Research of Compressive and Shear Stiffness of Laminated Elastomeric Structures

V Gonca[1] and S Polukoshko[2]
[1] Riga Technical University, Institute of Mechanics,
Ezermalas Str. 6, Riga, LV-1006, Latvia
[2] Ventspils University College, Engineering Research Institute "VSRC",
Inzenieru str. 101, Ventspils, LV-3601, Latvia

E-mail: vladimirs.gonca@rtu.lv

Abstract. Analytical and experimental research stiffness characteristics of laminated packet are presented in this work. Package consists of alternating thin elastomeric and metallic layers jointed by vulcanization or gluing. Such packages are used in machine building, shipbuilding, civil engineering as compensating devices. The analytical expression of "compressive force – displacement" dependence is derived for the flat thin-layered rubber – metal element on the basis of the variational principle; force is directed flatwise. The analytical expression of "shear force – deformation" is derived for the pre-compressed rubber-metal element. Analytical solution was confirmed by experimental data for flat packet in the shape of rectangular prism.

1. Introduction

Elastomeric (rubber and rubberlike materials) have very small volume compressibility and are able to maintain large elasticity deformation [1-5]. Laminated elastomeric structures consist of a large number of alternating thin layers of elastomeric and much more rigid reinforcing layers, usually metal, jointed by vulcanization or gluing. This allows to obtain the structures with axial compression stiffness much greater than shear stiffness. Packets of thin-layered rubber-metal elements (TRME) are successfully used as compensators, shock-absorbers, vibroisolators. These structures are used in machine building, shipbuilding, civil engineering, aviation and aerospace due to its unique mechanical properties. In practice the TRME packages of different geometric shapes are used: flat, cylindrical, conical, spherical and others; number of layers may be different, at least three. In many applications of TRME structures it is necessary to know its stiffness characteristics, in particular, if TRME packet is used for vibration isolation of the object from vibrating base. In this paper the compressive and share stiffness characteristics of flat TRME of rectangular shape are discussed (Fig. 1a).

TRME packet in Fig. 1 consists of square or rectangular elastomeric and steel layers with dimensions $a \times b \times h_e$ and $a \times b \times h_m$ respectively; further we considered square elements with $a = b$ and $a >> h_e, h_m$. TRME packet is compressed by the axial force P_z in normal to the layers z-direction on Δ-value (Fig. 1b), and then shear force P_x is applied to the margin plate in x-axis direction, causing the displacement of non-elastomeric layers on value f in this direction (Fig. 1c).

For TRME fabrication elastomeric with shear modulus $G_e \approx 0.1 \div 1.2\text{MPa}$ and bulk modulus $K_e \approx 1800 \div 3000\text{MPa}$ are used. Taking into account that $K_e = G_e \dfrac{2(1+\mu)}{1-2\mu}$ the Poisson ratio is equal $\mu \approx 0.485 \div 0.499$.

[1] Corresponding author

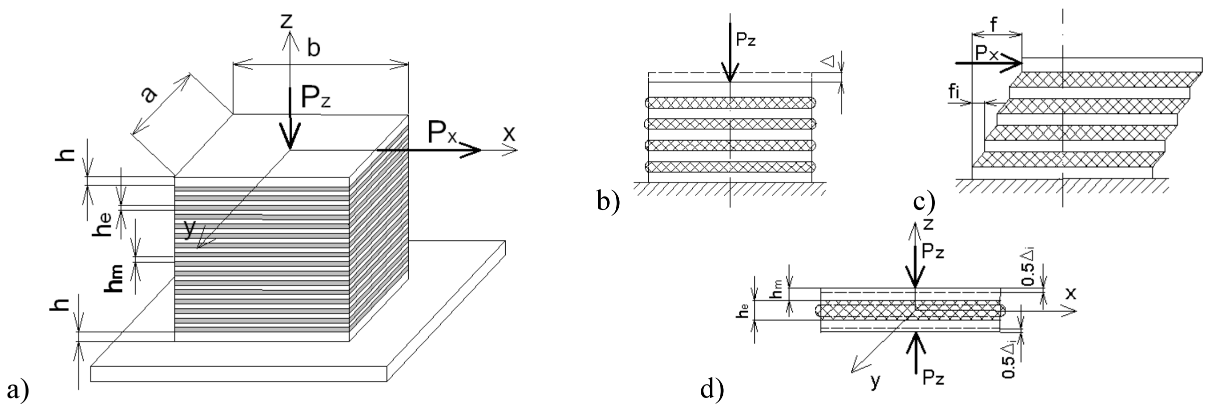


Figure 1. Scheme of testing object: a) flat rectangular TRME packet,
b) deformation of packet under compression, c) deformation of packet under shear force,
d) compressive deformation of rubber layer between two metal layers.
h_e – thickness of elastomeric layer, h_m – thickness of metal layer, h – thickness of margin metal layer.

2. Analytical study of stiffness characteristics of laminated packet

The dependences of "axial force – displacement" (P_z – Δ) and "shear force – shift" (P_x – f) will be received in two stages.

The first stage is axial compression of TRME packet; on this stage the weak compressibility of elastomeric layer, effect of deformation non-elastomeric layer, physical nonlinearity (the dependence of modules G_e and K_e on the hydrostatic pressure), the geometric nonlinearity (for deformation up to 50% ÷ 60%) are taken into account. The dependence of "force – displacement» (P_z – Δ) for deformations up to 10% ÷ 15% may be obtained from the principle of minimum total potential energy of deformation Π:

$$\Pi = \sum_{k=e,m} \left\{ G_k \iiint_{V_k} \left[\frac{1}{4}\left(u^k_{i,j}+u^k_{j,i}\right)\left(u^k_{i,j}+u^{\,k}_{j,i}\right) + \frac{3\mu^k}{1+\mu^k} S^k U^k_{i,i} - \frac{9}{4}\frac{1-2\mu^k}{(1+\mu^k)^2} S^{k2} \right] dV_k - \int_{F^k_\sigma} P^k_i u^k_i \, dF^{\,k}_{\sigma} \right\} \quad (1)$$

where: G_e, G_m – shear moduli of the elastomeric and non-elastomeric material layers, u^k_i – displacement function of elastomeric and non-elastomeric layers, S^k – hydrostatic pressure function for elastomeric and non-elastomeric layers.

Sign "," denotes the partial differentiation. The summation is performed over repeated subscripts.

Compressive stiffness of non-elastomeric layer is by several orders of magnitude larger than of the elastomeric layer. Therefore, we only consider non-elastomeric layers stretching in this plane layer, which is caused by the layer of elastomeric shear stress in the cross sections of their contacts (Fig. 1d). If elastomeric layer is vulcanised to metallic layer, the geometric boundary conditions for the functional (1) are:

$$u(x,y,\pm 0.5h_e) = u_m(\pm 0.5h_m),\;\; v(x,y,\pm 0.5h_e) = v_m(\pm 0.5h_m),\;\; w(x,y,\pm 0.5h_e) = \pm 0.5\Delta \quad (2)$$

where u, v, w – displacement on x, y, z – axis directions.

The hypothesis of plane sections for the elastomeric layer and uniform tensile strain for non-elastomeric layer are used when the displacements functions are selected. In view of introduced hypotheses and joint conditions (2) for a flat square TRME required displacements functions it is sufficient to choose the form of:

$$u_e = Ax(z^2 - 0.25h_e^2) + Lx,\ v_e = Ay(z^2 - 0.25h_e^2) + Ly,\ w_e = -C\left(\frac{z^3}{3} - \frac{zh_e^2}{4}\right) - 2Lz$$
$$S = D\left(z^2 - 0.25h_e^2\right),\ u_m = Lx,\ v_m = Ly,\ w_m = S_m = 0 \tag{3}$$

From the principle minimum potential energy $\frac{\partial \Pi(A,C,D,L)}{\partial(A,C,D,L)} = 0$ for TRME package we obtain the dependence:

$$\Delta = \frac{4P_z h_e N}{\pi^2 G_e a^2}\left[1 + \frac{5}{8}\frac{M}{\upsilon}\right]\left[1 + \frac{0.5M}{1 + (1-2\mu)M}\right]^{-1},\ \upsilon = \frac{G_m h_m}{G_e h_e},\ M = 1 + \frac{\pi^2\alpha^2}{24},\ \alpha = \frac{a}{h_e} \tag{4}$$

where N – the number of elastomeric layers.

When $\upsilon \to \infty$ and $\mu \to 0.5$ from (4) the known dependence of P_z – Δ excluding weak compressibility of the elastomer layer and deformation non-elastomeric layer follows [6]. At high specific axial forces the physical nonlinearity (dependence of G_e and K_e moduli on hydrostatic pressure) must be taken into account. The approximate method may be used, for which it is enough to have only a linear solution (4), in which the shear modulus G_e is replaced by the relationship $G_e(S)$:

$$G_e(S) = G_e(1 + \beta S),\ S \approx \frac{P_z}{F},\ F = a^2 \tag{5}$$

β – coefficient determined from volume compression of elastomer layer experiment [6]. Due to the lack of experimental data, it is assumed that:

$$\frac{G_e(S)}{G_e} \approx \frac{K_e(S)}{K_e} \approx (1 + \beta S) \tag{6}$$

When considering the average strain (up to 50% ÷ 60%) the delta- method is used [6]. In this case, the solution for small strains (4) conveniently to present as the sum of two solutions:

$$\Delta = \Delta_n + \Delta_o \tag{7}$$

where Δ_n – solution for an incompressible material ($\mu = 0.5$), Δ_o – elastomeric layer volume changes.

For TPME $a >> h_e$ and $\alpha = \frac{a}{h_e} >> 1$, in this case from (4) and (7) after simplifying, we have:

$$\Delta_n = \frac{4P_z h_e N}{\pi^2 G_e a^2}\left(1 + \frac{5\pi^2\alpha^2}{192\upsilon}\right)\left(1 + \frac{\pi^2\alpha^2}{48}\right)^{-1},\ \Delta_o = \frac{3(1-2\mu)}{2(1+\mu)}\frac{P_z h_e N}{G_e a^2} \tag{8}$$

Using the technique of the delta method for axial deformation up to 50% ÷ 60%, dependence (P_z – Δ_1) for one elastomer layer is obtained in the form:

$$P_z = G_e a^2\left[-3.6\ln\lambda + 0.25\frac{a^2}{h^2}\left(\lambda^{-2} - 1\right)\right]\left[1 + \frac{a^2\left(\lambda^{-2} - 1\right)}{4h^2\upsilon}\right],\ \lambda = 1 - \frac{\Delta_1}{h_e} \tag{9}$$

At high specific axial forces $\frac{P_z}{a^2}$ the shear modulus G_e in (9) must be replaced by

$G_e(S) = G_e(1+\beta S)$.

From (9) axial displacement of a single layer Δ_1 is found and displacement for package is equal $\Delta = \Delta_1 N$.

At the second stage the shear stiffness of the preliminary compressed by P_z force TRME packet is defined.

The problem is considered as two-dimensional: deformation in z-direction under P_x action is negligibly small in compare with the preloading deformation Δ. Without preloading the dependency ($P_x - f_1$) has the form: $P_x = G_e \dfrac{a^2 f_1}{h_e}$.

Using the linearized theory of superposition of small deformations [6], we obtain the dependence of ($P_x - f_1$) for one elastomeric layer under preliminary axial compression:

$$P_x = G_e \frac{a^2 f_1}{h_e} \lambda, \text{ where } \lambda = 1 - \frac{\Delta_1}{h_e} \qquad (10)$$

It follows from (10) that the shear stiffness of the elastomeric layer decreases continuously with preload compression Δ_1 increasing. This fact is confirmed by experiments [6, 7].

If all elastomeric layers are the same, shear displacements of TRME package under the action of compressive forces P_x is calculated by the formula: $f = f_1 N$.

3. Experimental research of compressive and shear stiffness of TRME packet

A series of experiments for evaluation of the compressive and shear stiffness stiffness were performed for flat TRME of prismatic shape with different thicknesses (from 0.1 to 0.58mm) and number of the rubber layers (10, 16 and 32) in scientific – research institute "VNIIMASH" (Moscow). The table 1 gives the results of share experiments for PRM-35 packet $(a = 35\text{mm}, h_e = 0.1\text{mm}, h_m = 0.1\text{mm})$, and PRM-60 $(a = 60\text{mm}, h_e = 0.28\text{mm}, h_m = 0.1\text{mm})$ elements with number of rubber layers $N = 16$, made by vulcanization, and analytical solution in accordance with Eq. (10).

Table 1. Experimental results of shear displacement of the 16-layer TRME, compressed with vertical force P_z = 15kN.

Horizontal force, P_x	Element PRM-35				Element PRM-60			
	Uniform pressure, P_x / A	Share stiffness eksperiment	Displacement eksperiment	Displacement teoretical	Uniform pressure, P_x / A	Share stiffness eksperiment	Displacement eksperiment	Displacement teoretical
kN	N/mm²	N/mm	mm	mm	N/mm²	N/mm	mm	mm
0.50	0.408	330	1.515	1.499	0.139	460	1.087	1.465
1.00	0.816	270	3.704	2.998	0.278	415	2.410	2.926
1.50	1.224	230	6.252	4.498	0.417	375	4.000	4.394
2.00	1.633	210	9.524	5.997	0.556	350	5.714	5.859
2.50					0.694	325	7.692	7.324
3.00					0.833	315	9.524	8.788

Plots of dependence of shear stiffness on horizontal displacement for PRM-35 PRM-60 elements are given in Fig. 2; plot of dependence of compression stiffness on vertical displacement for element PRM-35 PRM is presented in Fig. 3.

The shear stiffness may be approximated by the curves for FRM-35: $s(x) = 194 + 206e^{-0.265x}$ N/mm , for FRM-60: $s(x) = 290 + 220e^{-0.236x}$ N/mm ; compressive stiffness for FRM-35 $c(z) = 24660z - 993$ kN/mm .

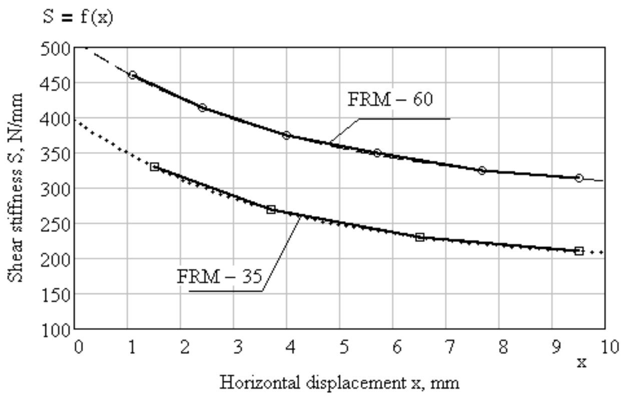


Figure 2. Plots of dependence of shear stiffness on horizontal displacement for elements:
FRM-60: ⊖⊖⊖ – experimental data,
— – – in accordance with approximating curve;
FRM-35: ⊟⊟⊟ – experimental data,
⋯⋯ – in accordance with approximating curve.

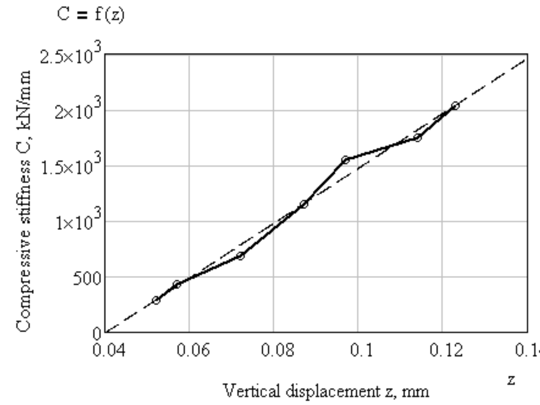
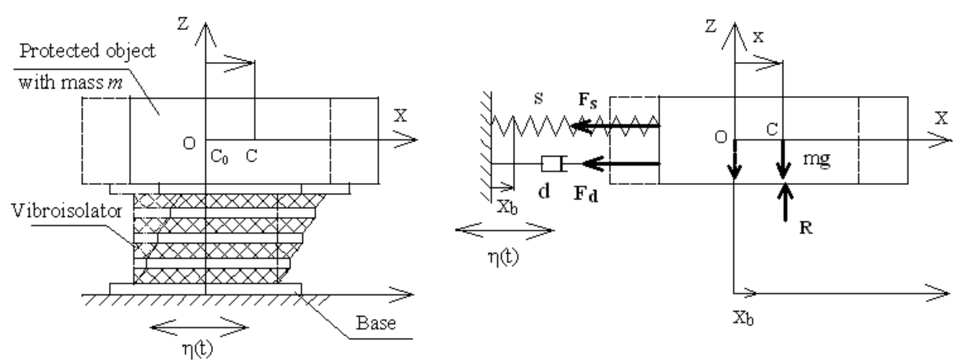


Figure 3. Plots of dependence of compressive stiffness on vertical displacement for element FRM-35: ⊖⊖⊖ – experimental data,
— – – in accordance with approximating curve.

4. Natural frequency of object protected with TRME packet

Static stiffness of TRME structures may be used for preliminary estimation of the natural circular frequency of the single-mass object in case of using TRME packet for protecting it against vibrating. Vibroisolator is placed between the object to be protected and a vibrating base. The lower plate of vibroinsolator is subjected to kinematic excitation. In Fig. 4 the example of TRME packet applying for the protecting the object against against the horizontal vibration of base is shown.



a) b)

Figure 4. TRME packet for vibroisolation:
a) scheme of protected object, b) analytical model of vibration object.

Differential equation of protected object motion: $m\ddot{x} = -s(x - x_b) - d(\dot{x} - \dot{x}_b)$, where x – mass centre displacement in respect to static equilibrium centre, $s = f(x - x_b)$. Equation of motion may be simplified if we denote coordinate of the relative motion of mass centre in respect to the base as $x^r = x - x_b$, then $\ddot{x}^r = \ddot{x} - \ddot{x}_b$, and $\ddot{x} = \ddot{x}^r + \ddot{x}_b$:

$$\ddot{z}^r + \frac{d}{m}\dot{z}^r + \frac{s(x^r)}{m}x^r = -\ddot{\eta}(t)\,,\ \omega_0 = \sqrt{\frac{s(x^r)}{m}} \tag{11}$$

ω_0 – natural circular frequency of vibrating mass, it may be approximately estimated from the "stiffness – displacement" analytical dependence. If exitation frequency is known, the resonance phenomenon possibility may be estimated.

5. Conclusion

The thin-layered rubber-metal structures using in practice are still limited because of the complicated theoretical calculations and the lack of simple calculation models. This paper presents a simple model of a flat TRME package, which takes into account the physical nonlinearity of elastomeric, provided that the task remains geometrically linear. Experimental studies indicate that, under these loads a significant non-linearity of the "force-displacement" stiffness characteristics associated with the physical nonlinearity of elastomeric materials takes place. Traditional methods of calculation do not allow to describe non-linear stiffness characteristics of these elements.

The analytical expressions for "compressive force – displacement" and "share force – deformation" characteristics of the flat TRME in the form of rectangular prism are derived on the basis of the variational principle; metallic plates-layers are assumed to be perfectly rigid.

Analytical solution in accordance with the received formulae shows the good coincidence with experimental data. This proves that the posed problem is completely solved and corresponds to the research of other scientists.

Approximating equations for compressive and share stiffness dependence on displacement was derived and used in the equation of motion of the object, protected against vibrating by means of TRME packet.

In future investigations it is necessary to clarify the dissipative properties of the rubber and to develop a model of dissipative forces; to clarify the limits of application of the formulas for calculating of static stiffness of the TRME dampers; to develop a model of dynamic compression stiffness and its verifying.

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