Analysis of dynamic properties of two-mass system with inertial exciter of limited power

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(Accepted 9 September 2015)

Abstract. In this paper a forced oscillations of two-mass system, induced by unbalance vibration exciter driven by asynchronous AC motor of limited power, are considered. The mathematical model of the system is obtained by taking into account static characteristic of the motor. Oscillations of the system in resonances zones are analyzed, and relations between power input and dissipated power are estimated depending on the system parameters.

Keywords: vibration, resonance, non-ideal source, two-mass system, vibrating machine.

1. Introduction

One of the problems of creating high-performance vibrating machines is to ensure stability of required oscillation regimes while reducing power consumption for their maintenance. In present time the most of vibrating machines are single-mass systems, which oscillation are excited in after-resonance regimes, which is due to its stability, however they have low efficiency of use a disturbing force [1-3]. One way to solve this problem is in application of multimass, for example, two-mass dynamic scheme of vibrating machines. Introduction of additional mass improves dynamic equilibrium of the machine, which leads to improving its vibration isolation and enables its operation at the resonant or near-resonant mode. At the same time, as it is known [2, 4-6], an oscillating system can significantly effect on an energy source, which depends on the ratio of input and dissipated energies, and can lead to “sticking” at a certain frequency or abrupt change in frequency and in oscillation amplitude - the so-called effect of Sommerfeld [1, 2, 4].

These effects may limit operational capabilities of vibrating machines and require a special study in every case. Questions of interaction of oscillatory systems with non-ideal source of energy has been widely discussed in many works [2, 3-6], which are mainly focused on the analysis of systems having single "basic" degree of freedom. General methods for research nonlinear systems with many degrees of freedom are also given [2, 4, 7]. In the literature devoted to the dynamics of vibrating technological machines, analysis of multimass dynamic schemes is carried out without taking into account interaction with an energy source [3, 8].

In this paper, we consider a model of vibrating machine as a system of two solids connected with each other and with fixed base by viscoelastic dampers. Each of the masses can perform only unidirectional oscillations which are excited by rotation of an unbalanced rotor of asynchronous motor of limited power. Dynamic properties of the motor are taken into account as static characteristics determined by Kloss formula [8]. Interaction of the oscillatory system with the motor in the resonance region is analyzed with evaluation of input and dissipated power relationship depending on the system parameters.

2. Mathematical model

Dynamic scheme of vibrating machine is shown in Fig. 1. It consists of working body with operational load on it which are presented as a single rigid body of mass m_1, and a platform with rigidly attached unbalance vibration exciter driven by asynchronous motor with cage rotor, which are modelled by a single rigid body of mass m_2. The working body and the platform are connected to each other by viscoelastic damper with linear characteristics of stiffness c_1 and viscous friction b_1, the platform is also connected to a fixed base by viscoelastic damper with linear characteristics of stiffness c_2 and viscous friction b_2. Working body and platform can move only along the vertical direction. The exciter has unbalanced mass m_r and eccentricity r. Gravity is not taken into consideration.

Motion of the system is considered relative to a fixed coordinate system xOy, the beginning of which coincides with the center of mass of the working body. Displacements of working body and platform are determined by displacements of its centers of masses y_1 and y_2, respectively, relative to their positions of static equilibrium. Angular position of debalance is determined by angle φ measured from its lower vertical position.

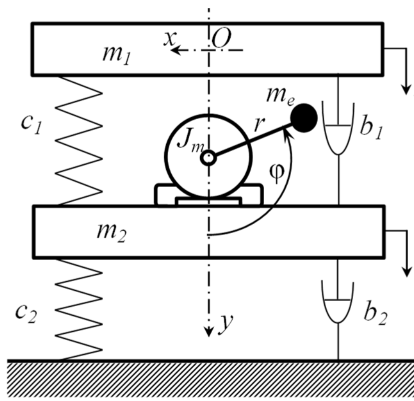

Fig. 1. Dynamic scheme of vibrating machine

According to the purposes of the study we consider steady or close to steady oscillations regimes, that is when the motor angular speed $\dot{\varphi} = d\varphi/dt$ changes quite slowly, so to describe driving moment we use static characteristics $L_m(\dot{\varphi})$ of the motor determined by Kloss formula [4, 8]. Friction in rotor supports is considered as a constant value L_f.

Then motion of the system is described by the following system of equations:

$$\begin{cases} m_1\ddot{y}_1 + b_1\dot{y}_1 + c_1y_1 = b_1\dot{y}_2 + c_1y_2, \\ (m_2 + m_e)\ddot{y}_2 + (b_1 + b_2)\dot{y}_2 + (c_1 + c_2)y_2 = b_1\dot{y}_1 + c_1y_1 + m_er(\ddot{\varphi}\sin\varphi + \dot{\varphi}^2\cos\varphi), \\ (J_m + m_er^2)\ddot{\varphi} = L_m(\dot{\varphi}) - L_f + m_er\ddot{y}_2\sin\varphi, \end{cases} \quad (1)$$

where $L_m(\dot{\varphi}) = 2M_k/(S_k(\nu_0 - \dot{\varphi})^{-1} + (\nu_0 - \dot{\varphi})S_k^{-1})$, M_k – maximum moment of the motor, S_k – slip corresponding to this moment, ν_0 – idle angular speed of the motor.

Let's introduce the dimensionless variables for time $\tau = t/T_*$ and displacements $u_1 = y_1/Y_*$, $u_2 = y_2/Y_*$ and choose scale factors for time T_* and displacements Y_* so that $T_* = \sqrt{m_1/c_1}$, $Y_* = \sqrt{J/m}$, where $J = J_m + m_er^2$ and $m = m_2 + m_e$. Then Eq. (1) could be represented in dimensionless form:

$$\begin{cases} \ddot{u}_1 + u_1 - u_2 = 2\lambda(\dot{u}_2 - \dot{u}_1), \\ \ddot{u}_2 - \beta u_1 + \alpha\beta u_2 = 2\lambda\beta(\dot{u}_1 - \eta\dot{u}_2) + \gamma(\ddot{\varphi}\sin\varphi + \dot{\varphi}^2\cos\varphi), \\ \ddot{\varphi} = \tilde{L}_m(\dot{\varphi}) - \tilde{L}_f + \gamma\ddot{u}_2\sin\varphi, \end{cases} \quad (2)$$

where:

$$\lambda = \frac{b_1}{2m}T_*, \ \alpha = \frac{c_1 + c_2}{c_1}, \ \beta = \frac{m_1}{m}, \ \eta = \frac{b_1 + b_2}{b_1},$$

$$\gamma = \frac{m_e r}{\sqrt{Jm}}, \ \ \tilde{L}_m(\dot{\varphi}) = \frac{T_*^2}{J} L_m(\dot{\varphi}), \ \ \tilde{L}_f = \frac{T_*^2}{J} L_f,$$

($\dot{}$) and ($\ddot{}$) in Eq. (2) are the first and the second derivative with respect to dimensionless time τ.

3. Solution method

One of the main objectives of the present study is to investigate the interaction of the oscillating system with the motor in the zones of "main" resonances. According to this let us turn to the analysis of particular solutions of the system Eq. (2) which correspond to single-frequency oscillations. Assuming that $\dot{\varphi}$ changes little over the maximum period of free oscillations of the system, the viscous resistance is low, the natural frequencies ω_i ($i = 1, 2$) are simple roots of the characteristic equation for a system consisting of left sides of the first two equations of Eq. (2), we use the method of Bogolyubov of perturbation theory [2, 4, 7], to obtain an approximate solution of Eq. (2). Following this method we represent the solution through its main coordinates z_1 and z_2: $u_1 = s_{11}z_1 + s_{12}z_2$, $u_2 = s_{21}z_1 + s_{22}z_2$, where, s_{ij} ($i, j = 1, 2$) – coefficients of form functions corresponding to eigenfrequencies ω_i.

For each of these coordinates we seek the solution in form $z_i = A_i\cos(\varphi + \Xi_i)$, taking into account $d\varphi/d\tau = \Omega$, $dz_i/d\tau = -A_i\omega_i\sin(\varphi + \Xi_i)$, and Ω, A_i, Ξ_i are slowly varying in time with respect to the main period of oscillations.

Let's consider the solution in the first approximation. Here we note that fluctuations of coordinate z_2 are small in comparison with z_1 when considering motion of the system near the frequency ω_1, so we can neglect z_2 when receiving solution in first approximation. Similar to this, fluctuations of coordinate z_1 are small in comparison with z_2 when considering motion of the system near the frequency ω_2.

Thus, we obtain the following expression for determining the amplitude and frequency of vibrations:

– near the resonance ω_1:

$$A_1^2 = \frac{\gamma^2 s_{12}^2 \Omega^4}{\omega_1^4\left[4\left(1 - \frac{\Omega}{\omega_1}\right)^2 - \left(\frac{h_1}{\omega_1}\right)^2\right]}, \tag{3}$$

$$\tilde{L}_m(\dot{\varphi}) - \tilde{L}_f - \frac{A_1^2\omega_1}{2\Omega} h_1(1 + s_{12}^2) = 0, \tag{4}$$

where $h_1 = 2\lambda(\beta\eta s_{12}^2 - (1+\beta)s_{12} + 1)/(1 + s_{12}^2)$;

– near the resonance ω_2:

$$A_2^2 = \frac{\gamma^2 s_{22}^2 \Omega^4}{\omega_2^4\left[4\left(1 - \frac{\Omega}{\omega_2}\right)^2 - \left(\frac{h_2}{\omega_2}\right)^2\right]}, \tag{5}$$

$$\tilde{L}_m(\dot{\varphi}) - \tilde{L}_f - \frac{A_2^2\omega_2}{2\Omega} h_2(1 + s_{22}^2) = 0, \tag{6}$$

where $h_2 = 2\lambda(\beta\eta s_{22}^2 - (1+\beta)s_{22} + 1)/(1 + s_{22}^2)$.

4. Results analysis

From Eqs. (4) and (6) one can see that oscillation frequency depends on characteristics of the motor $\tilde{L}_m(\dot{\varphi})$ and oscillation amplitude A_1, and oscillation can only occur with the frequency Ω

satisfying these equations. A clear understanding of the possible modes of oscillation provides a graphical representation of Eqs. (4) and (6) given in Fig. 2, where $H_i(\Omega)$ is the moment of forces of resistance to oscillatory motion, determined as: $H_i(\Omega) = \tilde{L}_f + \frac{A_i^2 \omega_i}{2\Omega} h_i(1 + s_{i2}^2)$. Curves $\tilde{L}_m^{(1)}$, $\tilde{L}_m^{(2)}$, $\tilde{L}_m^{(3)}$, $\tilde{L}_m^{(4)}$ represent characteristics of motors having different power, numbered in accordance with power increase. Moreover characteristics $\tilde{L}_m^{(1)}$, $\tilde{L}_m^{(2)}$ can be considered as adjusting characteristics of the same engine with natural characteristic $\tilde{L}_m^{(3)}$ in the implementation of so-called vector control, e.g. at simultaneous proportional change in frequency ω_s and amplitude U_s of the power supply so that maintaining constant the ratio U_s/ω_s. Frequency of forced vibration corresponding to each of the motors is determined by points of intersection of their characteristics with functions $H_i(\Omega)$. For example each of characteristics $\tilde{L}_m^{(1)}$ and $\tilde{L}_m^{(2)}$ intersect curves $H_i(\Omega)$ at only one point B and C respectively. Therefore there is only one possible oscillation mode for each motor with the frequency Ω_B or Ω_C respectively, and the more power of motor the higher oscillation frequency. Different situation occur when we use motor with characteristic $\tilde{L}_m^{(3)}$, which intersects $H_i(\Omega)$ at five points $D_1, D_2, \ldots, D_5$. Note that points D_1, D_3, D_5 correspond to stable oscillation modes and points D_2 and D_4 correspond to unstable modes.

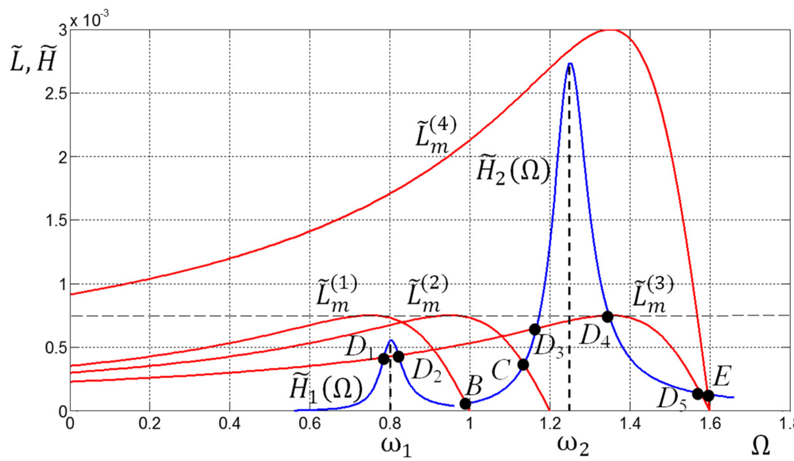

Fig. 2. Relations of driving moments and moments of forces of resistance to oscillatory motion

It is also interesting to note that although the nominal capacity of this engine is more than the first two, however, after it starts the system is unable to overcome first resonance, and come to steady mode of oscillation corresponding to point D_1 with frequency Ω_{D_1} which is less than Ω_B or Ω_C. Considering $\tilde{L}_m^{(1)}$ and $\tilde{L}_m^{(2)}$ as adjusting characteristics for latter motor it is evident that we can overcome first resonance by appropriate control the motor characteristics at starting mode. However the control system is unable to adjust system to after-resonance oscillation mode, that is when oscillation frequency is greater than ω_2, and we should apply the motor with higher power, e.g. the motor with the characteristic $\tilde{L}_m^{(4)}$. For this motor there is only one possible oscillation mode determined by point E of intersection of curves $\tilde{L}_m^{(4)}$ and $H_i(\Omega)$ corresponding to after-resonance oscillation mode with frequency $\Omega_E > \omega_2$.

Increase in motor power usually leads to increase in its mass, size and cost of the entire system. At the same time, necessity of its increase is usually due to the possibility of overcoming the resonance regions where there is the greatest power dissipation because of substantial increase in oscillation amplitude. Therefore it seems appropriate to consider reducing the amount of energy dissipated in the resonance region, which is most determined by dissipative terms in Eq. (2). For this purpose consider behavior of function $\tilde{H}_1(\Omega)$ at $\Omega = \omega_1$. From Eq. (4) taking into account

Eq. (3) we obtain:

$$\widetilde{H}_1(\omega_1) = \tilde{L}_f + \frac{\gamma^2 s_{12}^2 \omega_1^2}{4\lambda(\beta\eta s_{12}^2 - (1+\beta)s_{12} + 1)}. \tag{7}$$

One can see that resistance moment $\widetilde{H}_1(\omega_1)$ is inversely proportional to λ and η, that is an increase in any of these parameters will result in a monotonic decrease in the resistance moment. The change in λ has more significant impact on $\widetilde{H}_1(\omega_1)$ than η.

Similarly, considering function $\widetilde{H}_2(\Omega)$ at $\Omega = \omega_2$, from Eq. (6) taking into account Eq. (5) we obtain:

$$\widetilde{H}_2(\omega_2) = \tilde{L}_f + \frac{\gamma^2 s_{22}^2 \omega_2^2}{4\lambda(\beta\eta s_{22}^2 - (1+\beta)s_{22} + 1)}. \tag{8}$$

Comparing Eqs. (7) and (8) we can say that $\widetilde{H}_2(\omega_2)$ depending on the parameters β and η behaves similarly to $\widetilde{H}_1(\omega_1)$. Note that although increase in damping leads to the desired effect of reducing the resistance moment, it is not allowed to be increased indefinitely, and any other restrictions on the amount of damping, e.g. due to structural and technological parameters of the machine, should also be taken into account.

5. Conclusion

Presented here model of vibrating machine and analysis of its dynamics showed that for operation of two-mass vibration machine at after-resonant mode we need to apply more powerful motor than it's needed to operate at pre-resonant or inter-resonance modes. The results and solutions could be the basis for further study of dynamics of two-mass systems with limited power of drive to determine rational values of their parameters and choice of operating modes.

Acknowledgements

The reported study was funded by RFBR according to the research project No. 15-08-06961a.

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