# Kinematic Analysis of a Novel Vibratory Bowl Feeder

# Wen Hsiang Hsieh<sup>1</sup>, Guan-Heng Lin<sup>2</sup> and Chia-Heng Tsai<sup>3</sup>

<sup>1</sup> Professor, <sup>2</sup> Graduate Student, <sup>3</sup> Student Department of Automation Engineering, National Formosa University, No. 62, Wenhua Rd., Huwei Township, Yunlin 632, Taiwan, ROC

E-mail: allen@nfu.edu.tw

**Abstract.** This study aims to propose a kinematic analysis approach for a novel vibratory bowl feeder, and verify its feasibility by kinematic simulation. First, the novel design is presented, and its advantages compared with conventional feeders are described. Moreover, the equations for kinematic analysis are derived. Finally, the solid model for the proposed design is established, and then kinematic simulation is performed by ADAMS software. The simulation results indicate that the proposed design can effectively advances the part with high efficiency.

# 1. Introduction

A feeder (conveyor) refers to a machine that can continuously or intermittently transport parts according to the specified route. Since the feeder can perform the operations of screening, classifying, and orienting, and etc., it is extensively used in production and manufacturing, e.g., in electronic, mechanical, food, medical, and chemical industries. Nowadays, it is one of important and indispensable automatic machinery.



Figure 1. Vibratory bowl feeder [1].



Vibratory feeders consist of troughs or tubes flexibly supported and vibrated by mechanical or electrical means to convey objects or bulk materials.Vibratory feeders could be classified into the vibratory bowl feeder and the inline vibrating feeder by the moving path of the part or the shapes of the trough, as shown Fig. 1 and Fig. 2, respectively. The parts on feeders move along their tracks as a result of the vibration produced by the feeder. A vibratory feeder has the disadvantages of huge volume, short distance of transportation, large noise, and easy damage to the parts. In addition, its service life may be reduced due to the fatigue induced by vibration. Therefore, there have been many attempts to improve its performance.

In 1971, Gladwell and Masour [3] established the theoretical model for the vibrating feeders. In

<sup>&</sup>lt;sup>1</sup> To whom any correspondence should be addressed.

1972, Mansour [4] formulated the theoretical model of inline vibrating feeders, and investigated the effect of conveyor parameters on the motion of conveying parts. During 1990, El-Shakery et al. [5] conducts the analysis and design of inline feeders driven by linkages. In 2005, Ting et al. [6] proposed an inline conveyor driven by a bimorph piezoelectric actuator, and its dynamic modeling, driving control circuit design, motion trajectory analysis, and optimal transport feed are examined. In 2010 & 2012, Hsieh & Tsai [6-7] proposed a novel inline vibrating feeder, and. presented its approach of design and analysis, respectively.

The purpose of this study is to propose a novel vibratory bowl feeder, and verify its feasibility by kinematic simulation. In Section 2, the proposed new feeder will be presented. Then, Section 3 will be devoted to conduct the kinematic analysis. Furthermore, Section 4 will define the advancing efficiency for evaluating the performance of the feeder. In Section 5, kinematic simulation will be carried out with a specified example. Finally, conclusions will be drawn in Section 6.

## 2. New feeder

Fig. 3 shows the proposed new vibratory bowl feeder. The feeder is composed by a motor, a variable speed coupling (generalized Oldham coupling), and a bowl. The motor and the reducer are connected by the constant speed coupling, and then drive the bowl through the variable speed coupling. Since the variable speed coupling, in the proposed system, can be easily replaced with various dimensions of the coupling, or changed with different assembly configurations. Hence, the angular displacement, velocity, and acceleration of the output can be adjusted to meet different motion requirements with proper design.



Figure 5. Equivalent mechanism.

Since only a mechanism with one degree of freedom is used, the structure of the proposed system is simple and reliable, and the building cost is low. Moreover, the coupling can act as a flywheel, hence the proposed system is energy effective, and can produce larger impulsive forces. Compared to existed feeders, the proposed feeder can produce more complicated motion, if a proper design is conducted.

Fig. 4 and 5 show the generalized Oldham coupling [8] and it's kinematically equivalent

mechanism, respectively, employed in the proposed feeder. Points  $a_0$  and  $b_0$  denote the axes of rotation of the input and output disks, respectively. Points a and b denote the arc centers of the slots of the input and output disks, respectively. And point o is the point of intersection of the two arcs. Moreover, oa  $=r_1$  and ob  $=r_2$  are the radii of the circular slots of input disk and output disk, respectively.

## 3. Kinematic analysis

The main approaches for kinematic analysis of closed loop mechanisms are relative velocity/acceleration, vector loop, and matrix loop approaches. The vector loop approach can be easily computerized and is suitable for the analysis of planar mechanisms, therefore it is adopted for the kinematic analysis in this study.



Figure 6. Vector representation.

# 3.1. Position analysis

Fig. 6 shows the vector representation of the proposed design. It has two independent vector loops, and their equations can be formulated as

$$\vec{r}_2 + \vec{r}_3 - \vec{r}_4 - \vec{r}_1 = 0 \tag{1}$$

Eq. (1) can be resolved into its x and y components, respectively, as

$$r_2\cos\theta_2 + r_3\cos\theta_3 - r_4\cos\theta_4 - r_1\cos\theta_1 = 0$$
<sup>(2)</sup>

$$r_2 \sin \theta_2 + r_3 \sin \theta_3 - r_4 \sin \theta_4 - r_1 \sin \theta_1 = 0$$
(3)

where  $\theta_1$  is set as 0. Rearranging Eqs. (2) and (3), we have

$$r_3\cos\theta_3 = -r_2\cos\theta_2 + r_4\cos\theta_4 + r_1 \tag{4}$$

$$r_3 \sin \theta_3 = -r_2 \sin \theta_2 + r_4 \sin \theta_4 \tag{5}$$

The square sum of Eqs. (4) and (5) can be found to be

$$A\cos\theta_4 + B\sin\theta_4 = C \tag{6}$$

where

$$A = 2r_4(r_1 - r_2\cos\theta_2)$$
 (7)

$$B = -2r_2r_4\sin\theta_2\tag{8}$$

$$C = (r_3^2 - r_4^2 - r_1^2 - r_2^2) + 2r_1r_2\cos\theta_2$$
(9)

Dividing Eq. (6) by  $\sqrt{A^2 + B^2}$  and simplifying it, we obtain

$$\cos\phi\cos\theta_4 + \sin\phi\sin\theta_4 = \frac{C}{\sqrt{A^2 + B^2}}$$
(10)

where

$$\cos\phi = \frac{A}{\sqrt{A^2 + B^2}} \tag{11}$$

and

$$\sin\phi = \frac{B}{\sqrt{A^2 + B^2}} \tag{12}$$

Applying the adding formula of trigonometric functions to Eq. (10), then the closed form of  $\theta_4$  can be solved by

$$\theta_4 = \phi \pm \cos^{-1} \left[ \frac{C}{\sqrt{A^2 + B^2}} \right] \tag{13}$$

Dividing Eq. (5) by Eq. (4), and then followed by rearrangement, it yields

$$\theta_3 = \tan^{-1} \left[ \frac{r_4 \sin \theta_4 - r_2 \sin \theta_2}{r_1 + r_4 \cos \theta_4 - r_2 \cos \theta_2} \right]$$
(14)

## 3.2. Velocity analysis

Velocity analysis can be worked out by differentiating the equations deduced in position analysis with respect to time. Due to the limit on the number of pages, the results for velocity and acceleration analysis are presented directly without proofs. For velocity analysis, their deduced equations are

$$\dot{\theta}_3 = \frac{r_2}{r_3} \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \dot{\theta}_2 \tag{15}$$

$$\dot{\theta}_4 = \frac{r_2}{r_4} \frac{\sin(\theta_3 - \theta_2)}{\sin(\theta_3 - \theta_4)} \dot{\theta}_2 \tag{16}$$

#### 3.3. Acceleration analysis

Acceleration analysis can be performed by differentiating the equations derived in velocity analysis with respect to time. The derived equations are

$$\ddot{\theta}_3 = \frac{FH - EI}{DH - EG} \tag{17}$$

$$\ddot{\theta}_4 = \frac{DI - FG}{DH - EG} \tag{18}$$

where

$$D = -r_3 \sin \theta_3 \tag{19}$$

$$E = r_4 \sin \theta_4 \tag{20}$$

$$F = r_2 \sin \theta_2 \dot{\theta}_2 + r_2 \cos \theta_2 \dot{\theta}_2^2 + r_3 \cos \theta_3 \dot{\theta}_3^2 - r_4 \cos \theta_4 \dot{\theta}_4^2$$
(21)

95

$$G = r_3 \cos \theta_3 \tag{22}$$

$$H = -r_4 \cos \theta_4 \tag{23}$$

$$I = -r_2 \ddot{\theta}_2 \cos \theta_2 + r_2 \dot{\theta}_2^2 \sin \theta_2 + r_3 \dot{\theta}_3^2 \sin \theta_3 - r_4 \dot{\theta}_4^2 \sin \theta_4$$
(24)

## 4. Advancing efficiency

If the absolute value of the negative acceleration is higher than the maximum static friction  $F_s$ , between the part and the tough surface, then the part will advance on the trough. On the contrary, if the positive acceleration is higher than  $F_s$ , then the part will move backward. The former is much larger than  $F_s$ , and the later is only slightly higher, therefore the part will effectively advance on the trough. In this work, the net advance  $s_n$  is defined as

$$s_n = s_a - s_b \tag{25}$$

where  $s_a$  and  $s_b$  are the distances of advancing and retreating, respectively. Moreover, the advancing efficiency  $\eta$  of the conveyer is defined as the ratio of the net advance to the distance of advance forward, i.e.,

$$\eta = \frac{s_n}{s_a} \tag{26}$$

#### 5. Simulation

The kinematic dimensions of a design example are specified in Table 1. The solid model, as shown in Fig. 6, can then be drawn by CAD software. After that, the model is introduced into ADAMS software for kinematic simulation, where the input rotational speed of 100rpm and the static friction coefficient, between the part and the trough, of 0.3 are used in the simulation.

Fig. 8 shows the displacement of the advancing by the simulation. It can be found from the figure that  $S_a$  and  $S_b$  are 304.15 and 30.59mm, respectively. By substituting them into Eq. (25), it yields that  $s_n$  in a cycle is 273.56mm. Moreover, the advancing efficiency is 89.94, found by Eq. (26). Therefore, the proposed feeder is of relatively high efficiency.



Figure 7. ADAMS model.

KINEMATIC ANALYSIS OF A NOVEL VIBRATORY BOWL FEEDER. WEN HSIANG HSIEH, GUAN-HENG LIN AND CHIA-HENG TSAI



Figure 8. Advance of the part.

## 6. Conclusions

In this work, a novel design for vibratory bowl feeder has been proposed. The structure of the proposed design has been presented. Kinematic dimensions have been identified by investigating its kinematically equivalent linkage, and then kinematic analysis is conducted subsequently. Kinematic simulation has been conducted by utilizing ADAMS software, and the feasibility of the proposed design has been verified. The result has indicated that the proposed new design can effectively advance the part with high efficiency.

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