Study of the motion of a mechanical system due to the oscillatory motion of the side links

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Abstract. In this paper we study a five-link mechanical system, whose links are connected by cylindrical hinges equipped with actuators. The central link is the body and the side links execute oscillatory motion, which provides the required motion of the body. Such a system can be used as an ornithopter during antiphase oscillations of the side links with the same amplitudes and also during alternation of the stages of motion, in which forces of resistance acting on the links rise and fall.

Keywords: five-link system, cylindrical hinges, oscillations, synchronous movements, aerodynamic drag.

1. Introduction

Oscillatory movements of links, which lead to directed motion of the robot in space by their interaction with the surrounding environment, are widely used in mobile robots. These objects include crawling robots that move on rough surfaces, jumping, underwater ichthyoids, flapping wing flying robots many others [1-6].

Such oscillatory systems represent a special class of mechanical systems, whose directed motion is defined by periodically recurring movements of links. One of the main problems that need to be solved when designing such systems is the generation of the given oscillatory movements of the links of the system, which results in directed movement of the robot. This paper discusses a mathematical model of a five-link oscillatory system that describes the motion of a robot, for example, an ornithopter. We study the character of the controlled periodic movements of the external links under the action of piecewise constant control actions. The conducted analysis of motion parameters will be useful during the design of mobile robots of this type.

2. Modeling of the motion of the mechanical system

In this paper we look at an oscillating mechanical system, whose analytical model in the vertical plane Oyz of the absolute coordinate system is shown in Fig. 1. The system consists of five rigid undeformable links. Link 3 is the body, which is a parallelepiped with dimensions $l_3 \times h_3 \times b_3$ (in plane Oyz – a rectangle with dimensions $l_3 \times h_3$). The rest of the links have the form of plates with dimensions $l_i \times h_i \times b_i$, and $h_i \to 0$ (in plane Oyz – rods of lengths l_i, $i = 1$, 2, 4, 5). When the system is considered as an ornithopter the stated links play the role of the wings, each of which consists of two links (left wing – links 1 and 2, right wing – links 4 and 5). All the links have masses m_i and their centers of mass coincide with the centers of symmetry of the links-points C_i. each pair of adjacent links is connected to each other by cylindrical hinges 6-9 with a rotary actuator that generates torques: 6 – M_{21}, 7 – M_{32}, 8 – M_{34}, 9 – M_{45}. The position of this system in the plane Oyz is defined by seven generalized coordinates: projections of the center of mass on the coordinate axes y_{C3} and z_{C3}, and angles φ_1, φ_2, φ_3, φ_4 and φ_5 that characterize the anticlockwise rotations of all the links relative to the positive direction of axis Oy. The vector of generalised coordinates is given as follows:

$$\bar{q} = (z_{C3} \quad y_{C3} \quad \varphi_1 \quad \varphi_2 \quad \varphi_3 \quad \varphi_4 \quad \varphi_5)^T. \tag{1}$$

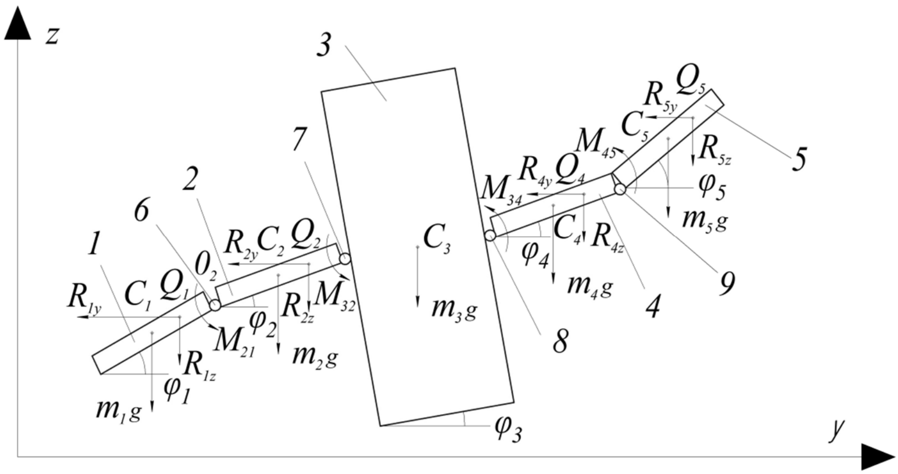

Fig. 1. Analytical model of the oscillatory mechanical system

We consider the case when the object moves horizontally along axis Ox ($x_{C3} \neq$ const) or is in a hovering mode at a certain height above the surface ($x_{C3} =$ const). This allows us to switch to a simplified mathematical model of the robot's motion, by assuming that the projections of the position of the center of mass of the body along axes Oy and Oz, as well as its angle of rotation about point C_3 remain unaltered ($y_{C3} =$ const, $z_{C3} =$ const, $\varphi_3 =$ const). We also assume that links 1 and 5, 2 and 4 move in antiphase with equal amplitudes ($\varphi_1 = -\varphi_5$, $\varphi_2 = -\varphi_4$) in order to achieve flapping motion of the wings. In this case the number of generalised coordinates is reduced to two and the vector q^* can be written as follows:

$$\bar{q}^* = (\varphi_4 \quad \varphi_5)^T. \tag{2}$$

During motion the studied system is subjected to the following forces: forces of gravity, $m_i g$, applied to the centers of mass of the links, torques M_{32}, $M_{21,}$ M_{34} and M_{45} generated by actuators and that provide relative oscillatory movements of the links, forces of resistance R_i, $i = 1, 2, 4, 5$, whose points of action, Q_i, are the centers of gravity of the diagram of the forces of resistance of the external environment, which are calculated as follows:

$$R_i = \frac{C_r \rho S_i V_i |V_i|}{2}, \tag{3}$$

where ρ – density of air; V_i – velocity of point Q_i; $S_i = l_i \times b_i$ – effective area of link i, C_r – dimensionless coefficient of aerodynamic force.

The given forces can be broken down into two components – horizontal, R_{iy} and vertical, R_{iz}, as well as into the components, $R_{iy}^{(i)}$, directed along link i, and $R_{iz}^{(i)}$, perpendicular to the link:

$$R_i = \sqrt{R_{iy}^2 + R_{iz}^2} = \sqrt{\left(R_{iy}^{(i)}\right)^2 + \left(R_{iz}^{(i)}\right)^2}. \tag{4}$$

Since the force R_i is proportional to the square of the velocity, V_i, of the its point of application, while the velocities of the points, Q_i, of links $i = 1, 2, 4$ and 5 will be constantly changing during motion, we can assume that at different points of motion the diagrams of the forces of resistance on the links will have different forms and the position of the points Q_i will move along the links. We assume that the points, Q_i on the links,$i = 1, 2, 4$ and 5, are seperated by distances k_i from the centers of mass, C_i along, lengths l_i. To calculate the distances, k_i we assume that the points,

Q_i are the centers of mass of the figures forces by the forces, $\bar{R}_{iz}^{(i)}$. Since $\varphi_1 = -\varphi_5$ and $\varphi_2 = -\varphi_4$, it is sufficient to determine the distances k_i only for links 4 and 5:

$$R_{iz}^{(i)} = a_i k_i^2 + b_i k_i + c_i, \tag{5}$$

where a_i, b_i and c_i are coefficients determined by the formulas:

$$a_{i=4,5} = C_R \rho S_i \dot{\varphi}_i^2 / 2, \tag{6}$$

$$b_4 = \frac{C_R \rho S_4 \dot{\varphi}_4^2 l_4}{2}, \quad b_5 = C_R \rho S_5 \dot{\varphi}_5 \frac{[2\dot{\varphi}_4 l_4 \cos(\varphi_4 - \varphi_5) + \dot{\varphi}_5 l_5]}{2}, \tag{7}$$

$$c_4 = \frac{C_R \rho S_4 \dot{\varphi}_4^2 l_4^2}{8}, \quad c_5 = C_R \rho S_5 \left[\dot{\varphi}_4 l_4 \cos(\varphi_4 - \varphi_5) + \frac{\dot{\varphi}_5 l_5}{2}\right]^2 / 2. \tag{8}$$

The distances, k_i for $i = 4$ and 5, are calculated as follows:

$$k_i = \frac{\int_{-li/2}^{li/2} k_i \cdot R_{iz}^{(i)} dk_i}{\int_{-li/2}^{li/2} R_{iz}^{(i)} dk_i} = \frac{b_i {l_i}^2}{a_i {l_i}^2 + 12 c_i}. \tag{9}$$

We assume that the linear and angular velocities of the robot body: $\dot{y}_{C3} = 0$, $\dot{z}_{C3} = 0$, $\dot{\varphi}_3 = 0$ and the angular velocities of the links: $\dot{\varphi}_4 = \dot{\varphi}_5 = 1$ rad/s, while $\varphi_3 = 0$ rad. Then at different values of angles φ_4 and φ_5 we get the following form of the distribution of aerodynamic forces on the relevant links (Fig. 2).

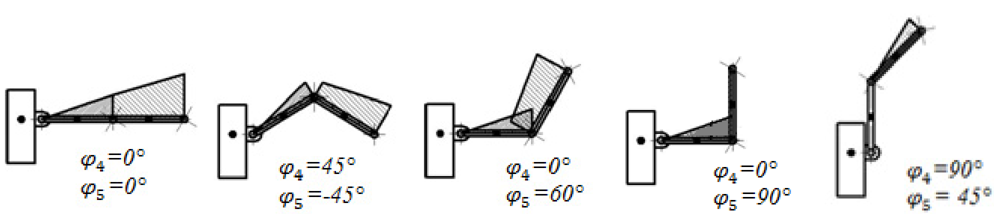

Fig. 2. Diagrams of distribution of forces of resistance along the lengths of the links 4 and 5

The system of differential equations describing the behaviour of the whole mechanical system in matrix form can be written down as follows:

$$A(q)\ddot{q} + B(q)D(\dot{q})\dot{q} = Q, \tag{10}$$

where $A(q)$, $B(q)$ are matrix coefficients, Q – matrix of generalized forces, which includes gravitational forces, forces of resistance and torques generated by the actuators, $D(\dot{q})$ – diagonal matrix of the first derivatives of the coordinates q.

For the particular case of horizontal flight or hovering, when the body is fixed, we will have two differential equations:

$$\begin{aligned} &\ddot{\varphi}_4 \left(J_4 + \frac{m_4 l_4^2}{4} + m_5 l_4^2\right) + \ddot{\varphi}_5 m_5 l_4 l_5 \frac{\cos(\varphi_4 - \varphi_5)}{2} + \dot{\varphi}_5^2 m_5 l_4 l_5 \frac{\sin(\varphi_4 - \varphi_5)}{2} \\ &= M_{34} - (2m_5 + m_4) g \frac{l_4}{2} \cos\varphi_4 - \left(\frac{l_4}{2} + k_4\right)\left(R_{4z}\cos\varphi_4 - R_{4y}\sin\varphi_4\right) \\ &-l_4\left(R_{5z}\cos\varphi_4 - R_{5y}\sin\varphi_4\right), \end{aligned} \tag{11}$$

$$\ddot{\varphi}_5\left(J_5+\frac{m_5 l_5^2}{4}\right)+\ddot{\varphi}_4 m_5 l_4 l_5 \frac{\cos(\varphi_5-\varphi_4)}{2}+\dot{\varphi}_4^2 m_5 l_4 l_5 \frac{\sin(\varphi_5-\varphi_4)}{2} = M_{45}-\left(\frac{l_5}{2}+k_5\right)\left(R_{5z}\cos\varphi_5-R_{5y}\sin\varphi_5\right)-m_5 g l_5 \frac{\cos\varphi_5}{2}, \quad (12)$$

where J_i is the central moment of inertia.

The control system that implements the given motion of the robot, in general form is shown in Fig. 3(a) ("Mechanical system"). Here the actuator torques play the role of reference signals and the angles of rotation of links $i = 1$, 2, 4 and 5 – the controlled variables. The coordinates of the center of mass of the body are generated at the output of the control system, but negative feedback channels are absent. In order to process the laws of oscillatory motion of links 4 and 5 when the boy is fixed the control system can be transformed into the form shown in Fig. 3(a) ("Links 4 and 5"). In it the number of generalized coordinates (φ_4, φ_5) obtained at the output correspond to the number of control parameters, which are torques M_{34} and M_{45}.

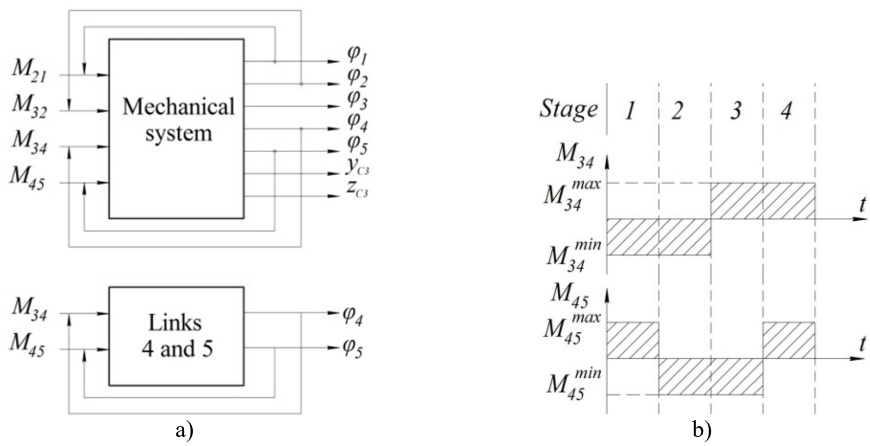

Fig. 3. a) Diagrams of control systems of the mechanical system and links 4 and 5 (horizontal flight and hovering modes), b) cylcogram of control torques of the automatic control system "Links 4 and 5"

This work studies one of the possible control strategies, under which the torques acting on links $i = 1$, 2, 4 and 5 are equal in magnitude and opposite in direction ($M_{32} = -M_{34}$, $M_{21} = -M_{45}$). To model the motion of the given system as an ornithopter it is necessary to alternate stages, in which the projections of the effective areas of the side links on axis Oz, as well as the velocity of the points of application of the forces of resistance rise up to the maximum values and vice-versa when the same parameters decrease up to minimum values.

We will model the motion of the robot during horizontal flight along axis Ox or in hover mode. Then, for the whole duration of the robot's motion $y_{C3} = \text{const}$, $z_{C3} = \text{const}$, $\varphi_3 = 0$ work, $\varphi_1 = -\varphi_5$, $\varphi_2 = -\varphi_4$. The sequence of the stages of motion is shown in Fig. 4. The purpose of modelling is the generation of such control torques that will provide oscillations of links 4 and 5 with a change of the projections of their areas on axis Oz and the velocities of points Q_i from the largest to the smallest and vice-versa.

Let the initial angles of links 4 and 5 be $\varphi_4 = \varphi_0$ and $\varphi_5 = -\varphi_0$, where φ_0 – a certain fixed value of the angle. In the first stage torques M_{34} and M_{45} act at points O_4 and O_5, as a result of which links 4 and 5 rotate up to the angles $\varphi_4 = 0$ rad, $\varphi_5 = \varphi_0$. With the onset of the second stage torque M_{45} changes its sign and at torque M_{34} continues to act at point O_4. As a result, the angles of the links attain the following values: $\varphi_4 = -\varphi_0$ and $\varphi_5 = 0$. In the third stage M_{34} changes its sign and M_{45} continues to act at point O_5. At the end of this stage the angles attain the values $\varphi_4 = 0$ and $\varphi_5 = -\varphi_0$. In the fourth stage torque M_{34} continues to act at point O_4, while

M_{45} reverses direction, as a result of which the angles are equal to the initial angles at the end of the stage.

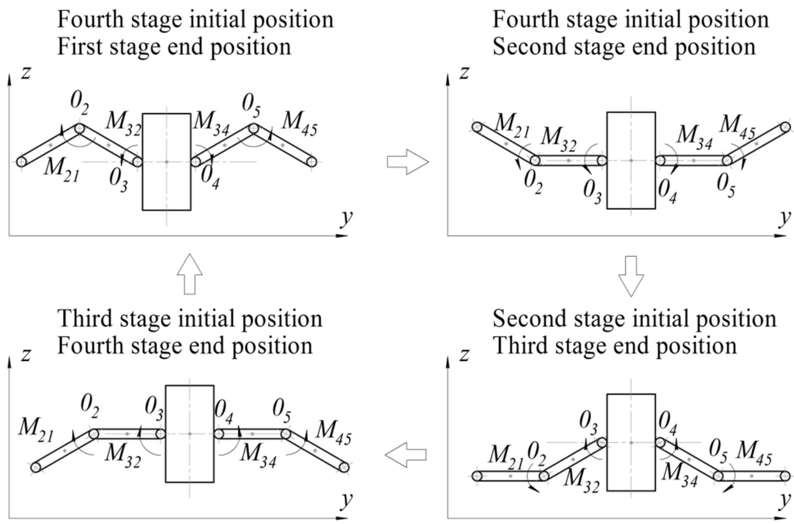

Fig. 4. Sequence of the stages of motion of the links of the mechanical system as an ornithopter

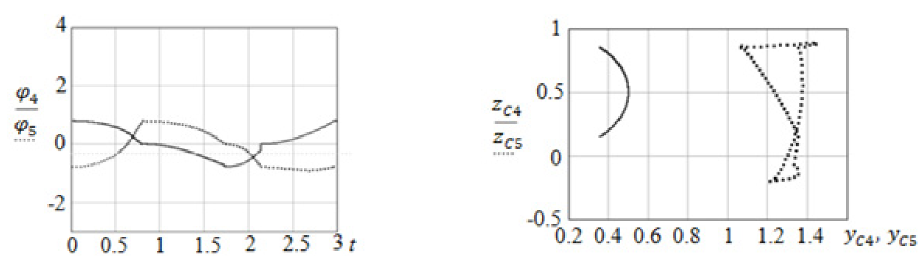

Fig. 5. Graphs of: a) $\varphi_4(t)$, $\varphi_5(t)$ and b) $z_{C4}(y_{C4})$, $z_{C5}(y_{C5})$

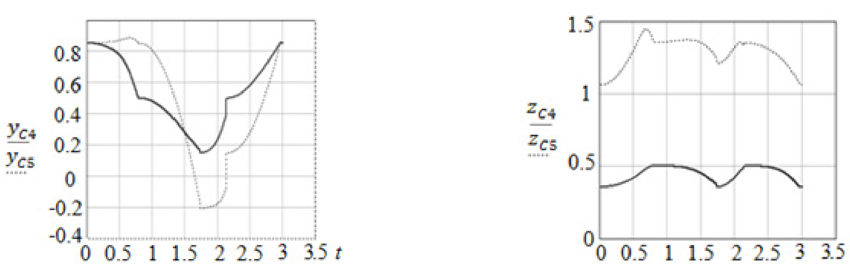

Fig. 6. Graphs of: a) $z_{C4}(t)$, $z_{C5}(t)$ and b) $y_{C4}(t)$, $y_{C5}(t)$

The parameters of the modeled object are as follows: $m_i = 1$ kg, $l_i = 1$ m, $\varphi_0 = \pi/4$ rad. The results of modelling are shown in Fig. 5 and Fig. 6. From the graphs it can be seen that the control system correctly processes the given sequence of movements: links 4 and 5 execute oscillatory movements in the range: $-\varphi_0$ to φ_0. It can be seen from the trajectory graphs that point C_4 executes rotational motion, while point C_5 moves on a more complex trajectory, executing rotational-translational motion. The proposed method of motion of the mechanical system can be used for viewing terrain, collection of data on the composition of air, its temperature, humidity, etc. for the purposes of monitoring environmental parameters using ornithopters.

3. Conclusions

This paper considers a mechanical system consisting of five links connected to each other by active cylindrical hinges, whose side links execute oscillatory motion. The motion of the system takes place in an environment with resistance represented by aerodynamic forces, whose points of application are assumed to be moving along the lengths of the side links and taken as the centers of mass of the distribution diagrams of the stated forces. Such a system can be used to simulate a flying robot of the ornithopter type if alternation of stages, in which the horizontal projections of the effective areas of the given links and the velocities of the points of application of the forces of resistance increase and decrease during antiphase oscillatory movements of links 1 and 5, 2 and 4 with equal amplitudes, occurs.

The given system is considered in this paper as an ornithopter in horizontal flight or hovering mode at a certain height, which allows us to impose restrictions on the mobility of the robot body completely by fixing it in plane Oyz. We develop a mathematical model of motion was for links 4 and 5, as well a negative feedback control system for their angles of rotation, where the torques generated by actuators are used as control actions and the controlled variables are the angles of rotation of the links. A sequence of the stages of motion of the stated links is also presented and a cyclogram of control torques is given. The time functions of the corresponding generalized coordinates are obtained as a result of the numerical modelling.

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