

# Identification of Physical and Mechanical Properties of Compound by Solving Inverse Problems

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**Abstract.** The paper presents a method of identification of physical and mechanical characteristics of the compound (calculated and experimental test method parameters). The idea of the method is that the studied material is connected with other materials, the characteristics of which are well known and differ from the characteristics of the material. Desired physical and mechanical properties should be considered unknown in the proposed calculation model, and the values of the parameters of trial constructions that can be measured accurately with experimental methods in the process of product operation, as well as characteristics of test materials and geometric dimensions of proposed structures are suggested to be placed in the same model as input information.

## 1. Introduction

The object of research was the process of determining physical and mechanical properties of the polymer compound that is used for sealing flat micromodules (FMMs).

Typical structure of flat micromodule, used in electronic engineering, is shown in Fig. 1. During the operation of FMMs the structure “electronic element-compound” is subject to dangerous stresses caused by the difference of the coefficients of linear thermal expansion, Poisson’s ratio and elastic moduli of these materials, which leads to the destruction of FMMs and the consequent destruction of expensive complex systems which contain them.

The analysis carried out by the authors showed that variations in the physical and mechanical properties of compounds in the production of FMMs can reach 300% and depends on many factors, such as the place where compound components are manufactured. In addition, the characteristics of compounds in the temperature range of 60°C–20°C are unknown, although the FMMs are operated in such temperatures.

Existing methods for determining the physical and mechanical properties of materials have a number of drawbacks which significantly narrows the range of applications. Speaking of static methods, it should be mentioned that since the studied materials are usually fragile, then, firstly, there are difficulties to comply with alignment when attaching samples in testing machines, and, secondly, the destruction of these samples occurs under low strain which reduces the accuracy of measurement. Moreover, the results of this experiment strongly dependent on the type of load, size of samples and their manufacture methods.

Dynamic methods can be applied only for materials with low absorption of oscillations. They depend on the temperature conditions under which the experiment is conducted and in order to be effective, they require the information about the speed of propagation of elastic waves in the material, the definition of which being a difficult task.

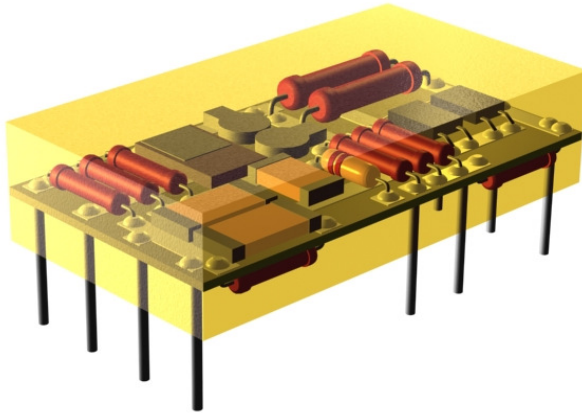
In addition, both existing static and dynamic methods allow to determine the physical and mechanical properties of generally homogeneous materials in which the value of these characteristics does not depend on the type of stress state of the material in a particular product. The considered compound exists as a sealant only in certain structures, such as FMM, and its physical and mechanical

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properties are different from the characteristics of the same compound in a different structure.

Thus, the problem of finding methods for identification of physical and mechanical properties of compounds directly in real structures in the temperature range  $-60^{\circ}\text{C} \dots 20^{\circ}\text{C}$  is relevant.



**Figure 1.** The design of a typical flat Micromodules.

## 2. Identification of physical and mechanical properties of the material

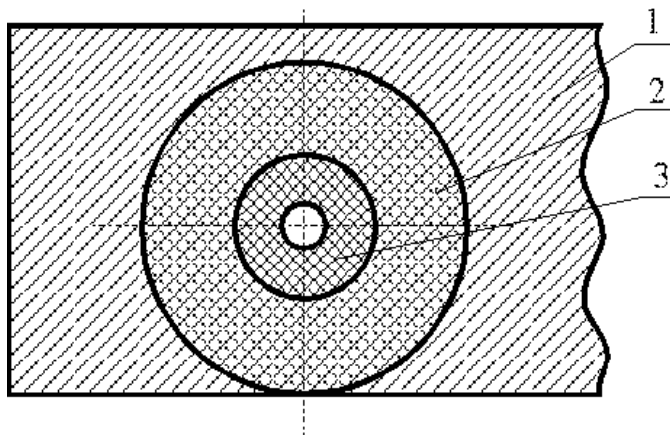
The study [1] is focused on the causes of FMM destruction and provides a mathematical model of stress calculation in the system “electronic element-compound”, represented by figures 2 and 3. The formula for calculating contact pressure in the materials of resistor and compound at a stable temperature looks the following way (1).

$$P_{1k} = \frac{(\alpha_1 - \alpha_2)t}{\frac{(1 - \mu_2)R_1^2 + (1 - 2\mu_1)R_2^2}{E_1(R_2^2 - R_1^2)} + \frac{(1 + \mu_2)R_1^2 + (1 - 2\mu_1)R_2^2}{E_2(R_3^2 - R_2^2)}} \quad (1)$$

where  $\alpha_1, \alpha_2$  – is the coefficient of linear thermal expansion in the material of the product and compound, respectively;  $E_1, E_2$  – their elastic moduli;  $\mu_1, \mu_2$  – their Poisson’s ratio.

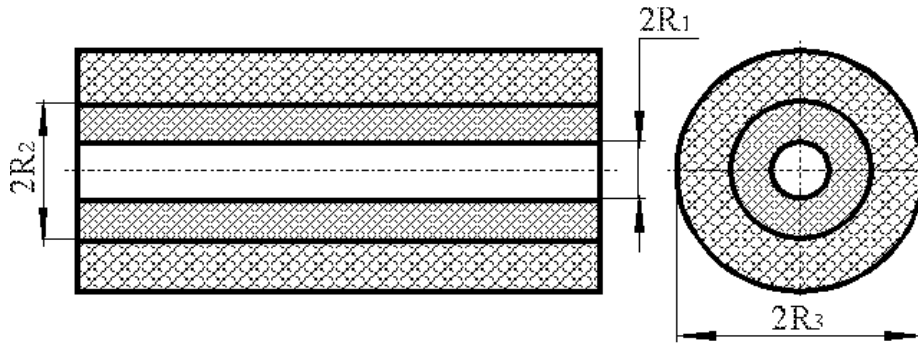
The model proved to be suitable for calculating the strength of parts of the system, and methods for determining the physical and mechanical characteristics, based on their identification by solving inverse problems of strength, appeared to be promising.

The authors developed an experimental method of calculation with testing parameters which can simultaneously identify the coefficient of linear thermal expansion, elastic modulus and Poisson's ratio of a compound.



**Figure 2.** Cross-section of an electronic element, compound polymerized with compound:

- 1 - compound,
- 2 – isolated compound cylinder,
- 3 - electronic element.



**Figure 3.** The electronic element surrounded by a layer of compound.

The idea of the method is that the studied material is connected with other trial materials, the properties of which are well known and differ from the properties of the material. Whereas the connecting parts should have specific forms that make it possible to describe the stress-strain state of the samples in real structures with the same equations.

In order to determine the physical and mechanical characteristics it is offered to consider the values of the identified parameters to be unknown in the calculation model. It is suggested that the values of the parameters of trial structures, which can be measured accurately by experimental means in the process of product operation, and characteristics of trial materials and geometric dimensions should be inserted in the same model as input information. Producing a number of test samples with values of physical and mechanical properties that are set so that the measured deformation values would be different, it is possible to write the number of linearly independent equations with the identified parameters based on the computation model which is required for their determination.

To identify the abovementioned physical and mechanical properties of the compound that seals and surrounds resistor we have made double-layer cylindrical structures “trial material - compound”, choosing the trial material with thoroughly investigated properties, different from the properties of compound. In these structures contact pressure occurs at the interface of materials at temperature extremes. The parameters that are included in the selected mathematical model and can be accurately measured by experimental methods are strains emerging under contact pressure on the outer surface of the testing cylinder, their values being connected with stress values by Hooke's law. The ratio determined by Hooke's law and the ratios specified by the authors of the developed model make it possible to find the value of contact pressure for known radial sizes of structures according to the experimentally determined values of strain.

According to the authors' model, the contact pressure can be considered as a structure parameter, which is determined experimentally and whose value can be substituted in the calculation model as input information in the form of

$$P_{1k} = -\varepsilon_i / \left( \frac{R_1^2}{R_2^2 - R_1^2} \cdot \frac{1 + \mu_1}{E_1} + \frac{R_2^2}{R_2^2 - R_1^2} \cdot \frac{1 - 2\mu_1}{E_1} \right) \quad (2)$$

where  $\varepsilon_i$  is relative circumferential deformation of the contact surface.

Let's explore the expression (1) for the contact pressure and turn it into the form suitable for the determination of the identified parameters. The analysis of the expression on the right side of (1) shows that its denominator is different from zero, since the ratio  $R_1 < R_2 < R_3$  is always preserved between the radial size of the studied structure, and the value of Poisson's ratio for any material lies within  $0 < \mu < 0,5$ .

Let's transform the right-hand side of (1) into the following

$$P_{1k} = \frac{(\alpha_1 - \alpha_2)t}{\frac{1-2\mu_1}{E_1} - \frac{1-2\mu_2}{E_2} + \frac{2-\mu_1}{E_1} \cdot \frac{R_1^2}{R_2^2 - R_1^2} + \frac{2-\mu_2}{E_2} \frac{R_3^2}{R_3^2 - R_2^2}} \quad (3)$$

After introducing new variables  $\chi_1 = (1 - 2\mu_2)/E_2$ ,  $\chi_2 = (2 - \mu_2)/E_2$ , expressed by parameters  $E_2$ ,  $\mu_2$ , subject to identification, we'll transform equation (3) in such a way that all the parameters that need definition, should be located in the left side of the resulting equation. After performing the transformation and introducing additional notation, we obtain

$$P_{1k} B \chi_2 - P_{1k} \chi_1 + \alpha_2 t = -(E + CD)P_{1k} + \alpha_1 t \quad (4)$$

where  $B = R_3^2 / (R_3^2 - R_2^2)$ ,  $C = (2 - \mu_1)/E_1$ ,  $D = R_1^2 / (R_2^2 - R_1^2)$ ,  $E = (1 - 2\mu_2)/E_1$ . What we get is a linear equation with respect to the unknown  $\chi_1$ ,  $\chi_2$ ,  $\alpha_2$ . For their determination it is necessary to write a system of three linear algebraic equations. For this purpose, we'll firstly assume that the sample is made of three different sets of values of the entire set of primary factors and the corresponding measured values of contact pressures. This will add the following system of equations

$$\begin{cases} P_{0ki} B_i \chi_2 - P_{1ki} \chi_1 + \alpha_2 t = -(E_i - C_i D_i) P_{1ki} + \alpha_1 t, \\ i = 1, 2, 3, \end{cases} \quad (5)$$

where the indices of coefficients  $B_i$ ,  $C_i$ ,  $D_i$ ,  $E_i$  and contact pressure  $P_{1ki}$  correspond to the vector of parameters  $\bar{x}_i = \{\alpha_{1i}, \mu_{1i}, E_{1i}, R_{1i}, R_{2i}, R_{3i}\}$  of the "i" trial sample ( $i = 1, 2, 3$ ).

Solution of equations (5) can find the properties of the material being identified.

However, the method of test parameters doesn't require setting various values of all parameters vector in order to achieve different values of contact pressures. It's enough to change one or more parameters in various combinations. Further we will explore such options or their combination, which, firstly, are most easily changed, and secondly, which ensure the existence, uniqueness and stability of solutions of equations (5).

The condition for the existence and unity of solutions of the mentioned equations is that its determinant  $\Delta$ , should be different from zero, i.e.

$$\Delta = \alpha_2 t [P_{1k1} P_{1k2} (B_2 - B_3) + P_{1k1} P_{1k3} (B_1 - B_3) + P_{1k2} P_{1k3} (B_3 - B_2)] \quad (6)$$

Analysis (6) shows that it cannot be met, for example, if the three studied samples have simultaneously equal constant surface radii and outer radii of compound. This conclusion can immediately exclude from consideration such common case as the use of different trial materials in three samples with preservation of the radial size of structures or changing contact pressure just by changing the internal diameter of the cylinder of the testing material. Further study of the system (3.12) shows that changing only the outer radius of the cylinder made of trial material with the preservation of the same values remaining coordinate vector of parameters in three samples leads to the fact that the equations have trivial solutions.

To ensure the stability of the solution (5), and thus given accuracy, it is necessary to approximate angles between the straight lines described by equations of the system (5), and 90°. From the analysis (5) it is clear that the most effective way is to change the geometric dimensions of devices included in the equation system.

Here are the most appropriate options for implementing the method of test parameters.

In the three studied samples only the values of the contact surface radius are variable. Indeed,  $R_2 \uparrow \Rightarrow B \uparrow$ , which improves the stability of solution. In this case, to calculate the identifiable characteristics it is necessary to put  $C_i = C$ ,  $E_i = E$ ,  $\alpha_{1i} = \alpha_1$ ,  $B_i = R_3^2 / (R_3^2 - R_{2i}^2)$ ,

$D_i = R_1^2 / (R_{2i}^2 - R_1^2)$ ,  $i = 1, 2, 3$  in (5). The condition  $R_2 \neq 0$  should also be met here.

In the three studied samples simultaneous alteration occurs only in the value of internal and external radii of the complex cylinder. To implement this option,  $C_i = C$ ,  $E_i = E$ ,  $\alpha_{i1} = \alpha_1$ ,  $B_i = R_{3i}^2 / (R_{3i}^2 - R_2^2)$ ,  $D_i = R_{1i}^2 / (R_2^2 - R_{1i}^2)$ ,  $i = 1, 2, 3$  must be substituted into (5).

### 3. Results and discussion

Practical implementation of the method of test parameters for determining the physical and mechanical properties of compounds requires, as mentioned earlier, the correct use of design model of complex bilayer cylinders.

This implies meeting the following conditions: cylinders in complex design should be thick; double-layer composite cylinders must be long enough to prevent the influence of the edge effects at the point where the deformation occurs.

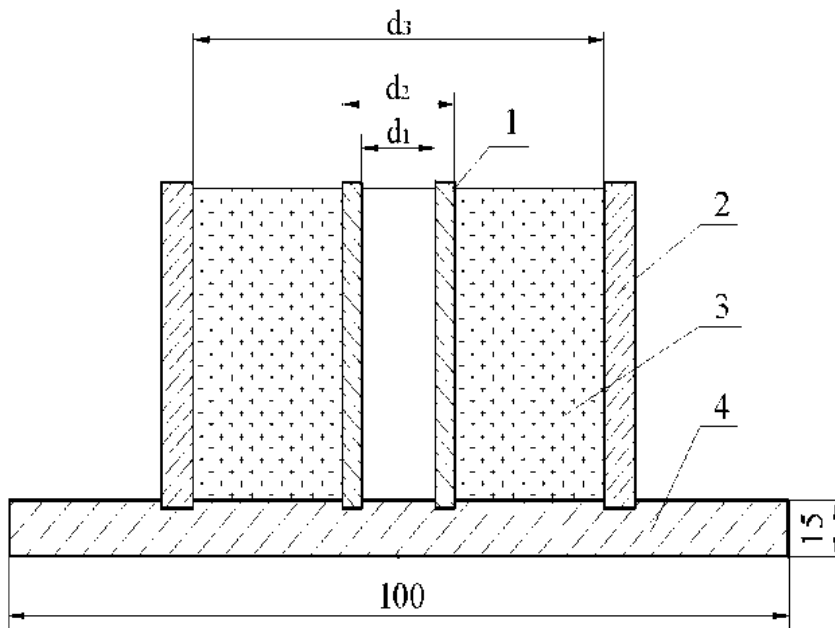
For practical implementation of the method a special device was developed and manufactured (Fig. 4). The main elements of adaptation is hollow cylinder (trial element), hollow steel cylinder cut according to two diametrically opposite generatrices, cylindrical bearing plate with two coaxial cylindrical grooves designed for fixation of trial and steel cylinders.

Three devices of this type have been made, differing from one another by cylinder diameters and the respective sizes of fixing anchor plate surfaces.

The sizes of cylinder diameters of these devices are shown in Table 1.

To ensure the stability of solutions averaged values of deformation were used for calculation.

According to the measured values of strain, the amount of contact pressure was determined, using formula (1), and then – the coefficient of linear thermal expansion, elastic modulus and Poisson's ratio of the compound in different ranges of negative temperatures (Table 2).



**Figure 4.** Scheme of the device for determining physical and mechanical properties of the compound:  
1 – hollow copper cylinder;  
2 – hollow steel cylinder;  
3 – compound;  
4 – bearing plate.

**Table 1.** Dimensions of cylinder diameters for determining the physical and mechanical properties of the compound.

Device #	Diameters, mm		
	d <sub>1</sub>	d <sub>2</sub>	d <sub>3</sub>
I	10	10.4	40
II	20	20.4	40
III	30	30.4	60

**Table 2.** The value of physical and mechanical properties of resistor ceramic.

Temperature range	Young's modulus E, N/mm <sup>2</sup> ·10 <sup>4</sup>	Poisson's ratio $\mu$	Local thermodynamic equilibrium $\alpha$ , 1/degree 10 <sup>-6</sup>
-20°C...-30°C	13.794	0.294	7.0023
-30°C...-40°C	13.658	0.282	6.376
-50°C...-60°C	12.95	0.276	5.898

#### 4. Conclusions

The method for calculation and testing of trial parameters to identify physical and mechanical properties of compound has been developed. A program in the package of MATLAB applications which realizes the identification has been created. Using the program, the authors identified the physical and mechanical properties of EZK-25 compound, specified in the selected mathematical model.

The value of the developed identification method is that it makes it possible to accurately determine the physical and mechanical properties for a particular batch of material, temperature, and especially the design scheme that describes the stress-strain state of “electronic element-compound” system. This is due to the fact that any mathematical model of the real product contains various assumptions and substituting it with inaccurate coefficients can compromise even the most accurate calculation scheme. Whereas the use of identification method contributes to the effectiveness of the offered mathematical model, since it will be substituted with identifiable characteristics that derived from the same mathematical model.

#### References

- [1] A M Lokoschenko, S A Petraschuk and V P Royzman 2012 Evaluate and ensure the strength of the system of "electronic element-compound" *Journal of Khmelnytsky National University* **5** 193–197