819. Nonlinear vibration of fluid-conveying double-walled carbon nanotubes under random material property

Tai-Ping Chang
National Kaohsiung First University of Science and Technology, Kaohsiung, Taiwan, R. O. C.
E-mail: *tpchang@ccms.nkfust.edu.tw*

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Abstract. In this study, one performs the stochastic dynamic analysis of nonlinear vibration of the fluid-conveying double-walled carbon nanotubes (DWCNTs) by considering the effects of the geometric nonlinearity and the nonlinearity of van der Waals (vdW) force. Based on the Hamilton's principle, the nonlinear governing equations of the fluid-conveying DWCNTs are derived. In order to truly describe the random material properties of the DWCNTs, the Young's modulus of elasticity of the DWCNTs is assumed as stochastic with respect to the position. By adopting the perturbation technique, the nonlinear governing equations of the fluid-conveying can be decomposed into two sets of nonlinear differential equations involving the mean value of the displacement and the first variation of the displacement separately. Then one uses the harmonic balance method in conjunction with Galerkin's method to solve the nonlinear differential equations successively. Some statistical dynamic response of the DWCNTs such as the mean values and standard deviations of the amplitude of the displacement are calculated. It is concluded that the mean value and standard deviation of the amplitude of the displacement increase nonlinearly with the increase of the frequencies for both cases of coupling between longitudinal displacement and transverse displacement and uncoupling between them. However, the coefficients of variation of the amplitude of the displacement remain almost constant and stay within certain range with respect to the frequency. The calculated stochastic dynamic response plays an important role in estimating the structural reliability of the DWCNTs.

Keywords: nonlinear vibration, fluid-loaded double-walled carbon nanotubes, random material properties, Galerkin's method.

Introduction

A landmark paper regarding Carbon nanotubes (CNTs) by Iijima [1] has attracted worldwide attention due to their potential use in the fields of chemistry, physics, nano-engineering, electrical engineering, materials science, reinforced composite structures and construction engineering. Carbon nanotubes (CNTs) are used for a variety of technological and biomedical applications including nanocontainers for gas storage and nanopipes conveying fluids [2-4]. The single-elastic beam model [5-6] were widely adopted to study the dynamic behaviors of fluid-conveying single-walled carbon nanotubes (SWCNTs) and multi-walled carbon nanotubes (MWCNTs). Normally speaking, the beam models mentioned above are linear; however, the vdW forces in the interlay space of MWCNTs are essentially nonlinear. Furthermore, the slender ratios are normally large if the beam models are adopted, that is, the large deformation will occur. Therefore, it is quite essential to consider two types of nonlinear factors, namely, the geometric nonlinearity and the nonlinearity of vdW force in investigating the dynamic behaviors of fluid-conveying MWCNTs. Salvetat et al. [7] measured the flexural Young's modulus and shear modulus using AFM test on clamped–clamped nanoropes, getting values with 50 % of error. Information related to statistical distributions of experimental data is also rare, and the important study from Krishnan et al. [8] provides one of the few examples available of histogram distribution of the flexural Young's modulus derived from 27 CNTs. The Young's modulus was estimated observing free-standing vibrations at room temperature using transmission electro-microscope (TEM), with a mean value of 1.3 TPa -0.4 Tpa / +0.6 TPa.

Uncertainty is also associated to the equivalent atomistic-continuum models adopted extensively in particular by the engineering and materials science communities. Therefore, to be realistic, the Young's modulus of elasticity of carbon nanotube (CNTs) should be considered as stochastic with respect to the position to actually describe the random property of the CNTs under certain conditions. In the present study, we investigate the stochastic dynamic behaviors of nonlinear vibration of the double-walled carbon nanotubes (DWCNTs) conveying fluid by considering the effects of the geometric nonlinearity and the nonlinearity of van der Waals (vdW) force. Based on the Hamilton's principle, the nonlinear governing equations of the fluid-conveying double-walled carbon nanotubes are formulated. Two different cases of nonlinearity are considered; case one is to include the coupling between the longitudinal displacement and transverse displacement of the DWCNTs, on the other hand, case two is to neglect the coupling between them. The Young's modulus of elasticity of the DWCNTs is considered as stochastic with respect to the position to actually characterize the random material properties of the DWCNTs.

Nonlinear beam model for fluid-loaded DWCNTs

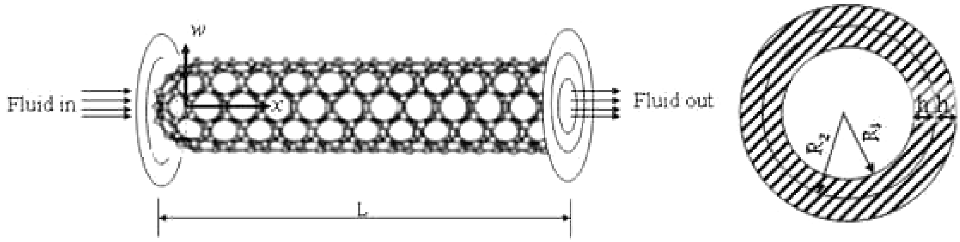

Fig. 1. Fluid-loaded double-walled carbon nanotubes

In Fig. 1, the double-walled carbon nanotubes (DWCNTs) are modeled as a double-tube pipe which is composed of the inner tube of radius R_1 and the outer tube of radius R_2. The thickness of each tube is h, the length is L, and Young's modulus of elasticity is E. It is noted that the Young's modulus of elasticity E is assumed as stochastic with respect to the position to actually describe the random material property of the DWCNTs. The internal fluid is assumed to flow steadily through the inner tube with a constant velocity U. Besides, the boundary conditions of the DWCNTs are assumed as clamped at both ends. Based on the theory of Euler-Bernoulli beam and a nonlinear strain-displacement relationship of Von Karman type, the displacement field and strain-displacement relation can be written as follows:

$$\begin{aligned} \bar{u}_i(x,z,t) &= u_i(x,t) - z\frac{\partial w_i}{\partial x} \\ \bar{w}_i(x,z,t) &= w_i(x,t) \\ \varepsilon_i &= \frac{\partial \bar{u}_i}{\partial x} + \frac{1}{2}\left(\frac{\partial \bar{w}_i}{\partial x}\right)^2 \end{aligned} \tag{1}$$

where x is the axial coordinate, t is time, $\bar{u}_i$ and $\bar{w}_i$ denote the total displacements of the ith tube along the x coordinate directions, u_i and w_i define the axial and transverse displacements of the ith tube on the neutral axis, ε_i the corresponding total strain, and the subscript $i = 1$ and $i = 2$. Notice that tube 1 is the inner tube while tube 2 is the outer tube. The potential energy V stored in a DWCNTs and the virtual kinetic energy T in the DWCNTs as well as the fluid inside the DWCNTs are individually written as follows:

$$V=\frac{1}{2}\int_0^L\left[\int_{A_1}E(x)\left(\frac{\partial u_1}{\partial x}+\frac{1}{2}\left(\frac{\partial w_1}{\partial x}\right)^2-z\frac{\partial^2 w_1}{\partial x^2}\right)^2 dA+\int_{A2}E(x)\left(\frac{\partial u_2}{\partial x}+\frac{1}{2}\left(\frac{\partial w_2}{\partial x}\right)^2-z\frac{\partial^2 w_2}{\partial x^2}\right)^2 dA\right]dx \quad (2)$$

$$\begin{aligned}T=&\frac{\rho_t}{2}\int_0^L\left\{\int_{A_1}\left[\left(\frac{\partial u_1}{\partial t}-z\frac{\partial \dot{w}_1}{\partial x}\right)^2+\left(\frac{\partial w_1}{\partial t}\right)^2\right]dA+\int_{A2}\left[\left(\frac{\partial u_2}{\partial t}-z\frac{\partial \dot{w}_2}{\partial x}\right)^2+\left(\frac{\partial w_2}{\partial t}\right)^2\right]dA\right\}dx\\&+\int_0^L\int_{A_f}\frac{1}{2}\rho_f\left[\left(\frac{\partial u_1}{\partial t}-U\cos\theta_1\right)^2+\left(\frac{\partial w_1}{\partial t}-U\sin\theta_1\right)^2+z^2\left(\frac{\partial^2 w_1}{\partial x\partial t}\right)^2\right]dAdx\end{aligned} \quad (3)$$

where $\theta_1=-\partial w_1/\partial x$, I_i and m_i are the moment of inertia and the mass of the ith tube per unit length; ρ_t is the mass density of the beam material; ρ_f is the mass density of the fluid inside tube 1; $A_1=\pi\left[\left(R_1+h\right)^2-R_1\right]^2$ and $A_2=\pi\left[\left(R_2+h\right)^2-R_2\right]^2$ are the cross-sectional areas of tube 1 and tube 2, respectively, and $A_f=\pi R_1^2$ is the cross-sectional areas of the fluid passage in tube 1. Based on Hamilton's principle, the variational form of the equations of motion for the DWCNTs can be given by:

$$\int_{t_0}^{t_1}\left(\delta V-\delta T-\delta\Psi\right)dt=0 \quad (4)$$

where the virtual work due to the vdW interaction and the interaction between tube 1 and the flowing fluid is given by:

$$\delta\Psi=\int_0^L\left[P-MU^2\frac{\partial^2 w_1}{\partial x^2}\cos\theta_1\right]dx\delta w_1+\int_0^L(-P)dx\delta w_2-\int_0^L MU^2\frac{\partial^2 w_1}{\partial x^2}\sin\theta_1 dx\delta u_1 \quad (5)$$

P is the nonlinear vdW force per unit length in the interlayer of the DWCNTs. The interlayer potential per unit area $\Pi(\delta)$ can be expressed in terms the interlayer spacing δ as follows:

$$\Pi(\delta)=K\left[\left(\frac{\delta_0}{\delta}\right)^4-0.4\left(\frac{\delta_0}{\delta}\right)^{10}\right] \quad (6)$$

the vdW force P is then obtained by considering the lowest-order nonlinear term in Taylor expansion of U, which is written as [9]:

$$P=2R_1\left[\left.\frac{\partial^2\Pi}{\partial\delta^2}\right|_{\delta=\delta_0}(\delta-\delta_0)+\frac{1}{6}\left.\frac{\partial^4\Pi}{\partial\delta^4}\right|_{\delta=\delta_0}(\delta-\delta_0)^3\right]=c_1(w_2-w_1)+c_3(w_2-w_1)^3 \quad (7)$$

where:

$K=-61.665$ meV/atom; $\delta-\delta_0=w_2-w_1$; $c_1=\left.\frac{\partial^2\Pi}{\partial\delta^2}\right|_{\delta=\delta_0}2R_1$; $c_3=\frac{1}{6}\left.\frac{\partial^4\Pi}{\partial\delta^4}\right|_{\delta=\delta_0}2R_1$ and $\delta_0=0.34$ nm is the equilibrium interlayer spacing.

By utilizing the Eqs. (4-5) and considering the boundary conditions of the clamped ends, and the assumption that all variables and derivatives are zero at $t = t_0$ and $t = t_1$, all the terms involving $\left[\cdot\right]_0^L$ and $\left[\cdot\right]_{t_0}^{t_1}$ vanish. In addition, considering the moderate large-amplitude deflection, $\cos\theta_1 \approx 1$ and $\sin\theta_1 \approx -\frac{\partial w_1}{\partial x}$ are adopted in the following derivation. In the present study, the boundary conditions of the DWCNTs are assumed as clamped, therefore, the following boundary conditions can be written for the axial displacement:

$$u_1\left(0,t\right) = u_1\left(L,t\right) = 0, \;\; u_2\left(0,t\right) = u_2\left(L,t\right) = 0 \tag{8}$$

Furthermore, we deal with two cases in the formulations. Case one is that the coupling between the axial displacement u_i and transverse displacement w_i is considered and case two is that the coupling between u_i and w_i is neglected. First of all, the case considering the coupling between u_i and w_i is investigated. By neglecting the rotation inertia and utilizing Eqs. (4-8), after some tedious derivations we can obtain the coupled nonlinear governing equations for the free vibration of DWCNTs conveying fluid as follows:

$$\begin{aligned} &I_1\frac{\partial^2(E(x)(\partial^2 w_1/\partial x^2))}{\partial x^2} + MU^2\frac{\partial^2 w_1}{\partial x^2} - \int_0^L\left(\frac{\partial w_1}{\partial x}\right)^2\left(\frac{E(x)A_1}{2L} + \frac{MU^2}{2L}\right)dx\frac{\partial^2 w_1}{\partial x^2} + \frac{3MU^2}{2}\left(\frac{\partial w_1}{\partial x}\right)^2\frac{\partial^2 w_1}{\partial x^2} \\ &+(M+m_1)\frac{\partial^2 w_1}{\partial t^2} + 2MU\frac{\partial^2 w_1}{\partial x\partial t} - MU\frac{\partial w_1}{\partial t}\frac{\partial w_1}{\partial x}\frac{\partial^2 w_1}{\partial x^2} = c_1(w_2 - w_1) + c_3(w_2 - w_1)^3 \end{aligned} \tag{9}$$

$$I_2\frac{\partial^2(E(x)(\partial^2 w_2/\partial x^2))}{\partial x^2} + m_2\frac{\partial^2 w_2}{\partial t^2} - \int_0^L\left(\frac{\partial w_2}{\partial x}\right)^2\left(\frac{E(x)A_2}{2L}\right)dx\frac{\partial^2 w_2}{\partial x^2} = -c_1(w_2 - w_1) - c_3(w_2 - w_1)^3 \tag{10}$$

For case two, the formulation neglecting the coupling between u_i and w_i is investigated. Once again, by neglecting the rotation inertia and utilizing Eqs. (4-8), after some tedious derivations we can obtain the coupled nonlinear governing equations for the free vibration of DWCNTs conveying fluid, for simplicity, the whole derivations for case two is not presented explicitly. In the following derivations, only case one is depicted explicitly.

Case one: considering the coupling between u_i and w_i

In the present study, the Young's modulus of elasticity *E*(*x*) is considered as stochastic with respect to the position to actually characterize the random properties of the DWCNTs and it is assumed as Gaussian distributed. Applying the perturbation technique on the Young's modulus of elasticity *E*(*x*), the following equations can be written:

$$E(x) = E^0(x) + \varepsilon E^I(x) + \cdots \tag{11}$$

where $E^0(x)$ is the mean value of the Young's modulus of elasticity *E*(*x*), ε is a zero-mean small parameter, and $\varepsilon E^I(x)$ is the first variation of the Young's modulus of elasticity *E*(*x*). Similarly, the displacement $w_1(x)$, $w_2(x)$ of the DWCNTs can be written as follows:

$$w_1(x) = w_1^0(x) + \varepsilon w_1^I(x) + \cdots \tag{12}$$

$$w_2(x) = w_2^0(x) + \varepsilon w_2^I(x) + \cdots \tag{13}$$

where $w_1^0(x)$, $w_2^0(x)$ are the mean values of displacement of the inner and outer tubes separately. Substituting Eqs. (11-13) into Eqs. (9-10), we can obtain the following two coupled equations based on the zero order of ε:

$$\begin{aligned} & E^0 \mathrm{I}_1 \frac{\partial^4 (w_1^0)}{\partial x^4} + MU^2 \frac{\partial^2 (w_1^0)}{\partial x} - \left(\frac{MU^2}{2L} + \frac{E^0 A_1}{2L} \right) \int_0^L \left(\frac{\partial (w_1^0)}{\partial x^2} \right)^2 dx \frac{\partial^2 (w_1^0)}{\partial x^2} + \frac{3MU^2}{2} \left(\frac{\partial w_1^0}{\partial x} \right)^2 \left(\frac{\partial^2 w_1^0}{\partial x^2} \right) \\ & + (M + m_1) \frac{\partial^2 (w_1^0)}{\partial t^2} + 2MU \frac{\partial^2 w_1^0}{\partial x \partial t} - MU \left(\frac{\partial w_1^0}{\partial t} \right) \left(\frac{\partial w_1^0}{\partial x} \right) \left(\frac{\partial^2 w_1^0}{\partial x^2} \right) = c_1 (w_2^0 - w_1^0) + c_3 (w_2^0 - w_1^0)^3 \end{aligned} \tag{14}$$

$$E^0 \mathrm{I}_2 \frac{\partial^4 (w_2^0)}{\partial x^4} + m_2 \frac{\partial^2 (w_2^0)}{\partial t^2} - \frac{E^0 A_2}{2L} \int_0^L \left(\frac{\partial w_2^0}{\partial x} \right)^2 dx \frac{\partial^2 (w_2^0)}{\partial x^2} = -c_1 (w_2^0 - w_1^0) - c_3 (w_2^0 - w_1^0)^3 \tag{15}$$

Based on the first order of ε, we can achieve the following two coupled equations:

$$\begin{aligned} & E^0 \mathrm{I}_1 \frac{\partial^4 (w_1^I)}{\partial x^4} + \mathrm{I}_1 \frac{\partial^2 (E^I (\partial^2 w_1^0 / \partial x^2))}{\partial x^2} + MU^2 \frac{\partial^2 (w_1^I)}{\partial x^2} - \frac{MU^2}{2L} \left[\int_0^L 2 \left(\frac{\partial w_1^0}{\partial x} \right) \left(\frac{\partial w_1^I}{\partial x} \right) dx \times \frac{\partial^2 (w_1^0)}{\partial x^2} + \int_0^L \frac{\partial^2 (w_1^0)}{\partial x^2} dx \frac{\partial^2 (w_1^I)}{\partial x^2} \right] \\ & - \frac{A_1}{2L} \left\{ \int_0^L \left[2E^0 \left(\frac{\partial w_1^0}{\partial x} \right) \left(\frac{\partial w_1^I}{\partial x} \right) + E^I \left(\frac{\partial (w_1^0)}{\partial x} \right)^2 \right] dx \times \frac{\partial^2 (w_1^0)}{\partial x^2} + \int_0^L E^0 \left(\frac{\partial^2 w_1^0}{\partial x^2} \right)^2 dx \frac{\partial^2 (w_1^I)}{\partial x^2} \right\} \\ & + \frac{3MU^2}{2} \left[2 \left(\frac{\partial w_1^0}{\partial x} \right) \left(\frac{\partial^2 w_1^0}{\partial x^2} \right) \frac{\partial w_1^I}{\partial x} + \left(\frac{\partial w_1^0}{\partial x} \right)^2 \frac{\partial^2 w_1^I}{\partial x^2} \right] + (M + m_1) \frac{\partial^2 (w_1^I)}{\partial x^2} + 2MU \frac{\partial^2 (w_1^I)}{\partial x \partial t} \\ & - MU \left[\left(\frac{\partial w_1^I}{\partial t} \right) \left(\frac{\partial w_1^0}{\partial x} \right) \left(\frac{\partial^2 w_1^0}{\partial x^2} \right) + \left(\frac{\partial w_1^0}{\partial t} \right) \left(\frac{\partial w_1^I}{\partial x} \right) \left(\frac{\partial^2 w_1^0}{\partial x^2} \right) + \left(\frac{\partial w_1^0}{\partial t} \right) \left(\frac{\partial w_1^0}{\partial x} \right) \left(\frac{\partial^2 w_1^I}{\partial x^2} \right) \right] \\ & = c_1 (w_2^I - w_1^I) + 3c_3 (w_2^0 - w_1^0)^2 (w_2^I - w_1^I) \end{aligned} \tag{16}$$

$$\begin{aligned} & E^0 I_2 \left(\frac{\partial^4 w_2^I}{\partial x^4} \right) + I_2 \left(\frac{\partial^2 (E^{\mathrm{I}} (\partial^2 w_2^0 / \partial x^2))}{\partial x^2} \right) + m_2 \left(\frac{\partial^2 w_2^I}{\partial t^2} \right) - \\ & - \frac{A_2}{2L} \left\{ \int_0^L \left[2E^0 \left(\frac{\partial w_2^0}{\partial x} \right) \left(\frac{\partial w_2^I}{\partial x} \right) + E^I \left(\frac{\partial w_2^0}{\partial x} \right)^2 \right] dx \frac{\partial^2 (w_2^0)}{\partial x^2} + \int_0^L E^0 \left(\frac{\partial w_2^0}{\partial x} \right)^2 dx \frac{\partial^2 w_2^I}{\partial x^2} \right\} \\ & = -c_1 (w_2^I - w_1^I) - 3c_3 (w_2^0 - w_1^0)^2 (w_2^I - w_1^I) \end{aligned} \tag{17}$$

First of all, we have to solve w_1^0, w_2^0 in Eqs. (14-15). By applying the harmonic balance method and Galerkin's method and substituting $w_i^0 = A_i \phi_1(x) \sin(\omega t)$ $(i = 1, 2)$ into Eqs. (14-15), after some tedious derivations the relationship between the amplitude A_i and the resonant frequency ω of the lowest-order mode $\phi_1(x)$ can be achieved as follows:

$$G_1 A_1 + G_2 A_1^3 + G_3 (A_2 - A_1) + G_4 (A_2 - A_1)^3 = 0 \tag{18}$$

$$G_5 A_2 + G_6 A_2^3 + G_3 (A_2 - A_1) + G_4 (A_2 - A_1)^3 = 0 \tag{19}$$

where:

$$G_1 = \left[(M + m_1)\omega^2 + MU^2\lambda_1^2 - E^0 I_1\lambda_1^4\right] \int_0^L \left[\phi_1(x)\right]^2 dx \times \int_0^{2\pi/\omega} \left[\sin(\omega t)\right]^2 dt$$

$$G_2 = -\frac{\left(E^0 A_1 + MU^2\right)\lambda_1^2}{2L} \int_0^L \left[\frac{d\phi_1(x)}{dx}\right]^2 dx \int_0^L \left[\phi_1(x)\right]^2 dx \times \int_0^{2\pi/\omega} \left[\sin(\omega t)\right]^4 dt \, +$$

$$+ \frac{3MU^2\lambda_1^2}{2} \int_0^L \left[\frac{d\phi_1(x)}{dx}\right]^2 dx \int_0^L \left[\phi_1(x)\right]^2 dx \times \int_0^{2\pi/\omega} \left[\sin(\omega t)\right]^4 dt$$

$$G_3 = c_1 \int_0^L \left[\phi_1(x)\right]^2 dx \int_0^{2\pi/\omega} \left[\sin(\omega t)\right]^2 dt$$

$$G_4 = c_3 \int_0^L \left[\phi_1(x)\right]^4 dx \int_0^{2\pi/\omega} \left[\sin(\omega t)\right]^4 dt$$

$$G_5 = \left[E^0 I_2\lambda_1^4 - m_2\omega^2\right] \int_0^L \left[\phi_1(x)\right]^2 dx \int_0^{2\pi/\omega} \left[\sin(\omega t)\right]^2 dt$$

$$G_6 = \frac{E^0 A_2\lambda_1^2}{2L} \int_0^L \left[\frac{\phi_1(x)}{dx}\right]^2 dx \int_0^L \left[\phi_1(x)\right]^2 dx \int_0^{2\pi/\omega} \left[\sin(\omega t)\right]^4 dt$$

where $\phi_1(x)$ is the first vibration mode of the corresponding linear system, which can be expressed as follows for the clamped-clamped boundary conditions as follows:

$$\phi_1(x) = \cosh(\lambda_1 x) - \cos(\lambda_1 x) - \frac{\sinh(\lambda_1 \mathrm{L}) - \sin(\lambda_1 \mathrm{L})}{\cosh(\lambda_1 \mathrm{L}) - \cos(\lambda_1 \mathrm{L})}\left(\sinh(\lambda_1 x) - \sin(\lambda_1 x)\right), \quad \lambda_1 L{=}4.73 \qquad (20)$$

After solving coupled Eqs. (18-19) for the amplitudes A_1, A_2, we can obtain w_1^0, w_2^0 readily. Substituting w_1^0, w_2^0 into Eqs. (16-17), and adopting the same technique for solving w_1^0, w_2^0, finally we can obtain w_1^I, w_2^I without any difficulties except the derivations are somewhat lengthy.

Numerical examples and discussion

In the numerical computations, the clamped-clamped boundary conditions are considered for the DWCNTs conveying fluid. The inner and the outer tubes are assumed to have the same Young's modulus, the same thickness and the same mass density. The numerical values of the parameters are adopted as follows: Mean value of Young's modulus E = 1 Tpa, tube thickness $h = 0.34$ nm, mass density $\rho = 2300\ \mathrm{Kg/m^3}$, the mass density of water flow is $\rho_f = 1000\ \mathrm{Kg/m^3}$, the inner radius $R_1 = 0.7$ nm and the outer radius $R_2 = 1.04$ nm and mean square values of ε is assumed as $E\left[\varepsilon^2\right] = 0.01$. The relations of the mean value of amplitude versus frequency are depicted in Fig. 2. The mean value of the amplitude increases with the increase of the frequencies for both cases of coupling between longitudinal displacement u and transverse displacement w and uncoupling between them. It is certainly reasonable that the relation between the mean value of the amplitude and the frequency is nonlinear; in addition, the mean value of the amplitude of the outer tube is larger than that of the inner tube. Furthermore, it is noted that the mean value of the amplitude with considering the coupling is larger than that with uncoupling for the fixed frequency. In Fig. 3, the standard deviation of the amplitude is plotted with respect to the frequency for both coupling and uncoupling cases. As it can be seen from the figure that the standard deviation of the amplitude increases nonlinearly with the increase of the frequencies, and it is noted that the standard deviation of the amplitude of the outer tube is larger than that of the inner tube. In Fig. 4, the coefficient of variation of the amplitude is depicted with respect to the frequency. It is interesting to notice that the coefficients of variation of the amplitude of the inner and outer tubes for both coupling and uncoupling cases are around 0.10 to 0.12.

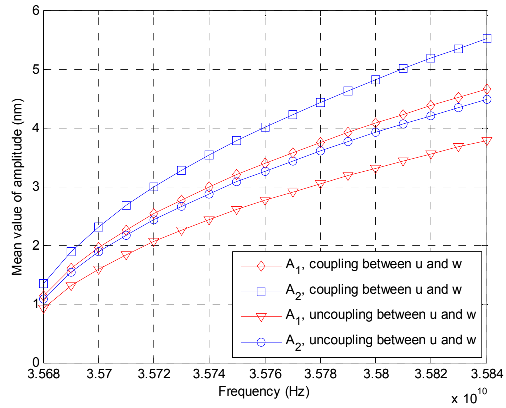

Fig. 2. Mean value of amplitude versus frequency

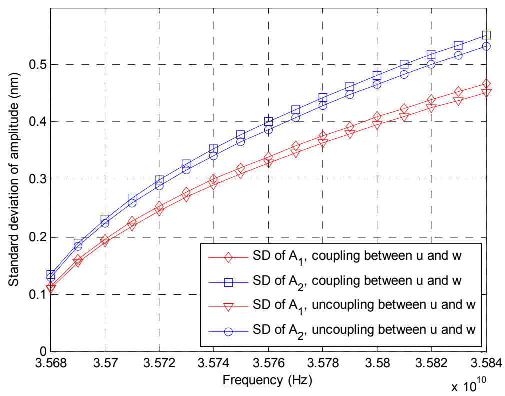

Fig. 3. Standard deviation of amplitude versus frequency

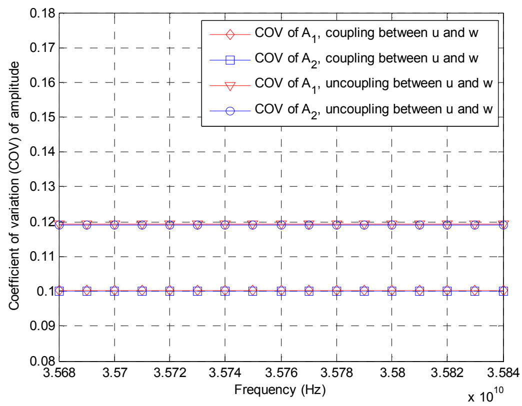

Fig. 4. Coefficient of variation of amplitude versus frequency

Conclusions

In the present study, we investigate the stochastic dynamic behaviors of nonlinear vibration of the double-walled carbon nanotubes (DWCNTs) conveying fluid by considering the effects of the geometric nonlinearity and the nonlinearity of van der Waals (vdW) force. Based on the Hamilton's principle, the nonlinear governing equations of the fluid-conveying double-walled carbon nanotubes are formulated. Two different cases of nonlinearity are considered; case one is to include the coupling between the longitudinal displacement and transverse displacement of the DWCNTs, on the other hand, case two is to neglect the coupling between them. The Young's modulus of elasticity of the DWCNTs is considered as stochastic with respect to the position to actually characterize the random material properties of the DWCNTs. By using the perturbation technique, the nonlinear governing equations of the fluid-conveying double-walled carbon nanotubes can be decomposed into two sets of nonlinear differential equations involving the mean value of the displacement and the first variation of the displacement separately. Then the harmonic balance method and Galerkin's method are used to solve the nonlinear differential equations successively. Some statistical dynamic response of the DWCNTs such as the mean values and standard deviations of the amplitude of the displacement are computed. It can be concluded that the mean value and standard deviation of the amplitude of the displacement increase nonlinearly with the increase of the frequencies for both cases of coupling between longitudinal displacement and transverse displacement and uncoupling between them. However, the coefficients of variation (COV) of the amplitude of the displacement remain almost constant and stay within certain range with respect to the frequency. It is noted that the computed stochastic dynamic response plays an important role in estimating the structural reliability of the DWCNTs.

Acknowledgements

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