Boundary Element Modeling of Acoustic Fields Generated During Ultrasonic Angioplasty

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Abstract. We investigated possibilities of application of boundary element method (BEM) to modelling of acoustic fields generated during ultrasonic angioplasty. It was shown that modelling by means of BEM can be more efficient comparing with traditionally used modelling by means of finite element method (FEM). We also considered test problem of calculation of acoustic field created by ultrasonic waveguide in semi-infinite fluid media and the problem was solved by means of BEM and FEM with comparative analysis of obtained results. Modelling by means of BEM additionally involves application of mirror source method for avoidance of treatment of infinite fluid boundary. On the basis of analysis of the test problem we shown that BEM can be used as efficient tool for modelling of acoustic fields generated during ultrasonic surgical procedures, particularly, during ultrasonic angioplasty. BEM has several advantages in comparison with FEM and can be used as alternative to traditionally used modelling by means of FEM or as supplementary method.

1. Introduction

Efficiency of destruction of thrombus material during ultrasonic angioplasty depends on degree of development of cavitation. Intensity of cavitation is in turn determined by the value of amplitude of acoustic pressure created by the waveguide tip in surrounding fluid media, including the blood. For onset of cavitation value of ultrasound intensity corresponding to the amplitude value of acoustic pressure should exceed cavitation threshold. Theoretical and experimental studies show that amplitude value of acoustic pressure created by the waveguide in fluid at constant amplitude of vibratory displacements depends on the shape of working end (tip) of the waveguide [1, 2]. Wylie et al. represented theoretical calculations by means of Finite Element Method (FEM), where it was shown that, if diameter of the tip is equal to 1 mm, frequency of ultrasound is equal to 23.5kHz and amplitude of vibratory displacements is equal to 60µm, spherical shape tip can create maximum amplitude of acoustic pressure equal to 300kPa, flat shape tip – 550kPa and concave spherical shape tip – 1100kPa [1]. Increase of the tip diameter results in growth of pressure. Effect of the tip's shape on amplitude of acoustic pressure is also indirectly confirmed according to results of experimental studies of thrombi destruction rate in vitro [2]. Particularly, figure 1 represents diagram of dependence between reduction of thrombus mass and intensity of ultrasound for different shapes of the tip at constant time of ultrasonic treatment.

Presented results provide possibility to arrange waveguides with different shapes of the tip in the order of decrease of their efficiency in the following way: Waveguide with Flat Head with Opening (WFHO), Waveguide with Flat Head (WFH), Waveguide with Spherical Head (WSH), Waveguide with Spherical Head with Opening (WSHO).

Theoretical calculations of parameters of acoustic field created by the waveguide in fluid are usually implemented by means of FEM [1, 3, 4]. However, it should be noted that solution of this class of problems (external problems of acoustics) by means of FEM is not optimal in terms of computation efficiency. Helmholtz differential equation, spatial discretization of which is implemented during

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solution of external acoustical problems by means of FEM, demands for application of boundary conditions on the surface of acoustic radiator and on infinity (Sommerfeld radiation conditions). The need for accounting boundary conditions on infinity requires finite-element discretization of large (in comparison with dimensions of radiator) volume of the space around the radiator with the aim of imitation of infinitely-extended real space. This results in substantial increase in the number of finite elements and computational time. More efficient way for solving external problems of acoustics consists in application of Boundary Element Method (BEM) [5, 6]. In this study we tried to investigate possibility of applying BEM for theoretical analysis of acoustic fields generated during ultrasonic angioplasty.

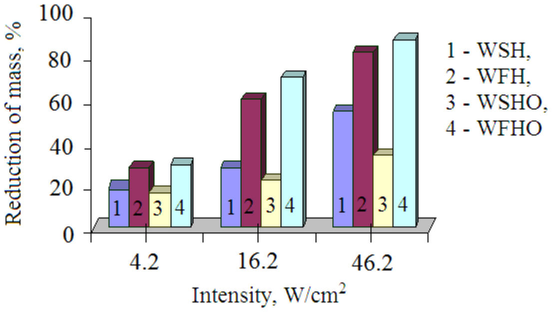

Figure 1. Dependence between reduction of thrombus mass and intensity of ultrasound for different shapes of the waveguide tip.

2. Methods of modelling

Modelling of acoustic field of the waveguide by means of BEM was implemented using open source software OpenBEM realized as a library of functions for Matlab [7]. In comparison with FEM BEM has the following advantages. For the first, boundary element discretization is necessary only on the surfaces of radiators and reflectors of acoustic waves. This can reduce total number of discrete elements and computational complexity of the problem. For the second, Sommerfeld radiation conditions are automatically satisfied, since they are taken into account during transition from Helmholtz differential equation to Kirchhoff-Helmholtz integral equation, spatial discretization of which is used for solution of acoustical problems by means of BEM.

To estimate efficiency of BEM application for modelling of ultrasonic angioplasty we considered simplified problem of calculation of acoustic field created by the waveguide in semi-infinite fluid medium. In case of necessity the problem under consideration could be approximated to real conditions (in vivo and in vitro experimental studies) by means of introduction of additional boundaries with impedance boundary conditions simulating walls of blood vessel or synthetic test tube.

Schematic drawing of the model under consideration is presented in the figure 2a.

In solution of three-dimensional problems by means of OpenBEM it is possible to use generation of boundary elements with the aid of external programs (for example, using FEM software like ANSYS with application of shell finite elements of the type SHELL) with subsequent import. For these purposes there are functions *readnodes*(‘*nlist.nod*’) and *readelements*(‘*elist.ele*’), where *nlist.nod* is the name of file containing numbers and coordinates of nodes, *elist.ele* is the name of file containing numbers of elements and numbers of nodes belonging to them. These functions also can operate with two-dimensional models, however, in solution of two-dimensional problems OpenBEM uses three-node boundary elements, but beam finite elements of the type BEAM applicable for meshing of boundaries in ANSYS are two-node elements. For these reasons we used generation of boundary elements by means of internal tools of OpenBEM. Geometry of the boundary of axially-symmetric radiator is described in OpenBEM as a sequence of straight and circular segments by means of matrix *segments* containing information on coordinates of boundary points and radii of curvature of the segments as well as information on the number of boundary elements used for meshing of each segment. Generation of boundary elements is implemented using function *nodegen*(*segments*). During solution of the problem under consideration segments were generated by means of ANSYS by meshing boundary of the radiator using finite elements of the type BEAM. Coordinates of the boundary points of segments were imported in OpenBEM by means of functions *readnodes* and *readelements*.

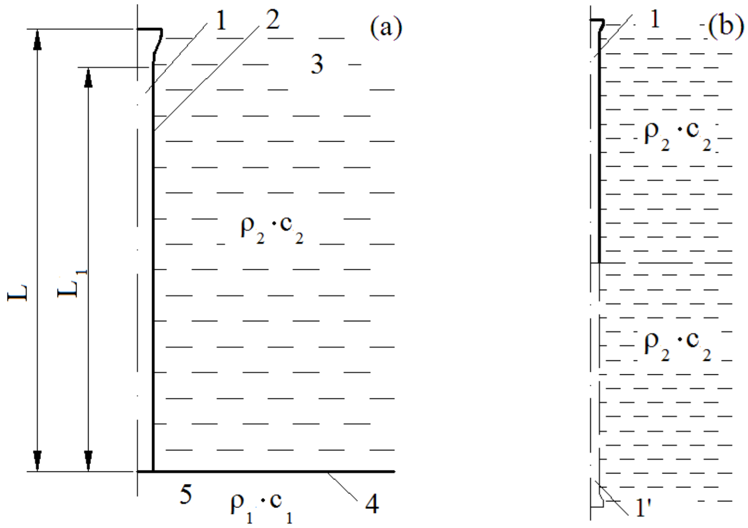

Figure 2. Two models of the waveguide radiation.

For validation of results obtained by means of BEM the problem was also modelled by means of FEM using Acoustics Module of COMSOL Multiphysics software. The problem was considered as axisymmetric problem of forced vibrations of the waveguide interacting with fluid (analysis type is "Acoustic-Structure Interaction -> Frequency response analysis"). Geometric model consisted of two subdomains: subdomain of the waveguide and fluid subdomain bounded by acoustically hard plane (interface between fluid and gaseous medium) and hemispherical surface with radiative boundary conditions for spherical waves. Radiative boundary conditions let outgoing wave to leave modelling domain without reflections [11, 12]. Amplitude of vibratory acceleration vector of fluid particles near the waveguide surface was accepted to be equal to amplitude of normal acceleration vector of the surface (boundary condition "Normal acceleration"). Vibration of the waveguide was considered with account for pressure exerted by the fluid on its surface (load of the type "Fluid load"). Forcing load was applied as prescribed displacement of the working surface of the waveguide head (boundary condition "Prescribed displacement"). Amplitude of displacement was accepted to be 10μm, frequency – 25kHz. Nodal plane of the waveguide coincident with interface between fluid and gaseous medium was constrained in all degrees of freedom.

3. Results of modelling and discussion

Results of BEM calculation of acoustic pressure amplitude on the surface of the waveguide with spherical head are presented in the figure 3a.

Numbering of nodes of boundary elements is modified in such way that they are arranged in ascending order according to values of arc coordinate measured along the boundaries of mirror and primary radiators in direction from point $(0,-L)$ to point $(0,L)$. Left portion of the plot corresponds to the mirror radiator, and right portion corresponds to primary radiator. Plots of acoustic pressure amplitude and its phase (figure 4) are symmetric, *i.e.* primary and mirror radiators generate co-phased acoustic pressures. This is in agreement with accepted boundary conditions. For comparison, figures 3b and 3c represent results of FEM calculation of acoustic field of the waveguide according to Wylie *et al.* [1] and present work. It follows from comparison of the presented plots that in all cases there are primary maximum of acoustic pressure amplitude (primary directional lobe) corresponding to the point $(0, L)$ and secondary maximum of acoustic pressure amplitude (lateral directional lobe) corresponding to the point on the rear portion of head surface ($n_y(y) < 0$). Amplitude of acoustic pressure calculated using BEM is in agreement with amplitude calculated by means of FEM (300kPa at amplitude of vibratory displacements $\xi_0 = 60\mu\text{m}$ according to Wylie *et al.* [1], 82.6kPa at amplitude 10µm – calculation using BEM, 96.0kPa at amplitude 10µm – calculation using COMSOL). Modelling using FEM shows that acoustic pressures in primary and lateral directional lobes are anti-phased (*i.e*. compression of medium at the front portion of head surface corresponds to expansion of medium at the rear portion). This is confirmed by results of BEM calculation of pressure phase presented in the figure 4.

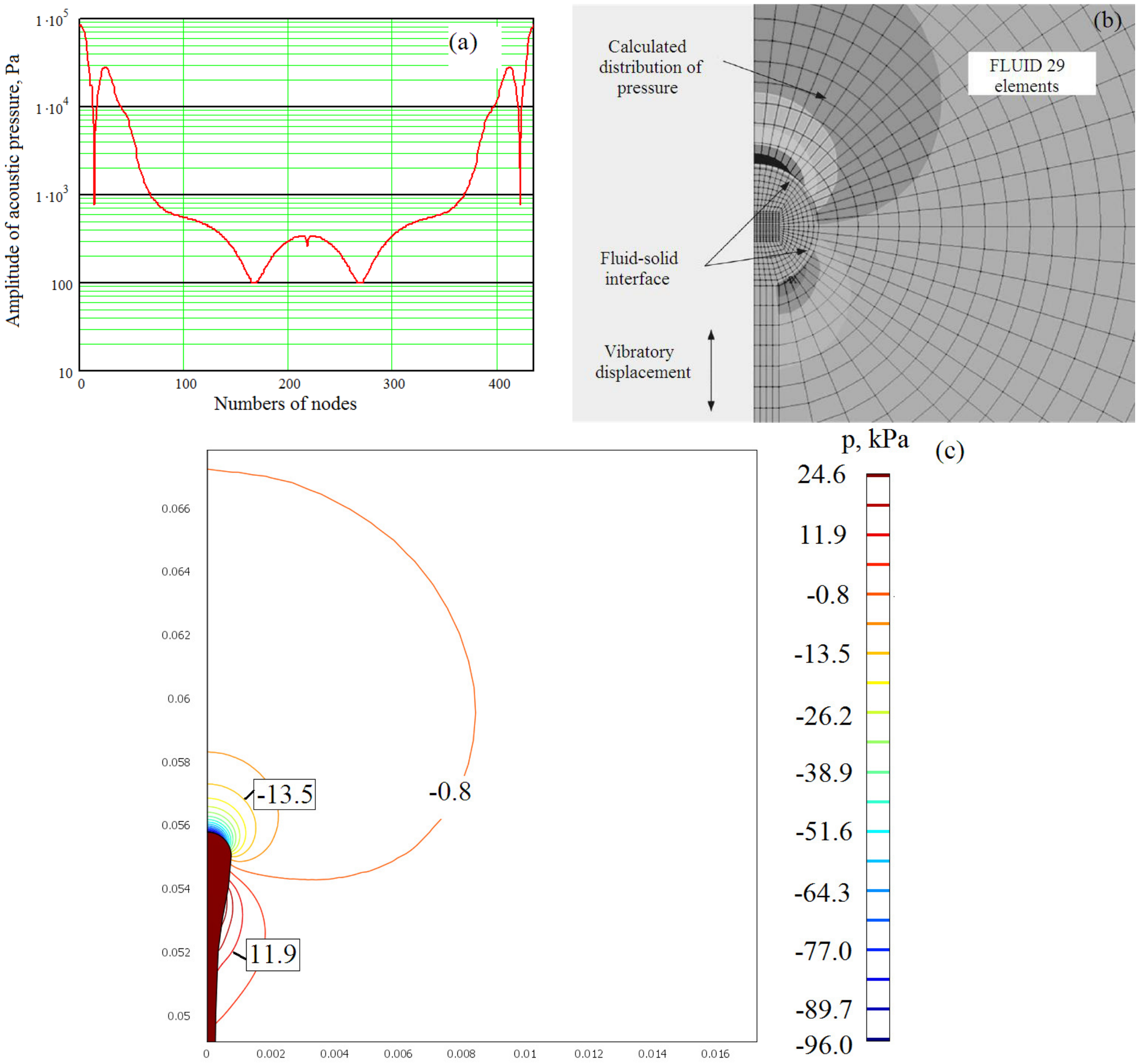

Figure 3. Results of calculation of acoustic pressure amplitude.

As it follows from the presented plot, there is phase shift equal to π between primary and secondary maximums of acoustic pressure amplitude. In order to simplify determination of positions of amplitude maxima plot 1 of phase was combined with arbitrarily scaled plot 2 of amplitude.

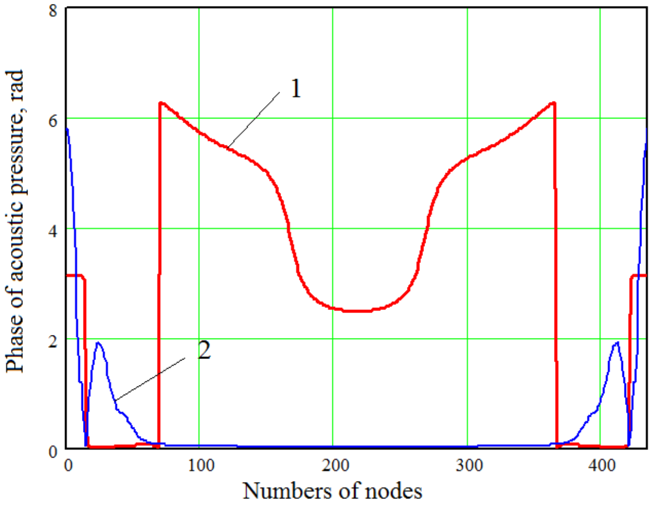

Figure 4. Plots of acoustic pressure amplitude and its phase calculated using BEM.

4. Conclusions

On the basis of analysis of simple test problem we shown that BEM can be used as efficient tool for modelling of acoustic fields generated during ultrasonic surgical procedures, particularly, during ultrasonic angioplasty. BEM has several advantages in comparison with FEM and can be used as alternative to traditionally used modelling by means of FEM or as supplementary method. Results of modelling by means of BEM and FEM are found to be in good qualitative and quantitative agreement. Presented technique of modelling provides possibility to study efficiency of waveguides with different shapes of the tip.

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References

[1] Wylie M P *et al.* 2010 A linear finite element acoustic fluid-structure model of ultrasonic angioplasty *in vivo Int J Numer Meth Biomed Eng* **26** 828–842
[2] Tun T 2006 *Efficiency of recanalization of arteries affected by atherosclerosis using various modifications of ultrasonic waveguides in vitro: PhD dissertation* (Minsk) (in Russian)
[3] Li T *et al.* 2011 *Advances in Piezoelectric Transducers* ed F Ebrahimi (New York: InTech) pp 3–26
[4] Li T *et al.* 2012 Development of piezoelectric ultrasonic thrombolysis device for blood clot emulsification *ISRN Mat Sci* **2012** 106484
[5] Juhl P M 1993 *The boundary element method for sound field calculations: PhD thesis* (Copenhagen)
[6] Kirkup S 1998 *The boundary element method in acoustics* (*S.l.*: Integrated Sound Software)
[7] http://www.openbem.dk/
[8] Wang W *et al.* 1997 A boundary integral approach for acoustic radiation of axisymmetric bodies with arbitrary boundary conditions valid for all wave numbers *J Acoust Soc Am* **101** 1468–78
[9] Brick H and Ochmann M 2008 *Proc. of Acoustics'08 (Paris, 29 June - 4 July 2008)*
[10] Tamm I E 1989 *Fundamentals of theory of electricity* (Moscow: Nauka) p 56 (in Russian)
[11] Bayliss A *et al.* 1982 Boundary conditions for the numerical solution of elliptic equations in exterior regions *SIAM J Appl Math* **42** 430–451
[12] COMSOL Multiphysics. Version 3.5. Acoustics Module. User's Guide. 2008.