Research on optimum design of temperature-vibration accelerated storage test plan

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Abstract. In allusion to the phenomenon that single stress is being used to conduct accelerated storage test currently, in order to establish a reasonable accelerated storage test program, an optimization design method of temperature and vibration step-down-stress accelerated storage test based on cumulative damage method was proposed. The minimum asymptotic variance of product's medium life estimated value at normal working stresses is taken as optimum target, each accelerated stress level as design variable. Meanwhile, employing maximum likelihood estimation theory for fisher matrix, the mathematic model of multi-stress accelerated storage test plan's optimal design was built. Comparing the minimum asymptotic variance of each scheme by an example of space electric connector, $k = 5$ is determined to be the optimal scheme. Using Monte-Carlo simulation to analyze volatility of minimum asymptotic variance and mean minimum asymptotic variance, it further proves the scheme when k is 5 to be optimal. The optimized test plan can not only solve optimization design and evaluation for small sample accelerated storage test, but also can ensure the accuracy of evaluation, reduce test times, shorten test time and save cost.

Keywords: electromechanical products, multiple stresses, step-down-stress, accelerated storage test, optimum design, the minimum asymptotic variance.

1. Introduction

With the development of social requirement and technology, more and more attention is paid to high-reliability and long-lifetime products of aviation, aerospace and other applications; especially the reliability of current products has become more sophisticated, and it takes a long time to gain the storage reliability of product in a natural environment. Therefore, accelerated life testing (ALT) methodology is becoming a central issue in the reliability testing domain. The single stress acceleration model has been widely used in engineering currently, such as Arrhenius model, etc. [1]. However, products during actual storage period will be affected by two or more environmental stresses at the same time, such as temperature, vibration, humidity, etc. In order to simulate the real environmental stresses when electromechanical products encountered in usage, Nelson put forward the accelerated life test plan and optimization method under multiple stresses [2]. Under the precondition of no interaction among stresses, Escobar and Meeker put forward the optimization design method and criterion used for censored data of double stresses accelerated life test plan [3]. The theory and method of multi-stress accelerated life testing scheme optimization design under temperature and vibration is given in Ref. [4-5]. Ref. [6] discussed multiple stresses accelerated life testing scheme optimization design theory and method based on temperature and vibration test regularly under constant stress. However, among all kinds of accelerated life test methods, constant stress accelerated life test is mostly applied at present, while step-down-stress method is least seen both at home and abroad. The multi-stress accelerated storage test based on step down stress, as a test method of more efficient and closer to actual working environment [7-8], has become a development direction of accelerated storage test in the practical engineering application.

In this paper, taking electromechanical products as research object, a general model for describing failure processes is established by converting degradation processes at each stress level

to the first stress level based on accumulative damage theory [9]. Taking the minimum asymptotic variance of electromechanical products medium life as optimization goal [12-15], this paper put forward an optimization design method of temperature-vibration multi-stress accelerated storage test scheme, which provides technical support for high reliability and long life electromechanical products multi-stress accelerated storage test based on step down stress.

2. Reliability statistical model of electromechanical products

2.1. The failure distribution function

Under the impact of environmental stresses during the long storage period, the life distribution of electronic equipment obeys Weibull distribution [9], denoted as Weibull (m, η).

2.2. Accelerated model

The general Eyring model [10], which is applicable to the condition where the failure is caused by temperature stress and vibration stress at the same time, can be represented as follows:

$$t_L = A \cdot S^B \exp\left(\frac{E_a}{kT}\right), \tag{1}$$

where, t_L denotes characteristics of products; E_a is activation energy, usually in electron-volts; k stands for the Boltzmann's constant; T represents the absolute Kelvin temperature; S represents vibration strength (unit: g²/Hz); A and B are parameters to be estimated, $B < 0$.

Taking logarithm function on both sides, let $\theta_0 = \ln A$, $\theta_1 = Ea/1000k$, $x = 1000/T$, $\theta_2 = B$, $y = \ln H$ the distribution parameters of product lifetime is the function of stress x and y, namely:

$$\ln \eta = \mu(x, y) = \theta_0 + \theta_1 x + \theta_2 y. \tag{2}$$

3. Stress accumulation effect

Assuming that: The remaining lifetime of specimens only depends on the currently cumulative fraction failed and current stress regardless how the fraction accumulated [11]. Specifically, the cumulative failure probability $F_i(\tau_i)$ cumulated under stress level S_i by time τ_i is equal to the cumulative failure probability $F_j(\tau_{ij})$ cumulated under stress level S_j by time τ_j. Under Weibull distribution, there is:

$$1 - \exp\left\{-\left(\frac{\tau_i}{\eta_i}\right)^{m_i}\right\} = 1 - \exp\left\{-\left(\frac{\tau_{ij}}{\eta_j}\right)^{m_j}\right\}. \tag{3}$$

According to the basic assumption $m_i = m_j$, the acceleration factor of S_i relative to S_j can be represented as:

$$\tau_{ij} = \frac{\eta_j \tau_i}{\eta_i} = \tau_{S_i \sim S_j} \tau_i = \tau_i e^{[\theta_1(x-x_i)+\theta_2(y-y_j)]}. \tag{4}$$

Under step-down-stress accelerated storage test, the failure time is not the product's real lifetime under S_i besides $i = 1$. Therefore the real lifetime data under stress level S_i can be represented as:

$$t'_{ij} = \sum_{h=1}^{i} a_i + t_{ij}, \quad 1 \leq j \leq r_i, \quad 1 \leq i \leq k, \tag{5}$$

where $a_1 = 0$, $a_i = \tau_{1i} + \tau_{2i} + \cdots + \tau_{i-1,i} = \sum_{j=1}^{i-1}(\eta_i \tau_j / \eta_j)$, $i = 2,\ldots, k$. a_i is the compensation of failure time $t_{i1} \leq t_{i2} \leq \cdots \leq t_{ir_i} \leq \tau_i$ converted by working time $\tau_1, \tau_2, \ldots, \tau_{i-1}$ under $S_1, S_2, \ldots, S_{i-1}$.

4. The maximum likelihood estimation (MLE) of test optimization scheme

4.1. Likelihood function of censored samples

In order to simplify the formula, let $\delta = \ln t$, the Weibull distribution can be transformed to extreme value distribution. Its probability density function is:

$$f(\delta) = \left(\frac{1}{\sigma}\right) \exp\left[\frac{\delta - \mu}{\sigma}\right] \exp\left\{-\exp\left[\frac{\delta - \mu}{\sigma}\right]\right\}, \tag{6}$$

where, $\mu = \ln\eta$. For ith test specimen, the failure time under stress combination is δ_i. Then:

$$I_i = \begin{cases} 1, & \delta_i < \tau_i \quad \text{(the failure time is smaller than } \tau_i\text{)}, \\ 0, & \delta_i \geq \tau_i \quad \text{(the failure time is bigger than } \tau_i\text{)}. \end{cases} \tag{7}$$

Therefore, the likelihood function of ith test specimen can be denoted as follows:

$$L_i = I_i \ln f(\delta_i) + (1 - I_i) \ln R(\tau_i). \tag{8}$$

Let $z_i = (\delta_i - \mu)/\sigma$, $\xi_i = (\tau_i - \mu)/\sigma$, where, $\mu = \theta_0 + \theta_1 x_i + \theta_2 y_i$. The likelihood function of the whole test sample can be represented as follows:

$$L = \sum_{i=1}^{n} \{I_i[-\ln\sigma + z_i - \exp(z_i)] + (1 - I_i)[-\exp(\xi_i)]\}. \tag{9}$$

4.2. Fisher information matrix and covariance matrix

According to the maximum likelihood estimation theory, the variance and covariance matrix of model parameter estimates can be presented as:

$$\boldsymbol{\Sigma} = \begin{pmatrix} \text{Var}\hat{\theta}_0 & \text{Cov}(\hat{\theta}_0, \hat{\theta}_1) & \text{Cov}(\hat{\theta}_0, \hat{\theta}_2) & \text{Cov}(\hat{\theta}_0, \hat{\sigma}) \\ & \text{Var}\hat{\theta}_1 & \text{Cov}(\hat{\theta}_1, \hat{\theta}_2) & \text{Cov}(\hat{\theta}_1, \hat{\sigma}) \\ & & \text{Var}\hat{\theta}_2 & \text{Cov}(\hat{\theta}_2, \hat{\sigma}) \\ \text{symmetrical} & & & \text{Var}\hat{\sigma} \end{pmatrix}, \tag{10}$$

where, $\boldsymbol{\Sigma}$ is the inverse matrix of information matrix, namely, $\boldsymbol{\Sigma} = \mathbf{F}^{-1}$. While, the information matrix is the expectation of likelihood function's negative second order partial derivative matrix. Combined with Eq. (10), there is:

$$\mathbf{F} = \sum_{i=1}^{k} \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ & A_{22} & A_{23} & A_{23} \\ & & A_{33} & A_{34} \\ \text{symmetrical} & & & A_{44} \end{pmatrix}, \tag{11}$$

where, A_{ij} is mathematical expectation of parameters $\theta_0, \theta_1, \theta_2, \sigma$.

5. Mathematical model of testing scheme optimization design

5.1. Objective function

Assuming that temperature stress levels and vibration stress levels are both k, which satisfies $T_1 > T_2 ... > T_k$, $S_1 > S_2 ... > S_k$. And test sample is n. According to uniform design theory, the SDSAST only need to be conducted k steps [12]. For linear extremum model, under the stress combination x_i, y_j, the P_{th} order quantile of product life distribution is $\hat{g}_P(x_0, y_0) = \hat{\theta}_0 + \hat{\theta}_1 x + \hat{\theta}_2 y + z_P \hat{\sigma}$ [13]. When $P = 0.5$, $z_p = -0.3665$, the P_{th} order quantile $\hat{g}_P(x_0, y_0)$ of product lifetime is the product's median life [2]. Under the normal stress level combination x_0, y_0, the asymptotic variance can be represented as:

$$\text{Var}[\hat{g}_P(x_0, y_0)] = (1, x_0, y_0, z_p)\mathbf{F}^{-1}(1, x_0, y_0, z_p)' = (1, x_0, y_0, z_p)\mathbf{\Sigma}(1, x_0, y_0, z_p)'. \tag{12}$$

Regarding the minimum asymptotic variance of normal stress level combination as objective function, namely solving $\min\text{Var}[\hat{g}_P(x_0, y_0)]$ [14].

5.2. Selection of design variables

Now that the maximum temperature stress level, the highest vibration stress level, normal temperature and vibration stress level are all given, the P_{th} order quantile estimates of normal life distribution only depends on the rest two groups of k-1 stress levels and failure time considering stress accumulation effect, namely, the optimization design variables are mainly $T_1 > T_2 ... > T_k$, $S_1 > S_2 ... > S_k$, and accumulation failure time $t_1(\theta_1, \theta_2)$, $t_2(\theta_1, \theta_2),\ldots, t_r(\theta_1, \theta_2)$.

5.3. Constraint conditions

1) Stress level: Before stress combination, temperature stress levels must satisfy $T_1 > T_2 ... > T_k$ and vibration stress levels must satisfy $S_1 > S_2 ... > S_k$.

2) In order to guarantee a certain acceleration effect, the acceleration factor $\alpha_{i\sim0}$ of all levels relative to normal stress level must be bigger than a certain value, namely: $\alpha_{i\sim0} = \exp\{\theta_1(x_0 - x_i) + \theta_2(y_0 - y_i)\} \geq \alpha_{min} = \eta_0/\eta_i$ and $a_{1\sim0} > a_{2\sim0} > \cdots > a_{k\sim0}$, $i = 1, \ldots, k$.

3) The combination of temperature and vibration stress should satisfy uniform design table.

5.4. Optimization model

Summing up conditions above, the mathematical model of multi-stress ALT optimization design scheme can be concluded as follows:

$$\begin{cases} \min\text{Var}[\hat{g}_P(x_0, y_0)] = (1, x_0, y_0, z_P)\mathbf{\Sigma}(1, x_0, y_0, z_P)^T, \quad P = 0.5, \\ T_1 > T_2 ... > T_k, \quad S_1 > S_2 ... > S_k, \\ T_i, S_j \quad \text{satisfies the uniform design table.} \end{cases} \tag{13}$$

6. Numerical example

6.1. Multi-stress accelerated storage test optimization design of an electric connector

6.1.1. Multi-stress accelerated storage test of an electric connector

Space electric connectors obey Weibull distribution. There are 6 sets of products to be conducted SDSAST. Temperature and vibration are the accelerated stresses. Test is started from

the highest stress combination and turned to the next stress combination when there is a failure product and will be continued unless there is a failure product under the stress combination of k. Under the storage condition, the normal temperature and vibration stress level are $T_0 = 25°C$ and $S_0 = 0.06$ g^2/Hz respectively. The initial temperature accelerated stress T_1 can be determined by work limit stress through reliability enhancement testing, which is $T_1 = 158°C$. By referring other test experience, the required highest vibration stress is determined to be $S_1 = 1.0$ g^2/Hz. The least temperature level T_k and vibration stress level S_k are $T_k = 95°C$ and $S_k = 0.20$ g^2/Hz.

6.1.2. Optimized design results

The estimates of $\hat{\theta}_0, \hat{\theta}_1, \hat{\theta}_2, \hat{\sigma}$ can be obtained by referring prior experience. Employing Monte-Carlo simulation, the failure data of electromechanical products by simulation can be obtained. The asymptotic variances of product's median life $\hat{g}_{0.5}(x_0, y_0)$ is minimum when $k = 5$, which is minVar$[\hat{g}_P(x_0, y_0)] = 0.0034$. So, it is determined that the scheme is optimal when $k = 5$. The optimized multi-stress accelerated storage test scheme is shown as Table 1.

**Table 1.** Optimization design scheme of accelerated storage test when $k = 5$

No.	Temperature (K)	Temperature level	Vibration (g^2/Hz)	Vibration level
1	$T_1 = 431.15$	2.3194	$S_2 = 0.80$	–0.2231
2	$T_2 = 413.46$	2.4186	$S_5 = 0.20$	–1.6094
3	$T_3 = 397.17$	2.5178	$S_3 = 0.60$	–0.5108
4	$T_4 = 382.11$	2.6170	$S_1 = 1.00$	0
5	$T_5 = 368.15$	2.7163	$S_4 = 0.40$	–0.9163

6.2. Monte-Carlo simulation and verification

In order to illustrate the rationality of optimization scheme, 100 groups of experimental data are generated employing Monte-Carlo to simulate each accelerated storage test schema.
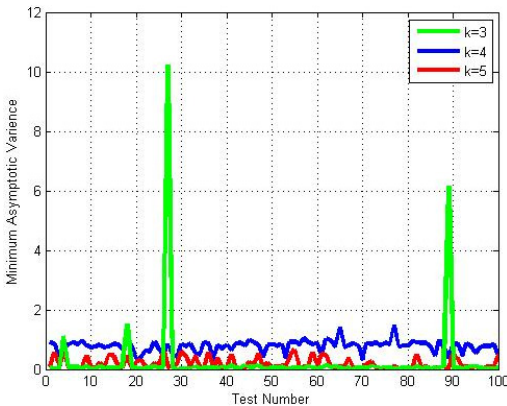
Fig. 1. Minimum asymptotic variance value volatility of each program
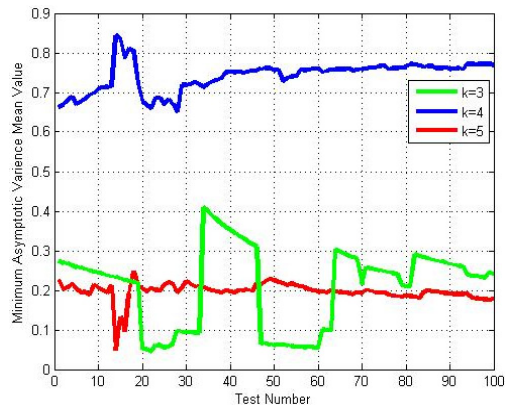
Fig. 2. Minimum asymptotic variance mean value volatility of each program

Drawing the variation tendency figure of the minimum asymptotic variance and the minimum asymptotic variance mean value changing with experiment times, which are shown as Fig. 1 and Fig. 2. From Fig. 1, it is easy to see that the asymptotic variance of median life is minimum when $k = 5$. From Fig. 2, we see that the minimum asymptotic variance mean value is maximum when $k = 4$, median when $k = 3$, but has a bigger fluctuation. While the minimum asymptotic variance mean value is minimum when $k = 5$ and has a smaller fluctuation. In conclusion, the combination scheme is optimal when $k = 5$ in the combination schemes.

7. Conclusion

1) This paper develops an optimum design scheme of multi-stress SDSAST based on uniform design theory, which tackles the problem that testing is difficult to conduct when the factors and stress levels are bigger. Using this method, it can reach the result very close to full times test with less experiment times under the premise of estimation precision basically maintaining unchanged.

2) Considering cumulative damage effect, this paper put forward a method of establishing multi-stress SDSAST optimization design mathematical model of electromechanical equipment regarding the minimum asymptotic variance of equipment median life estimation under normal stress as the goal.

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