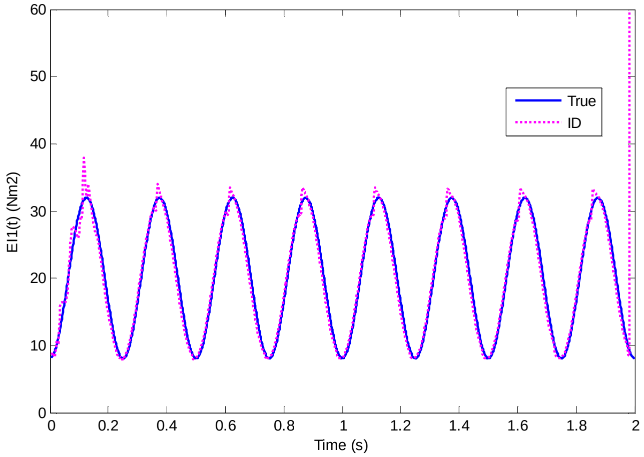

Fig. 3. Comparison of identified result and true value for Example 2

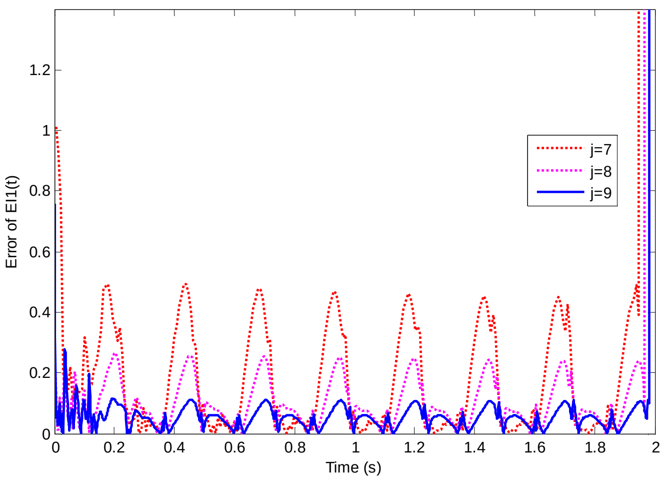

Fig. 4. Identification error versus resolution and time for Example 2

Example Three: Abrupt change of flexural rigidity in multiple elements.

The abrupt change in multiple elemental flexural rigidities of the cantilever beam is further investigated to evaluate the ability of the proposed algorithm to detect multiple damages. The flexural rigidity of the 1-st, 2-nd and 6-th elements are assumed to change abruptly at $t = 1.0$s from the original value of 20.834Nm^2 to new values of 10.417Nm^2, 12.5004Nm^2 and 16.6672Nm^2 respectively. The flexural rigidities of other elements are assumed constant at their original values. The identification is conducted in the same way with $j = 9$ as for the last two examples. Fig. 5 shows the estimated flexural rigidity of the 1-st, 2-nd and 6-th elements respectively alongside the true values. The proposed algorithm is demonstrated to be capable of tracking the abrupt changes of flexural rigidities in the three elements of the beam structure.

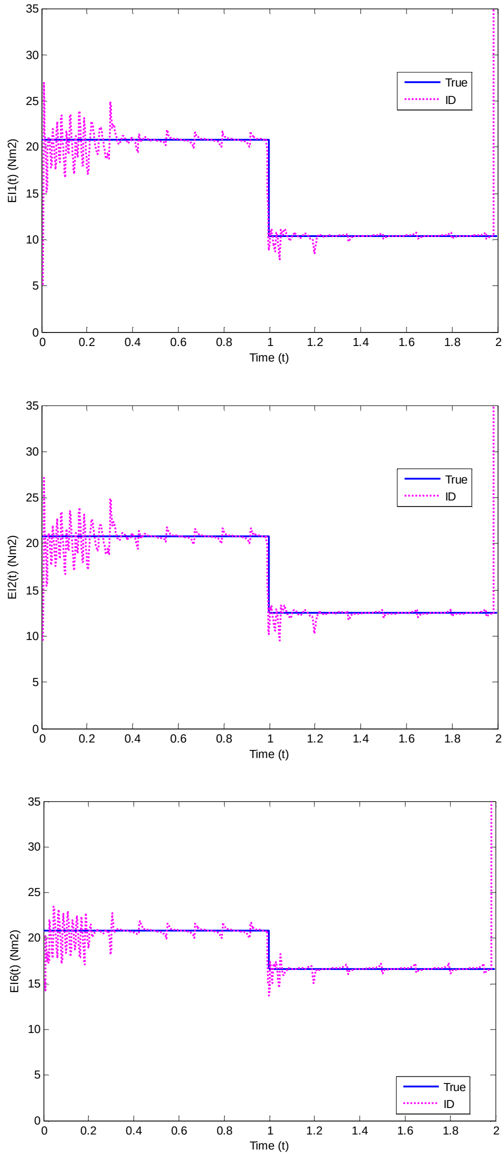

Fig. 5. Comparison of identified results and true values for Example 3

Example Four: Abrupt and periodical changes of flexural rigidity in multiple elements.

The flexural rigidities of the 1-st, 2-nd and 6-th elements change abruptly at $t = 0.6\text{s}$, and then vary as a periodic process. The expressions to describe these abrupt and periodic changes are denoted as:

$$\begin{cases} EI_1(t) = EI, & t \le 0.6 \\ EI_2(t) = EI, & t \le 0.6, \\ EI_6(t) = EI, & t \le 0.6 \end{cases} \quad \begin{cases} EI_1(t) = EI\left[1 - 0.5\cos(8\pi t)\right] \mathrm{Nm}^2, & t > 0.6 \\ EI_2(t) = EI\left[1 - 0.6\cos(8\pi t)\right] \mathrm{Nm}^2, & t > 0.6 \\ EI_6(t) = EI\left[1 - 0.8\cos(8\pi t)\right] \mathrm{Nm}^2, & t > 0.6 \end{cases}$$

The flexural rigidities of other elements are constant at their original values. The unknown parameters to be identified are the time-varying flexural rigidities of all the elements in the structure, same as those for Example 3. The identification is performed in a similar way with $j = 9$ as for the last example. The estimated results are given in Fig. 6, and they are noted to be very close to the true values. The error of identification on the unknown parameters from using different resolution in the analysis with $j = 7$, 8 and 9 are shown in Fig. 7. Higher resolution is noted once more to be responsible for a higher accuracy in the identified results.

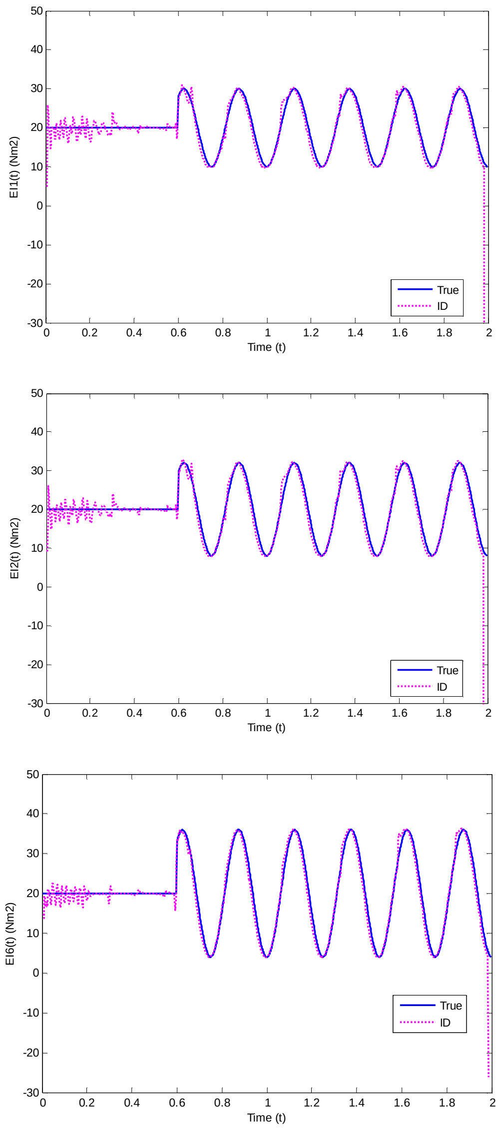

Fig. 6. Comparison of identified results and true values for Example 4

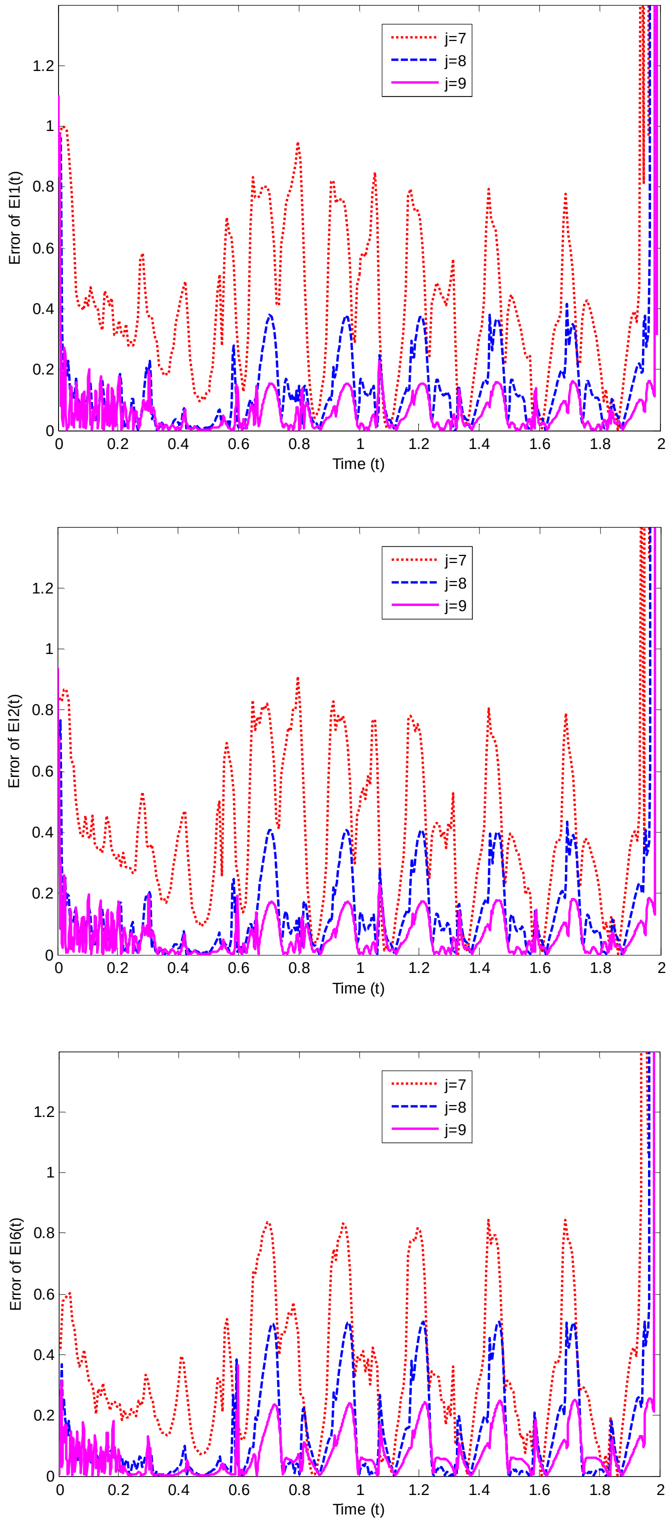

Fig. 7. Identification error versus resolution and time for Example 4

5. Discussions

(1) A resolution scale j equal to 9, implying a sampling rate of 512 Hz, is adopted in the conducted analysis. The estimated results demonstrate that such a low sample rate is sufficient to estimate the unknown physical parameters including the first five modal frequencies. Resolution scale $j < 9$ implies a sampling rate smaller than 512 Hz, which would result in a smaller number of modal responses captured and would lead to a larger identification error as noted in Figures 4 and 7 for examples 2 and 4.
(2) In all of the examples, there is no false alarm in the identified results for all other elements with time-invariant flexural rigidity, while small fluctuations occur at intervals in all the time histories of identified results of all elements. These time instances correspond to the moments when the response of the whole structure is at zero crossing, which indicates that larger errors may occur by using response data close to zero time. The structure undergoing the present type of excitation has a single dominating response frequency leading to such a phenomenon. That means such error will be reduced or will not exist if the responses are not at zero crossing simultaneously.
(3) The error of identification in the whole time period is very small except at the beginning and end of the time duration, where there are large fluctuations. This may be due to the fact that the wavelet representation of the response in the first and last few data points doesn't satisfy the periodic assumption as required in Equation (12). Further research is required to reduce this type of error.
(4) The proposed identification algorithm requires measurements of displacement responses theoretically, while most practical applications require acceleration responses for the identification. Several methods [36] have been developed to reduce the numerical integration error from acceleration to displacement. The development of the present algorithm with direct use of acceleration responses should be further studied.
(5) Results not shown indicate that small local changes of 2~3 % can still be identified using the proposed approach. However, when normal random noise is included in the responses, the proposed method does not perform satisfactorily. This is because the solution is based on least-squares method and the random error in the measurement remains in the identified results as errors in the identified time histories. The solution could be obtained using an over-determined set of equations with regularization on the solution [37].
(6) The present study makes use of the whole measurement for the identification. For the case of incomplete measurement, the number of equation in Equation (14) will be reduced with the number of measurement, nn, which may not be sufficient to identify a large number of unknowns. This problem can be overcome by assuming the characteristics of the system to be time-invariant over several consecutive time instances, mm, and the linear equations from Equation (14) for the mm time instances can be grouped up to form a set of $nn \times mm$ simultaneous equations in Equation (16) for a least-squares solution. A higher sampling rate would provide sufficient information from a few sensors to form the over-determined set of Equation (16).

6. Conclusions

This paper addresses the identification of time-varying physical parameters from the forced vibration response of beam structures based on wavelet transform. The use of incomplete measurement in the proposed method can be realized with the use of a high sampling rate. A parameter identification algorithm is established based on the Wavelet-Galerkin method, and the complete identification procedure is demonstrated using four scenarios with a cantilever beam. Simulation results indicate that the proposed technique has an excellent capability for tracking smooth, periodic and abrupt changes of parameters in the beam structure.

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