1716. PD control for global stabilization of an n-TORA system

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(Received 27 May 2015; accepted 2 August 2015)

Abstract. This paper concerns a global stabilization problem for an n-TORA (Translational Oscillator with a Rotational Actuator) system which consists of n carts connected to the fixed walls and each other by $n+1$ linear springs with each cart having an eccentric rotational proof-mass actuator moving in the horizontal plane. First, this paper derives the motion equation of the n-TORA system. Then, by using Lyapunov stability theory and physical properties of mechanical parameters of the n-TORA system, this paper proves that the global stabilization of the n-TORA system can be achieved by the PD control of the angle of the rotational proof-mass of each TORA. This paper presents numerical simulation results for 2- and 3-TORA systems to validate the result of the global stabilization.

Keywords: underactuated system, global stabilization, translational oscillator, rotational actuator, PD control, Lyapunov stability theory.

1. Introduction

In the last two decades, there has been a lot of interest in the control design and analysis of underactuated systems, defined as mechanical systems in which the dimension of the configuration space exceeds that of the control input space; that is, with fewer control inputs than degrees of freedom. The TORA (Translational Oscillator with a Rotational Actuator) system which is a simplified model of a dual-spin spacecraft for investigating the resonance capture phenomenon, has been proposed as a benchmark problem for nonlinear system design [1, 2]. Since the translational position of the cart is unactuated, the TORA system has also been studied as a typical example of underactuated systems.

The global stabilization problem of the TORA system has attracted many researchers [1, 3-7]. It was shown that the TORA system is globally stable under the PD control of the angle of the rotational proof-mass [8]. A system consisting of n rotor-type oscillators mounted on a straight-line spring-mass system was proposed in [9] to investigate self-synchronized phenomena which are found in mechanics, electricity, chemistry, and biology [10], where each unit of the rotor-type oscillator is the same with the rotor moving in the vertical plane.

Motivated by the above researches, in this paper, we concern an n-TORA system consisting of n-TORAs with a spring connecting two neighboring TORAs (Fig. 1), and we study the global stabilization problem for a general n-TORA system in which all TORAs are not assumed to be the same rather than the synchronization problem. To the best of our knowledge, such a stabilization problem has not been studied. First, this paper derives the motion equation of the n-TORA system. Then, by using Lyapunov stability theory and physical properties of mechanical parameters of the n-TORA system, we prove that the global stabilization of the n-TORA system can be achieved by the PD control of the angle of the rotational proof-mass of each TORA.

The remainder of this paper is organized as follows: Section 2 presents the motion equation of the n-TORA system. Section 3 derives a PD controller for the global stabilization and presents a global motion analysis of the n-TORA system under the derived PD controller. Section 4 gives numerical simulation results for 2- and 3-TORA systems, and Section 5 makes some concluding remarks.

2. Motion equation of n-TORA system

We consider the n-TORA system (Fig. 1) consists of n carts connected to the fixed walls and each other by $n+1$ linear springs with each cart having an eccentric rotational proof-mass actuator moving in the horizontal plane. All carts are constrained to have one-dimensional travel in the horizontal plane.

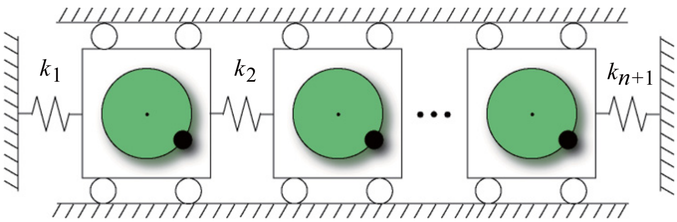

Fig. 1. n translational oscillators with n rotational actuators: an n-TORA system

For the ith TORA (Fig. 2), q_i is the translational position of the ith cart, θ_i is the angular position of the rotational proof-mass, k_i is the spring stiffness, M_i is the cart mass, m_i is the eccentric mass, e_i is the distance between eccentricity and axis of rotation (AOR), I_i is the moment of inertia around the AOR, and N_i denotes the control torque applied to the rotational proof-mass.

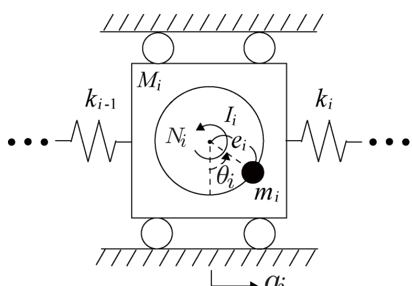

Fig. 2. ith translational oscillator with ith rotational actuator

Below, we derive the motion equation for the n-TORA system by using the Euler-Lagrange equation [11]. The total kinetic energy of the n-TORA system is:

$$T = \sum_{i=1}^{n} T_i, \tag{1}$$

where T_i is the total kinetic energy of the ith TORA. To obtain T_i, we express (X_i, Y_i), the coordinates of the ith actuator as:

$$X_i = q_i + e_i \sin\theta_i, \quad Y_i = -e_i \cos\theta_i. \tag{2}$$

T_i can be expressed as:

$$T_i = \frac{1}{2}\left(M_i \dot{q}_i^2 + m_i\left(\dot{X}_i^2 + \dot{Y}_i^2\right) + I_i \dot{\theta}_i^2\right), \tag{3}$$

where $M_i \dot{q}_i^2/2$ is the translational kinetic energy of the ith cart, $m_i\left(\dot{X}_i^2 + \dot{Y}_i^2\right)/2$ is the translational kinetic energy of the ith actuator, and $I_i\dot{\theta}_i^2/2$ is the rotational kinetic energy of the ith actuator. Direct computation yields:

$$T_i = \frac{1}{2}\left((M_i + m_i)\dot{q}_i^2 + 2m_i\dot{q}_i e_i\dot{\theta}_i \cos\ \theta_i + (m_i e_i^2 + I_i)\dot{\theta}_i^2\right). \tag{4}$$

Let us consider the potential energy P of the n-TORA system. We obtain:

$$P = \frac{1}{2}\sum_{i=1}^{n+1} k_i\,(q_i - q_{i-1})^2, \tag{5}$$

where $q_0 = q_{n+1} = 0$.

The Lagrangian function of the n-TORA system is:

$$\begin{aligned} L = T - P = \frac{1}{2}\sum_{i=1}^{n}\left((M_i + m_i)\dot{q}_i^2 + 2m_i\dot{q}_i e_i\dot{\theta}_i \cos\theta_i + (m_i e_i^2 + I_i)\dot{\theta}_i^2\right) \\ -\frac{1}{2}\sum_{i=1}^{n+1} k_i\,(q_i - q_{i-1})^2. \end{aligned} \tag{6}$$

Using the Euler-Lagrange equation for q_i, θ_i and N_i expressed below:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0,\ \ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \frac{\partial L}{\partial \theta_i} = N_i,\ \ i = 1, 2, \dots, n,$$

yields:

$$\begin{aligned} &\frac{\partial L}{\partial \dot{q}_i} = (M_i + m_i)\dot{q}_i + m_i e_i\dot{\theta}_i \cos\theta_i, \\ &\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = (M_i + m_i)\ddot{q}_i + m_i e_i\left(\ddot{\theta}_i \cos\ \theta_i - \dot{\theta}_i^2 \sin\theta_i\right), \\ &\frac{\partial L}{\partial q_i} = -k_i(q_i - q_{i-1}) - k_{i+1}(q_i - q_{i+1}), \\ &\frac{\partial L}{\partial \dot{\theta}_i} = m_i e_i\dot{q}_i \cos\theta_i + (m_i e_i^2 + I_i)\dot{\theta}_i, \\ &\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) = m_i e_i\ddot{q}_i \cos\theta_i - m_i e_i\dot{q}_i\dot{\theta}_i \sin\theta_i + (m_i e_i^2 + I_i)\ddot{\theta}_i, \\ &\frac{\partial L}{\partial \theta_i} = -m_i e_i\dot{q}_i\dot{\theta}_i \sin\theta_i. \end{aligned}$$

Thus, we obtain:

$$\begin{aligned} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = (M_i + m_i)\ddot{q}_i + m_i e_i\left(\ddot{\theta}_i \cos\theta_i - \dot{\theta}_i^2 \sin\theta_i\right) \\ +(k_i + k_{i+1})q_i - k_i q_{i-1} - k_{i+1}q_{i+1}, \end{aligned} \tag{7}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \frac{\partial L}{\partial \theta_i} = (I_i + m_i e_i^2)\ddot{\theta}_i + m_i e_i\ddot{q}_i \cos\theta_i, \tag{8}$$

which yields:

$$\begin{cases}(M_i+m_i)\ddot{q}_i+(k_i+k_{i+1})q_i-k_iq_{i-1}-k_{i+1}q_{i+1}=-m_ie_i\left(\ddot{\theta}_i\cos\theta_i-\dot{\theta}_i^2\sin\theta_i\right),\\ (m_ie_i^2+I_i)\ddot{\theta}_i=-m_ie_i\ddot{q}_i\cos\theta_i+N_i.\end{cases} \tag{9}$$

From Eq. (1) and Eq. (5), the total mechanical energy of the n-TORA system, E, satisfies:

$$\begin{aligned}E&=T+P\\ &=\frac{1}{2}\sum_{i=1}^{n}\left((M_i+m_i)\dot{q}_i^2+2m_i\dot{q}_ie_i\dot{\theta}_i\cos\theta_i+(m_ie_i^2+I_i)\dot{\theta}_i^2\right)+\frac{1}{2}\sum_{i=1}^{n+1}k_i\,(q_i-q_{i-1})^2.\end{aligned} \tag{10}$$

A property of the n-TORA system is:

$$\dot{E}=\sum_{i=1}^{n}N_i\,\dot{\theta}_i. \tag{11}$$

To show this, one can compute $\dot{E}$ directly by using Eq. (9). An alternative and direct way is to use on the principle of virtual work [11] to obtain:

$$\delta E=\sum_{i=1}^{n}N_i\,\delta\theta_i, \tag{12}$$

where δE is small variation of E and $\delta\theta_i$ is small variation of θ_i. Dividing two sides of Eq. (12) by δt and taking $\delta t\to 0$, we obtain Eq. (11).

3. PD control for global stabilization

Define:

$$q=[q_1,\quad q_2,\quad \dots,\quad q_n]^T, \tag{13}$$
$$\theta=[\theta_1,\quad \theta_2,\quad \dots,\quad \theta_n]^T. \tag{14}$$

The control objective is to stabilize the n-TORA system to the origin $(q,\theta,\dot{q},\dot{\theta})=(0,0,0,0)$; that is, to design N_i ($i=1,2,\dots,n$) such that:

$$\lim_{t\to\infty}q_i=0,\quad \lim_{t\to\infty}\dot{q}_i=0, \tag{15}$$
$$\lim_{t\to\infty}\theta_i=0,\quad \lim_{t\to\infty}\dot{\theta}_i=0. \tag{16}$$

We provide the following theorem about the global motion analysis of the n-TORA system under the PD controllers.

Theorem 1. Consider the n-TORA system given in Eq. (9). Under the PD controller:

$$N_i=-k_{P_i}\theta_i-k_{D_i}\dot{\theta}_i,\;\; i=1,2,\dots,n, \tag{17}$$

where k_{P_i} and k_{D_i} are positive constants, Eq. (15), (16) hold for any initial condition of the n-TORA system in Eq. (9); that is, the global asymptotical stabilization is achieved.

Proof. Take the following Lyapunov function candidate:

$$V=E+\frac{1}{2}\sum_{i=1}^{n}k_{P_i}\theta_i^2, \tag{18}$$

where each k_{P_i} is a positive constant. Using Eq. (11) and taking the time derivative of V along Eq. (9), we obtain:

$$\dot{V} = \sum_{i=1}^{n} \dot{\theta}_i \left(N_i + k_{P_i}\theta_i\right). \tag{19}$$

Using Eq. (17) yields:

$$\dot{V} = -\sum_{i=1}^{n} k_{D_i} \dot{\theta}_i^2 \leq 0. \tag{20}$$

Now, we use LaSalle's invariant principle [12] to analyze the motion of the closed-loop system consisting of Eq. (9) and Eq. (17). To this end, we need to define a compact set that is positively invariant with respect to the closed-loop system. Under controller Eq. (17), owing to $\dot{V} \leq 0$ in Eq. (20), V is bounded. This shows that E and θ_i are bounded. Since E in Eq. (10) is bounded, so q_i, $\dot{q}_i$, and $\dot{\theta}_i$ are bounded. Define:

$$\Phi_c = \{(q, \theta, \dot{q}, \dot{\theta}) \mid V(q, \theta, \dot{q}, \dot{\theta}) \leq c\}, \tag{21}$$

where c is a positive constant. Then any closed-loop solution $(q, \theta, \dot{q}, \dot{\theta})$ starting in Φ_c remains in Φ_c for all $t \geq 0$. Let W be the largest invariant set in:

$$S = \left\{(q, \theta, \dot{q}, \dot{\theta}) \in \Phi_c \middle| \ \dot{V} = 0\right\}. \tag{22}$$

Using LaSalle's invariant principle, we know that every $(q, \theta, \dot{q}, \dot{\theta})$ starting in Φ_c approaches W as $t \to \infty$. Since $\dot{V} = 0$ holds identically in W, V is a constant in W, and from Eq. (20) we can see that $\dot{\theta}_i = 0$ holds in W and θ_i is also a constant in W denoted as θ_i^*.

From Eq. (17) with $\theta_i = \theta_i^*$ and $\dot{\theta}_i = 0$, we conclude that N_i is a constant in W denoted as N_i^*. Putting $\theta_i = \theta_i^*$ and $N_i = N_i^*$ into Eq. (9) and Eq. (17) yields:

$$N_i^* + k_{P_i}\theta_i^* = 0, \tag{23}$$
$$(M_i + m_i)\ddot{q}_i + (k_i + k_{i+1})q_i - k_i q_{i-1} - k_{i+1} q_{i+1} = 0, \tag{24}$$
$$m_i e_i \ddot{q}_i \cos\ \theta_i^* = N_i^*. \tag{25}$$

Integrating Eq. (25) with respect to time t gives:

$$m_i e_i \dot{q}_i \cos\theta_i^* = N_i^* t + \eta_i, \tag{26}$$

where η_i is a constant. Since $\dot{q}_i$ is bounded due to the boundedness of E in Eq. (10) and since Eq. (26) holds for all t, we conclude that:

$$N_i^* = 0, \tag{27}$$

must hold; otherwise there is a contradiction since the left-side term of Eq. (26) is bounded while the right-side term is unbounded as $t \to \infty$. With Eq. (23), we have $\theta_i^* = 0$. This shows that $\theta_i = 0$ holds in W; that is, Eq. (16) holds.

Putting Eq. (27) and $\theta_i^* = 0$ into Eq. (25), we have $\ddot{q}_i = 0$, which yields:

$$q_i = \lambda_i t + \gamma_i, \tag{28}$$

where λ_i and γ_i are constants. Since E in Eq. (10) is bounded, from the boundedness of q_i in the left-hand side of the above equation, we can see $\lambda_i = 0$; otherwise, the absolute value of the right-hand side of the above equation goes to infinity as $t \to \infty$. Thus, $q_i = \gamma_i$ is a constant denoted as q_i^*. Thus, from Eq. (24), we obtain:

$$(k_i + k_{i+1})q_i^* - k_i q_{i-1}^* - k_{i+1} q_{i+1}^* = 0. \tag{29}$$

Writing out Eq. (29) with $i = 1, 2, \ldots, n$ yields:

$$\Gamma_n(k_1, k_2, \ldots, k_n, k_{n+1})q^* = 0, \tag{30}$$

where:

$$q^* = [q_1^*, \quad q_2^*, \quad \ldots, \quad q_n^*]^T, \tag{31}$$

$$\Gamma_n(k_1, k_2, \ldots, k_n, k_{n+1}) = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 & \cdots & 0 & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -k_3 & k_3 + k_4 & -k_4 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & k_{n-2} + k_{n-1} & -k_{n-1} & 0 \\ 0 & 0 & 0 & 0 & \cdots & -k_{n-1} & k_{n-1} + k_n & -k_n \\ 0 & 0 & 0 & 0 & \cdots & 0 & -k_n & k_n + k_{n+1} \end{bmatrix}. \tag{32}$$

Perform column manipulation to delete $-k_2$ in (1, 2) of $\Gamma_n(k_1, k_2, \ldots, k_n, k_{n+1})$ yields the following matrix:

$$\begin{bmatrix} k_1 + k_2 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ -k_2 & \hat{k}_2 + k_3 & -k_3 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -k_3 & k_3 + k_4 & -k_4 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & k_{n-2} + k_{n-1} & -k_{n-1} & 0 \\ 0 & 0 & 0 & 0 & \cdots & -k_{n-1} & k_{n-1} + k_n & -k_n \\ 0 & 0 & 0 & 0 & \cdots & 0 & -k_n & k_n + k_{n+1} \end{bmatrix}, \tag{33}$$

where:

$$\hat{k}_2 = \frac{k_1 k_2}{k_1 + k_2}. \tag{34}$$

Thus:

$$|\Gamma_n(k_1, k_2, \ldots, k_n, k_{n+1})| = (k_1 + k_2)|\Gamma_{n-1}(\hat{k}_2, k_3, \ldots, k_n, k_{n+1})|, \tag{35}$$

where:

$$\Gamma_{n-1}(\hat{k}_2, k_3, \ldots, k_n, k_{n+1}) = \begin{bmatrix} \hat{k}_2 + k_3 & -k_3 & 0 & \cdots & 0 & 0 & 0 \\ -k_3 & k_3 + k_4 & -k_4 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & k_{n-2} + k_{n-1} & -k_{n-1} & 0 \\ 0 & 0 & 0 & \cdots & -k_{n-1} & k_{n-1} + k_n & -k_n \\ 0 & 0 & 0 & \cdots & 0 & -k_n & k_n + k_{n+1} \end{bmatrix}. \tag{36}$$

From the above relation, by performing the above column manipulation repeatedly, we can obtain:

$$|\Gamma_n(k_1, k_2, \ldots, k_n, k_{n+1})| = \left(\hat{k}_1 + k_2\right)\left(\hat{k}_2 + k_3\right) \cdots \left(\hat{k}_n + k_{n+1}\right) > 0, \tag{37}$$

where:

$$\hat{k}_i = \begin{cases} k_1 > 0, & i = 1, \\ \dfrac{\hat{k}_{i-1} k_i}{\hat{k}_{i-1} + k_i} > 0, & i = 2, \ldots, n. \end{cases} \tag{38}$$

Thus, $\Gamma_n(k_1, k_2, \ldots, k_n, k_{n+1})$ in Eq. (32) is an invertible matrix. From Eq. (30), we have:

$$q^* = 0. \tag{39}$$

This shows that $q = 0$ holds in W; that is, Eq. (15) holds. This completes the proof of Theorem 1.

We present the following two remarks:

Remark 1. Theorem 1 shows that the global stabilization of n-TORA system given in Eq. (9) can be achieved by using the PD control of the angle of the rotational proof-mass of each TORA. This means that the local control of each TORA (using the angular position and velocity of each TORA) yields the global stabilization of the whole system.

Remark 2. The fact that $\Gamma_n(k_1, k_2, \ldots, k_n, k_{n+1})$ in Eq. (32) determined by all $n + 1$ springs is an invertible matrix is important in the proof of Theorem 1. We can show that the PD control can globally stabilize the n-TORA system with $k_1 = 0$ and/or $k_{n+1} = 0$; that is, the spring (respectively, springs) connected to one side (respectively, two sides) of the wall (respectively, walls) of the n-TORA system is (respectively, are) absent. Indeed, from Eq. (37), we can show that $|\Gamma_n(k_1, k_2, \ldots, k_n, k_{n+1})| \neq 0$ for $k_1 = 0$ and/ or $k_{n+1} = 0$.

4. Numerical simulation results

The validity of the developed theoretical results was verified via numerical simulation investigation for 2- and 3-TORA systems.

4.1. 2-TORA system

First, we took the mechanical parameters in [2] shown in Table 1 for TORA 1 and took different mechanical parameters for TORA 2, and the stiffness of each spring is described as: $k_1 = 186.3$ N/m, $k_2 = 100.3$ N/m, $k_3 = 186.3$ N/m.

It was mentioned in [2] that the physical constraint of displacement of the cart is its absolute value not greater than 0.025 m. Thus, we took an initial condition $\left(q_i, \theta_i, \dot{q}_i, \dot{\theta}_i\right) = (0.025, 0, 0, 0)$, $i = 1$, 2, and we chose the control gains $k_{P_1} = 0.08$, $k_{P_2} = 0.07$, $k_{D_1} = 0.002$, $k_{D_2} = 0.001$. The simulation results of the PD control are shown in Figs. 3-6. From Fig. 3, we can see that V is non-increasing; from Figs. 5-6, we can see that the state variables of the 2-TORA system converge to the origin.

Table 1. Mechanical parameters of a 2-TORA system

TORA i	TORA 1	TORA 2
M_i [kg]	1.3608	0.8608
m_i [kg]	0.096	0.056
e_i [m]	0.0592	0.0392
I_i [kg·m^2]	0.0002175	0.0001175

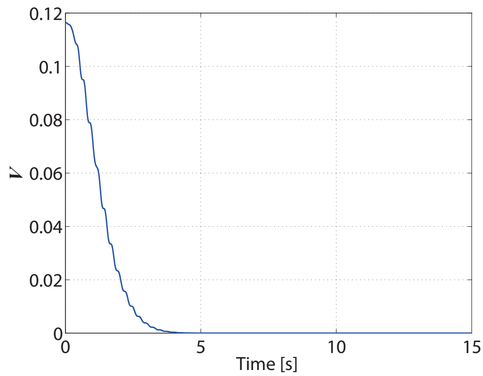

Fig. 3. Time response of V of the 2-TORA system

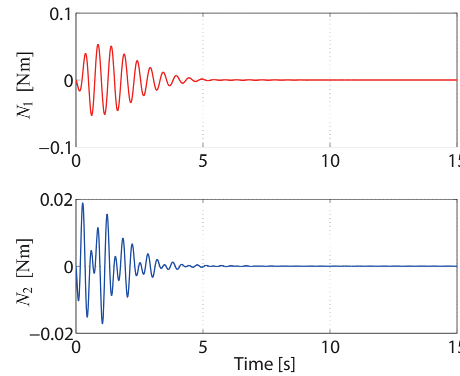

Fig. 4. Time responses of N_1 and N_2 of the 2-TORA system

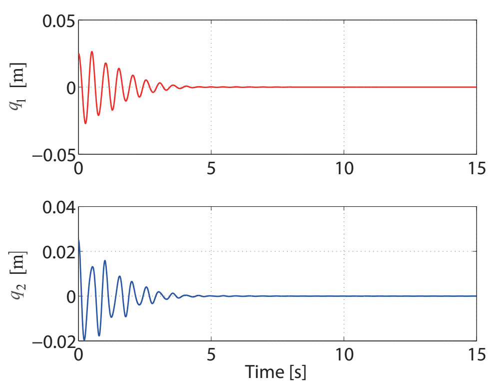

Fig. 5. Time responses of q_1 and q_2 of the 2-TORA system

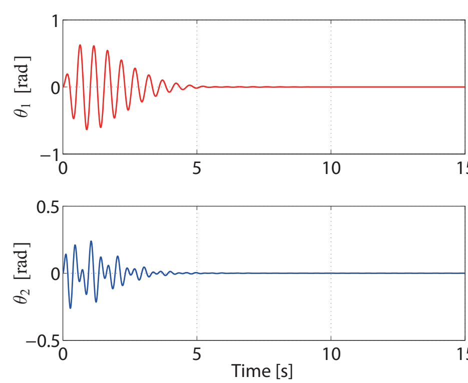

Fig. 6. Time responses of θ_1 and θ_2 of the 2-TORA system

4.2. 3-TORA system

We took the mechanical parameters shown in Table 2 and the stiffness of each spring is described as: $k_1 = 186.3$ N/m, $k_2 = 100.3$ N/m, $k_3 = 130.3$ N/m, and $k_4 = 160.3$ N/m.

Table 2. Mechanical parameters of a 3-TORA system

TORA i	TORA 1	TORA 2	TORA 3
M_i [kg]	1.3608	0.8608	0.5608
m_i [kg]	0.096	0.056	0.036
e_i [m]	0.0592	0.0392	0.0192
I_i [kg·m^2]	0.0002175	0.0001175	0.00009175

For an initial condition $(q_i, \theta_i, \dot{q}_i, \dot{\theta}_i) = (0.025, 0, 0, 0)$, $i = 1, 2, 3$ and we chose the control gains $k_{P_1} = 0.055$, $k_{P_2} = 0.055$, $k_{P_3} = 0.055$, $k_{D_1} = 0.001$, $k_{D_2} = 0.001$, $k_{D_3} = 0.0004$. The simulation results of the PD control are shown in Figs. 7-10. From Fig. 7, we can see that V is non-increasing; from Figs. 9-10, we can see that the state variables of the 3-TORA system converge to the origin.

By comparing the simulation results for the 2 and 3-TORA systems, we find that it takes more time to suppress the oscillation in the system as the number of TORAs increases.

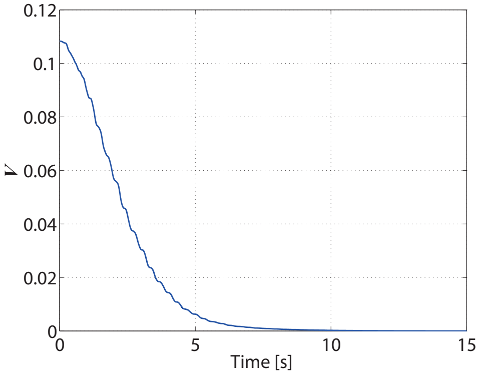

Fig. 7. Time response of V of the 3-TORA system

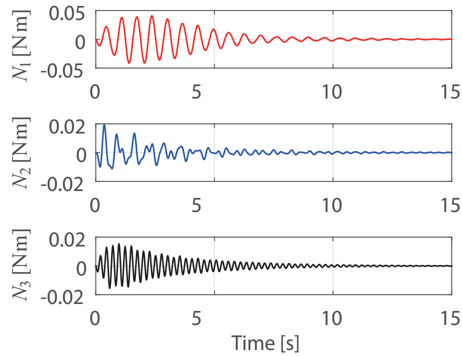

Fig. 8. Time responses of N_1, N_2, and N_3 of the 3-TORA system

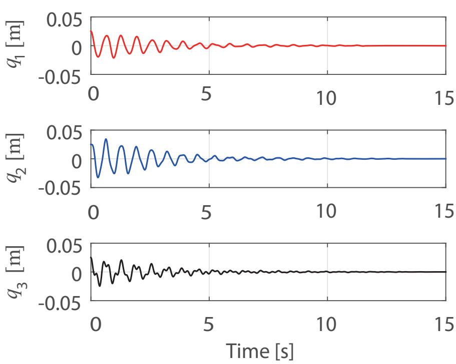

Fig. 9. Time responses of q_1, q_2, and q_3 of the 3-TORA system

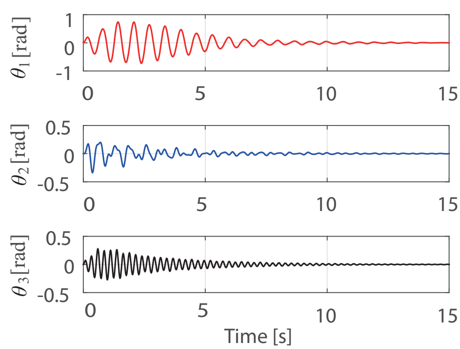

Fig. 10. Time responses of θ_1, θ_2, and θ_3 of the 3-TORA system

5. Conclusions

In this paper, we studied the global stabilization problem of an n-TORA system consisting of n TORAs with a spring connecting two neighboring TORAs. First, we derived the motion equation of the n-TORA system. Then, by using Lyapunov stability theory, we designed the PD controller and proved that the proposed PD controller can globally stabilize the n-TORA system. Finally, we presented the simulation results for 2- and 3-TORA systems to validate the effectiveness of the proposed controller. Our numerical investigation showed that the proposed controller can stabilize the 2- and 3-TORA systems to the equilibrium point for the all initial conditions simulated.

A future research subject is to study the relationship between the rate of convergence to the origin and the values of the control gains.

Acknowledgement

This work was supported in part by JSPS KAKENHI Grant Number 26420425.

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