781. Synthesis as designing of mechatronic vibrating mixed systems

Andrzej Buchacz[1], Damian Gałęziowski[2]
Silesian University of Technology, Gliwice, Poland
E-mail: [1]*andrzej.buchacz@polsl.pl*, [2]*damian.galeziowski@interia.pl*

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Abstract. In this paper synthesis of mechatronic vibrating mixed systems has been studied. Mechatronic structures have been built from mechanical discrete systems connected to piezoelectric actuator and external L_x R_x C_x network. By use of non-dimensional transformations mechatronic mixed structures (branched and cascade part), which must comply with the dynamical properties in the form of frequency spectrum: resonant and anti-resonant frequencies, have been received. The problem has been presented on few different examples of four degrees of freedom.

Keywords: piezo, synthesis, mechatronics systems, vibration.

Introduction

In this research work mechatronic structures have been considered as mechanical discrete systems connected to piezoelectric actuator and external L_x R_x C_x network. Piezo actuator is in the form of piezo stack, monolithic ceramic construction of many thin piezoceramics layers which are connected electrically in parallel. The small motions of each layer contribute to overall displacements. The main characteristics of the stack can be described as follows: low voltage operation, huge energy conversion efficiency, a large force and a fast response.

By different configuration of external electric network, passive (Fig. 1A) or semi-active (Fig. 1B) damping can be performed [1, 2].

Piezoelectric actuator is modeled in mechanical part by the stiffness c_{pm} and in electrical part by capacitance C_{ps}, which has been shown in Fig. 1.

Voltage on the electrodes of piezostack u_p and current flowing through the circuit i_p have been found as the following functions:

$$u_p = f(e, F_{pe}), \; i_p = f(e, \dot{x}). \tag{1}$$

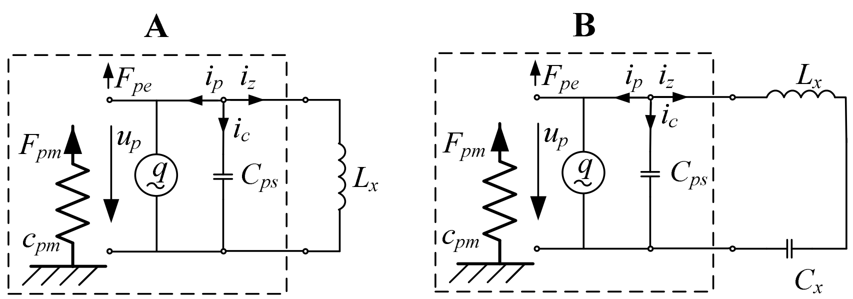

Fig. 1. Examples of different configurations of piezostack connected to electric network

Parameters of piezoelectric material have been determined from commonly known piezoelectric equations:

$$\sigma = K_E s - eE, \quad D = es - \varepsilon_s E, \tag{2}$$

where: σ – mechanical stress, K_E – Young Modulus, s – mechanical strain, D – electrical displacement, ε_s – electrical permittivity, E – electrical field, e – piezoelectric constant.

Problem of mechanical and electrical systems synthesis is well known [3, 4]. However, due to ambiguity of transformation from theoretical model to the real one Gliwice Research Center had started at the beginning of the 80's to study synthesis and designing of mechanical continuous [5], discrete [6] and continuous-discrete systems [7].

Based on [1-7] and previous authors' papers, which present methodology of using reverse task as a designing of mechatronic structures [8-10], this work is an extension of current achievements for structures containing branched and cascade parts. Requirements in each case have been defined as dynamical properties in the form of frequencies spectrum: resonant and anti-resonant frequencies. By use of non-dimensional transformation and retransformation [11], all parameters of received mechatronic structures has been physically executed and shown on different examples.

Synthesis as designing of mechatronic mixed structures – passive damping

As shown in the Fig. 1A, passive damping can be represented by the piezo element connected to external electric network only with inductance L_x.

System must comply with dynamical properties in form of:

- resonant frequencies ω_1, ω_3, ω_5, ω_7 in rad/s,
- anti-resonant frequencies ω_0, ω_2, ω_4, ω_6 in rad/s.

By using mixed method of synthesis, dynamical characteristic has been in first step distributed into contiunous fraction and then distributed to partial fraction. Slowness function has been written in the form of:

$$U(s) = \frac{c_1}{s} + m_1 s + \cfrac{1}{\cfrac{s}{c_2} + \cfrac{1}{\cfrac{c_3}{s} + m_2 s + \cfrac{1}{\cfrac{1}{\cfrac{s}{c_4} + m_3 s} + \cfrac{1}{\cfrac{s}{c_5} + \cfrac{1}{m_4 s + \cfrac{s}{c_6}}}}}} \tag{3}$$

where: c_i – mechanical spring [N/m], m_i – inertial elements [kg].

By use of non-dimensional transformation:

$$\tau = \omega_2 t, \quad \ddot{x}_i = \omega_2 x_i'', \quad \dot{x}_i = \omega_2 x_i', \quad i = 1...4 \tag{4}$$

and parameters defined as:

$$\omega_1^2 = \frac{c_1}{m_1}, \ \ \omega_2^2 = \frac{c_2}{m_2}, \ \ \omega_3^2 = \frac{c_3}{m_3}, \ \ \omega_4^2 = \frac{c_4}{m_4}, \ \ \gamma = \frac{c_2}{c_1}, \ \ \alpha_1 = \frac{c_3}{c_2}, \ \ \alpha_2 = \frac{c_4}{c_2}, \ \ \alpha_1 = \frac{c_5}{c_2}, \tag{5}$$

$$\beta = \frac{c_4}{c_3}, \quad \chi_1 = \frac{c_5}{c_4}, \quad \chi_2 = \frac{c_6}{c_4}, \quad F(t) = F_0 \cos(\Omega t), \quad \eta = \frac{\Omega}{\omega_2}, \quad x_0 = \frac{F_0}{c_1}, \tag{6}$$

non-dimensional replacement model has been defined as:

$$\begin{bmatrix} 1+\gamma-\eta^2 & -\gamma & 0 & 0 \\ -1 & \begin{matrix}1+\alpha_1+\\+\alpha_2+\alpha_3-\eta^2\end{matrix} & -\alpha_2 & -\alpha_3 \\ 0 & -\beta & \beta-\eta^2 & 0 \\ 0 & -\chi_1 & 0 & \chi_1+\chi_2-\eta^2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}+\begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 \\ 0 & 0 & 0 & \lambda_3 \end{bmatrix}\begin{bmatrix} x_1'' \\ x_2'' \\ x_3'' \\ x_4'' \end{bmatrix}=\begin{bmatrix} 0 \\ x_0\cos(\eta\tau) \\ 0 \\ 0 \end{bmatrix}. \quad (7)$$

Electric charge has been found from (1), (2) as:

$$q_p = e\frac{A_p}{l_p}x_1 + \varepsilon_s\frac{A_p}{l_p}u_p, \quad (8)$$

where: A_p – surface area of electrodes, l_p – height of piezoceramics, x_1 – displacement.

It is known that piezoelectric constant e has been defined as:

$$e = K_E d_{33}, \quad (9)$$

where: d_{33} – charge density per unity stress under constant electric field, K_E – Young modulus.

The capacitance of piezo C_{ps} has been described as:

$$C_{ps} = \varepsilon_s\frac{A_p}{l_p}. \quad (10)$$

The stiffness c_{pm} and c_3 have been found as:

$$c_{pm} = K_E\frac{A_p}{l_p},\ c_3 = c_0 + c_{pm}. \quad (11)$$

Defining additional variable:

$$x_1 = x_2 + u_p\frac{\varepsilon_s}{e}, \quad (12)$$

knowing (1), (2) and by use of (4-6), it is possible to determine inductance of external network:

$$L_x = \frac{\lambda_2}{C_{ps}\omega_2^2}[\mathrm{H}]. \quad (13)$$

In this case the simplest configuration of external network is possible to perform.

Synthesis as designing of mechatronic mixed structures – semi active damping

To receive structure with semi-active damping function, having the same slowness function (3), stiffness c_7 has been added to mass m_3 and fixed to the ground.

Due to additional stiffness element c_7, to parameters (4-6), it is mandatory to add and change:

$$\beta_1 = \frac{c_4}{c_3}, \quad \beta_2 = \frac{c_6}{c_2}, \quad \chi_1 = \frac{c_5}{c_4}, \quad \chi_2 = \frac{c_7}{c_4}. \tag{14}$$

After dimensionless transformation and retransformation, which should be done with the same algorithm as presented in previous example, mechatronics equations of the system have been written in matrix form as:

$$\begin{bmatrix} \left(\frac{C_{ps}}{C}+1\right)\frac{\varepsilon_s}{e} & \frac{C_{ps}}{C} & 0 & 0 \\ -\frac{\varepsilon_s}{e}c_2 & c_2+c_3+c_4+c_5 & -c_4 & -c_5 \\ 0 & -c_4 & c_4+c_6 & 0 \\ 0 & -c_5 & 0 & c_5+c_7 \end{bmatrix} \begin{bmatrix} u_p \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \\ + \begin{bmatrix} LC_{ps}\frac{\varepsilon_s}{e} & LC_{ps} & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{bmatrix} \begin{bmatrix} u_p \\ \ddot{x}_2 \\ \ddot{x}_3 \\ \ddot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 \\ F(t) \\ 0 \\ 0 \end{bmatrix} \tag{15}$$

The inductance and additional capacitance element in electric network have been calculated like previously from (1), (2), (4-6) and (14):

$$L_x = \frac{\lambda_1}{C_{ps}\omega_2^2}[\mathrm{H}], \quad C_x = \frac{C_{ps}}{\gamma}[\mathrm{F}]. \tag{16}$$

In comparison to the previous model, additional capacitor C_x has been found.

Synthesis as designing of mechatronic mixed structures – with damping proportional to inertial element

To obtain a structure with damping proportional to inertial element, to the passive model L_x, damping element d_1, proportional to mass m_3 has been added:

$$d_1 = 2hm_3, \tag{17}$$

where: h = idem – parameter which describes damping in the system [rad/s], with range of:

$$0 < h < |\omega_{\min}|, \ |\omega_{\min}| \neq 0. \tag{18}$$

For this structure it is important to add to (4-6) further parameter:

$$2D = \frac{d_1\omega_2}{c_4}. \tag{19}$$

This allows us to write dynamical equation in dimensionless matrix form as:

$$\begin{bmatrix} 1+\gamma-\eta^2 & -\gamma & 0 & 0 \\ -1 & 1+\alpha_1+\alpha_2+\alpha_3-\eta^2 & -\alpha_2 & -\alpha_3 \\ 0 & -\beta & \beta-\eta^2 & 0 \\ 0 & -\chi_1 & 0 & \chi_1+\chi_2-\eta^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 \\ 0 & 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} x_1'' \\ x_2'' \\ x_3'' \\ x_4'' \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2D & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{bmatrix} = \begin{bmatrix} 0 \\ x_0\cos(\eta\tau) \\ 0 \\ 0 \end{bmatrix}. \quad (20)$$

and again, basing on (1), (2), (4-6) and (19), parameters of external network have been written as:

$$L_x = \frac{\lambda_2}{C_{ps}\omega_2^2}[\mathrm{H}], \quad R_x = \frac{d_1}{C_{ps}c_4}[\Omega]. \quad (21)$$

In comparison to the first model, additional resistant element R_x has been found.

Mechatronic structures models

In each case, mechanical replacement model has been transformed to mechatronic form. These structures have been presented in Fig. 2, and all of them are corresponding to the same requirements in form of resonant and anti-resonant frequencies.

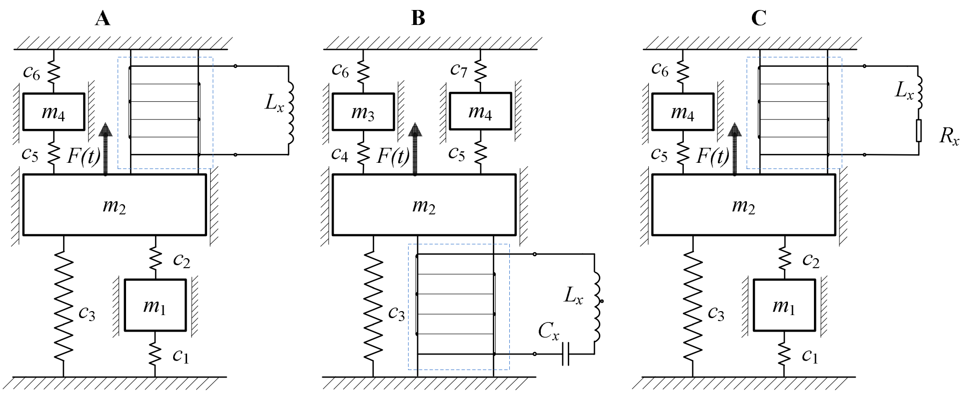

Fig. 2. Mechatronic structures: A – with passive damping "*L*", B – with semi-active damping "*LC*", C – with damping proportional to inertial element – passive "*LR*"

Conclusions

The synthesis can be one of the methods used for designing of mechatronic vibrating mixed structures. These structures can be built from mechanical discrete models connected to piezo actuators with external electric network. What is important and shown in this paper, synthesis leads to various structures with different configuration of L_x R_x C_x and different type of damping for the same requirements given.

By use of non-dimensional transformation and retransformation, physical parameters of the system can be easily derived.

It is possible to design more complex structures with more degrees of freedom and more piezoelectric actuators. In next research works further investigations will be done: range of requirements for cascade, branched and mixed structures for applications, calculation examples.

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