

Nonlinear filtering and identification algorithms for correlation-extremum dynamic systems with random structure

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Abstract. The problem of adaptive estimation using spatial-time-varying filtering in dynamic systems with random structure is investigated. The proposed approach of the extension of state estimation in the classical stochastic dynamic systems with deterministic structure to the case of signal processing and parameter identification in stochastic systems with random structure or with switching parameters using the correlation-extremum methods and the theory of Markov processes provides the system operation in varying and uncertain external conditions.

Keywords: signal processing, filtering, identification, Markov processes, optimization of stochastic systems with random structure.

1. Introduction

In many practical cases the state estimate of the observable object coordinates cannot be obtained using the optimal filtering algorithms. In particular, this relates to the case of radar tracking of an airborne target when the vehicle dynamics is described by nonlinear differential equations. The known nonlinear filtering algorithms are based on the assumption of linearization of the nonlinear functions in the state and measurements equations relative to estimation errors. These algorithms remain true when the estimation errors are small enough to satisfy a linearization.

Some previous work has already dealt with the linearization problem in the case of great estimation errors. One of the first approaches was taken in [1], where the two-feedback filtering system with changing operating conditions depending on the estimation error value of the observable object coordinates was suggested. But the estimation method discussed in [1] was not a result of a rigorous solution of such system synthesis problem that did not allow the two-feedback system to be considered as an optimal one.

The other filtering problems to be solved are connected with the moving objects parameters estimation, particularly, in the cases of discontinuous jump of the estimated process features, abrupt increasing of the measurements noise, informative signal interruption, and etc.

In most practical applications of filtering theory the uncertain parts of the dynamic system model are sometimes represented by an unknown parameter vector and the variations of these parameters and their identification play an important role.

Many scientific researches have been performed in the class of adaptive filtering scheme of the state vector estimation together with parameter identification (for example, [2, 3]). In [2] the problem of state estimation for discrete systems with parameters which may be switching within a finite set of values was considered for the general case and for the Markov parameter case. The approach to unknown parameters estimation taken in [3] was based on the numerical differentiation method with B^0 splines to assure better noise-resistance.

In the known earlier researches the theory of Markov processes and systems with random structure has been developed for signal processing when the observable signal has been considered only as a function of time and the correlation-extremum methods have not been used. This limitation did not allow the system with random structure to use the information of spatial-time-varying signals of different physical nature (obtained by radar, optical, infrared and other image sensors).

The solution of the above-mentioned problems applied to the spatial-time-varying signals processing was originally proposed in [4] where the correlation-extremum systems theory was extended to systems with random structure and the new suboptimal filtering algorithms were derived. The case when the change of the structure is supposed to be Markov process with two states is considered in this paper and the solution of the linearization problem based on the derived filtering algorithm is proposed, particularly, for the case of great estimation errors.

There are a number of potential military and civil application areas of the above proposed [4] nonlinear information processing algorithms. In this paper the solution for a target tracking system (TTS) is considered as an example.

The problem of target tracking is under consideration since many previous researches (see, for example, [5], where the treatment was based on a closed-form quasi-linearized solution of the proportional navigation (PN) equations followed by the small-angle approximation only to line of sight (LOS) angle rate). In [5] only the dynamic system state model without noise and measurements was discussed, and the state estimation problem was not solved. The other solutions of the state estimation problem for target tracking systems are based on Kalman filtering schemes (linear or suboptimal extended one) which can process only time-varying signal functions.

A new approach of the extension of Kalman filter adaptive estimation using the theory of Markov processes and systems with random structure or with switching parameters was proposed in [6] to the case of signal processing and parameter identification in stochastic magnetic navigation systems, where the parameters to be identified are the Poisson's coefficients.

The purpose of this paper is to present the solutions of the problem of nonlinear filtering and identification algorithms synthesis and analysis for correlation-extremum systems, based on Markov processes and the theory of optimization of stochastic systems with random structure for military and civil applications.

1.1. Problem statement

The problem under consideration is nonlinear estimation for the dynamic state process described by a stochastic differential equation (Eq. (1)):

$$\dot{\Lambda}(t) = \mathbf{F}^{(l)}(\Lambda, \mathbf{u}, t) + \mathbf{W}^{(l)}(t), \quad \Lambda(t_0) = \Lambda_0, \quad l = \overline{1, p}, \quad (1)$$

where $\Lambda(t)$ is the state vector, which contains the random, unknown, and time-varying parameters vector $\mathbf{a}^T = (a_1, \dots, a_q)$, $\mathbf{u}(t)$ is the known control vector, which may depend on the state vector components, $l(t)$ is a stationary Markov process taking values in the set $\{1, 2, \dots, p\}$ (number of the state). Here $\Lambda(t)$ is n -dimensional vector, in general case, with initial Gaussian value $\Lambda(t_0)$, $\mathbf{F}^{(l)}(\Lambda, \mathbf{u}, t)$ is the nonlinear deterministic vector function $\mathbf{F}^{(l)}(\Lambda, \mathbf{u}, t) = \|f_i^{(l)}(\Lambda, \mathbf{u}, t)\|$, ($i = \overline{1, n}$) which satisfies Lipschitz conditions. $\mathbf{W}^{(l)}(t)$ is a vector process of the state Gaussian white noise with diagonal intensity matrix $\mathbf{Q}^{(l)}(t) = \|q_i^{(l)}(t)\|$, ($i = \overline{1, n}$).

The observable signal $\mathbf{r}(x, y, t)$ is the m -dimensional spatial-time-varying process described by the following measurement equation (Eq. (2)):

$$\mathbf{r}(x, y, t) = \mathbf{S}^{(l)}(x, y, \Lambda, t) + \mathbf{N}^{(l)}(x, y, t), \quad (l = \overline{1, p}), \quad (2)$$

where x, y are the space variables – space coordinates at any point – $x \in X = [x_0, x_X]$, $y \in Y = [y_0, y_Y]$, t is the time variable $t \in T = [t_0, t_T]$, $\mathbf{S}^{(l)}(x, y, \Lambda, t)$ is the vector of spatial-time-varying signals of different physical nature, $\mathbf{N}^{(l)}(x, y, t)$ is the spatial-time-varying vector process of the measurements Gaussian white noise with diagonal intensity matrix $\mathbf{C}_0^{(l)}$ and correlation function $h(\Delta x, \Delta y, \Delta t) = \mathbf{C}_0^{(l)} \delta(\Delta x, \Delta y, \Delta t)$, δ is the delta function.

The signal position on the image plane XOY can be determined by parameters vector:

$$\lambda_x(t) = \varphi_x(\mathbf{\Lambda}, t), \quad \lambda_y(t) = \varphi_y(\mathbf{\Lambda}, t).$$

Then the signal may be represented as $\mathbf{S}^{(l)}(x, y, \mathbf{\Lambda}, t) = \mathbf{S}^{(l)}(x - \lambda_x, y - \lambda_y, t)$. The suboptimal estimator in this case represents a tracker system. The estimation errors are formed by receiving the derivatives of the cross-correlation function of the observed spatial-time-varying signal $\mathbf{r}(x, y, t)$ and the reference image $\mathbf{S}^{(l)}(x, y, \widehat{\mathbf{\Lambda}}, t)$.

The changes of the structure are supposed to be Markov process with p -finite states and the transition intensities $\nu_{jl}(t)$ and $\nu_l(t)$, where $j, l = \overline{1, p}$. The behavior of the system may be explicated as follows. The system begins in a particular mode of operation, say $l(t) = 1$, then at a random time the system jumps to one of the other $(p - 1)$ possible modes of operation and may or may not remain in this state. The model has an interpretation as a problem of a maneuvering target tracking interruption or a case of great estimation errors.

The nonlinear filtering or estimation problem defined above for Eqs. (1-2) is to find the finite-dimensional dynamical system whose output is the best minimum variance estimate of the joint Markov process $(\mathbf{\Lambda}(t), l(t))^T$, for $t \geq 0$ given the observed data $\mathbf{r}(x, y, t)$.

1.2. Solution of the algorithms synthesis problem

The synthesis of the filtering algorithms for correlation-extremum systems with random structure [4] was based on the generalized Fokker-Plank-Kolmogorov-Stratonovich equation for the evolution of joint conditional probability density of the state dynamics $\mathbf{\Lambda}(t)$ and the system structure $l(t)$ $\widehat{\omega}(\mathbf{\Lambda}, l, t) = \widehat{\omega}^{(l)}(\mathbf{\Lambda}, t)$ (Eq. (3)):

$$\begin{aligned} \frac{\partial \widehat{\omega}^{(l)}(\mathbf{\Lambda}, t)}{\partial t} = & - \operatorname{div} \widehat{\mathbf{\pi}}^{(l)}(\mathbf{\Lambda}, t) \\ & - \frac{1}{2} \widehat{\omega}^{(l)}(\mathbf{\Lambda}, t) \left[\Phi^{(l)}(\mathbf{\Lambda}, \mathbf{r}, t) - \int_{-\infty}^{\infty} \Phi^{(l)}(\mathbf{z}, \mathbf{r}, t) \widehat{\omega}^{(l)}(\mathbf{z}, t) d\mathbf{z} \right] \\ & + \sum_{j=1(j \neq l)}^p \nu_{jl}(t) \frac{\widehat{P}_j(t)}{\widehat{P}_l(t)} \left[\widehat{\omega}^{(j)}(\mathbf{\Lambda}, t) - \widehat{\omega}^{(l)}(\mathbf{\Lambda}, t) \right], \quad \omega(\mathbf{\Lambda}_0, t_0), \quad l = \overline{1, p}, \end{aligned} \quad (3)$$

where $\widehat{\mathbf{\pi}}^{(l)}(\mathbf{\Lambda}, t)$ is the probability density flow vector:

$$\widehat{\mathbf{\pi}}^{(l)}(\mathbf{\Lambda}, t) = \mathbf{K}_1^{(l)}(\mathbf{\Lambda}, t) \widehat{\omega}^{(l)}(\mathbf{\Lambda}, t) - \frac{1}{2} \operatorname{div} [\mathbf{K}_2^{(l)}(\mathbf{\Lambda}, t) \widehat{\omega}^{(l)}(\mathbf{\Lambda}, t)],$$

where $\mathbf{K}_1^{(l)}(\mathbf{\Lambda}, t)$ is the local rate vector, $\mathbf{K}_2^{(l)}(\mathbf{\Lambda}, t)$ is the diffusion matrix, $\widehat{P}_l(t)$ is the a posteriori probability of the l th state, $\omega(\mathbf{\Lambda}_0, t_0)$ is the initial value of probability density of the state dynamics $\mathbf{\Lambda}(t_0)$, $\Phi^{(l)}(\mathbf{\Lambda}, \mathbf{r}, t)$ is the derivative of the likelihood function logarithm in the l th state (\mathbf{z} is the variable $\mathbf{z} \in \mathbf{\Lambda}$, the sign \wedge means the a posteriori function value).

The a posteriori probability density for the whole dynamics process is determined as:

$$\widehat{\omega}(\mathbf{\Lambda}, t) = \sum_{l=1}^p \widehat{P}_l(t) \widehat{\omega}^{(l)}(\mathbf{\Lambda}, t).$$

The suboptimal estimate of Markov process $\mathbf{\Lambda}(t)$ on the assumption of mean square loss function is the conditional mathematical expectation. The optimal estimate of discrete process $l(t)$ by the a posteriori probability criterion will be such a value of l that makes the value of $\widehat{P}_l(t)$ maximum.

The estimate of the state is the probabilistically weighted average $\hat{\Lambda}(t) = \sum_{l=1}^p \hat{P}_l(t) \hat{\Lambda}^{(l)}(t)$.

The main distinction of signal processing in systems with the random structure consists in the presence of the state probability estimate equations (Eq. (4)) in the filtering algorithms and in the relation between these equations:

$$\frac{d\hat{P}_l(t)}{dt} = -\nu_l(t)\hat{P}_l(t) + \sum_{j=1(j \neq l)}^p \nu_{jl}(t)\hat{P}_j(t) + \frac{1}{2}\hat{P}_l(t) \left\{ \int_{-\infty}^{\infty} \Phi^{(l)}(\mathbf{z}, \mathbf{r}, t) \hat{\omega}^{(l)}(\mathbf{z}, t) d\mathbf{z} - \sum_{k=1}^p \hat{P}_k(t) \int_{-\infty}^{\infty} \Phi^{(k)}(\mathbf{z}, \mathbf{r}, t) \hat{\omega}^{(k)}(\mathbf{z}, t) d\mathbf{z} \right\} \quad (4)$$

The a priori state probabilities $P_l(t)$ can be found according to Kolmogorov equations.

In [4] the solution of the problem was obtained in Gaussian approximation of the a posteriori probability density. In the algorithms synthesis it was supposed that the signal or image position along one of the axes (e.g., in y direction) was known to simplify the derivation, denoting the signal $\mathbf{S}^{(l)}(x, \mathbf{A}, t) = \mathbf{S}^{(l)}(x - \lambda_x, t)$, and the state parameter λ_x without index $\lambda_x = \lambda$.

For the case when the parameters changes form Markov process with two states ($l = \overline{1,2}$) and the transition intensity $\nu(t)$ the solution of nonlinear filtering problem for systems with random structure derived in [4] is presented by the following correlation-extremum algorithms for computing the a posteriori probabilities of state (Eq. (5)), the state estimates (Eq. (6)), and the covariance (Eq. (7)).

The differential equation for the a posteriori probabilities of state is:

$$\frac{d\hat{P}_1(t)}{dt} = - \left\{ \nu - \frac{1 - \hat{P}_1(t)}{C_X^{(1)}} \left[k_1 + B^{(1)}(\Delta\lambda_{(1)}) + \frac{1}{2} \sigma_{(1)}^2(t) \frac{\partial^2 B^{(1)}(\Delta\lambda_{(1)})}{\partial \Delta\lambda_{(1)}^2} \right] \right\} \hat{P}_1(t) + \left\{ \nu - \frac{\hat{P}_1(t)}{C_X^{(2)}} \left[k_2 + B^{(2)}(\Delta\lambda_{(2)}) + \frac{1}{2} \sigma_{(2)}^2(t) \frac{\partial^2 B^{(2)}(\Delta\lambda_{(2)})}{\partial \Delta\lambda_{(2)}^2} \right] \right\} [1 - \hat{P}_1(t)], \quad (5)$$

$$\hat{P}_2(t) = 1 - \hat{P}_1(t),$$

where $\hat{P}_2(t)$ is the a posteriori probability of the second state, $\Delta\lambda_{(l)}(t)$ is the state estimate error $\Delta\lambda_{(l)}(t) = \lambda(t) - \hat{\lambda}^{(l)}(t)$, $\sigma_{(l)}^2(t)$ is the variance of the a posteriori probability density function $\sigma_{(l)}^2(t) = \langle [(\lambda(t) - \hat{\lambda}^{(l)}(t))]^2 \rangle$, $l = \overline{1,2}$; $B^{(l)}(\Delta\lambda_{(l)}, t)$ is the spatial correlation function in the l th state $B^{(l)}(\Delta\lambda_{(l)}, t) = \langle \mathbf{S}^{(l)T}(x - \hat{\lambda}^{(l)}(t), t) \mathbf{S}^{(l)}(x - \lambda, t) \rangle$, $C_X^{(l)}$ is the specific spectral intensity $C_X^{(l)} = C_0^{(l)}/X$ of the spatial-time-varying noise $\mathbf{N}^{(l)}(x, t)$.

The Eq. (5) has been derived using the assumption of the “unpowered” parameters. This assumption means that the integrals $\int_{-X}^X [S^{(l)}(x - \hat{\lambda}^{(l)}(t))]^2 dx$ and $\int_{-X}^X r^2(x, t) dx$, which represent signal energy and are explicitly independent of the estimate parameter, may be included in the k_1 and k_2 coefficients.

The state estimate equation (Eq. (6)) is presented below:

$$\frac{d\hat{\lambda}^{(l)}(t)}{dt} = f^{(l)}(\hat{\lambda}^{(l)}, u, t) - \frac{\sigma_{(l)}^2(t)}{C_X^{(l)}} \frac{\partial B^{(l)}(\Delta\lambda_{(l)}, t)}{\partial \Delta\lambda_{(l)}} + \frac{\sigma_{(l)}^2(t)}{C_0^{(l)}} N_X^{(l)} + \nu \frac{\hat{P}_j(t)}{\hat{P}_l(t)} [\hat{\lambda}^{(l)}(t) - \hat{\lambda}^{(j)}(t)], \quad \hat{\lambda}^{(l)}(t_0) = \hat{\lambda}_0^{(l)}, \quad (l, j = \overline{1,2}, j \neq l), \quad (6)$$

where $N_x^{(l)} = \int_{-x}^x \frac{\partial S^{(l)T}(x-\hat{\lambda}^{(l)}, t)}{\partial \hat{\lambda}^{(l)}} \mathbf{N}^{(l)}(x, t) dx$, (as a remark: in many cases the measurements signals and noises are uncorrelated). The suboptimal state estimate for the whole process can be obtained by using a weighted sum $\hat{\lambda}(t) = \hat{P}_1(t)\hat{\lambda}^{(1)}(t) + \hat{P}_2(t)\hat{\lambda}^{(2)}(t)$.

The variance equation is:

$$\begin{aligned} \frac{d\sigma_{(l)}^2(t)}{dt} &= 2\sigma_{(l)}^2(t) \frac{\partial f^{(l)}(\hat{\lambda}^{(l)}, u, t)}{\partial \hat{\lambda}^{(l)}} + \frac{\sigma_{(l)}^4(t)}{C_x^{(l)}} \frac{\partial^2 B^{(l)}(\Delta\lambda_{(l)})}{\partial \Delta\lambda_{(l)}^2} + q^{(l)}(t) \\ &+ v \frac{\hat{P}_j(t)}{\hat{P}_l(t)} \left[\sigma_{(j)}^2(t) - \sigma_{(l)}^2(t) + (\hat{\lambda}^{(j)}(t) - \hat{\lambda}^{(l)}(t))^2 \right], \quad \sigma_{(l)}^2(t_0), \quad (l, j = \overline{1, 2}, j \neq l), \end{aligned} \quad (7)$$

and the variance of the estimate error for the whole process is $\sigma^2(t) = \hat{P}_1(t)\sigma_{(1)}^2(t) + \hat{P}_2(t)\sigma_{(2)}^2(t)$. The correlation-extremum filtering algorithm with random structure for estimation of signal position along the y axis has been derived, similarly.

The proposed algorithms (Eqs. (5-7)) can be applied to the solution of the linearization problem in the case of great estimation errors and provide the estimator adaptive capability due to the extension of the correlation-extremum filtering and identification methods for systems with random structure.

The research is continuing in different areas of the proposed algorithms application and, particularly, for the systems with interrupted signal information, and for the case when the measurement noise vector $\mathbf{N}^{(l)}(x, y, t)$ is the spatial-time-varying colored Gaussian-Markov process new algorithms have been derived.

1.3. Applications to TTS

A guidance kinematic scheme under consideration can be described by a system of nonlinear stochastic differential equations (Eq. (8)) for the state dynamics model:

$$\begin{aligned} \dot{\varepsilon}^{(l)}(t) &= \frac{1}{D^{(l)}(t)} (V_c(t) \sin(\varepsilon^{(l)}(t) - \theta_c^{(l)}(t)) - V_T(t) \sin(\varepsilon^{(l)}(t) - \theta_T^{(l)}(t))) + W_\varepsilon(t), \\ \dot{D}^{(l)}(t) &= -V_c(t) \cos(\varepsilon^{(l)}(t) - \theta_c^{(l)}(t)) + V_T(t) \cos(\varepsilon^{(l)}(t) - \theta_T^{(l)}(t)) + W_D(t), \\ \dot{\theta}_c^{(l)}(t) &= N_c^{(l)} \dot{\varepsilon}^{(l)}(t), \quad \varepsilon^{(l)}(t_0) = \varepsilon_0^{(l)}, \quad D^{(l)}(t_0) = D_0^{(l)}, \quad \theta_c^{(l)}(t_0) = \theta_{c0}^{(l)}, \end{aligned} \quad (8)$$

where $\varepsilon(t)$ is a LOS angle relative to reference line, $\theta_c(t)$ is an angular position of an aircraft (pursuer) velocity vector $V_c(t)$, $\theta_T(t)$ is an angular position of a target velocity vector $V_T(t)$, $D(t)$ is a current range from an aircraft to a target, $W_\varepsilon(t)$ and $W_D(t)$ are the system uncorrelated zero-mean Gaussian white noises with spectral densities $q_\varepsilon(t)$ and $q_D(t)$, respectively; $\varepsilon^{(l)}(t_0)$, $D^{(l)}(t_0)$, $\theta_c^{(l)}(t_0)$, $V_c(t_0)$, $V_T(t_0)$, $\theta_T^{(l)}(t_0)$ are the initial values; $N_c^{(l)}$ is the proportional navigation law constant N_c in the l th state. As a remark, the form of the equations description (Eq. (8)) for systems with random structure is first presented in this paper.

The measurements equation (Eq. (9)) represents a spatial-time-varying signal $S(x, \varepsilon, t)$ observed by means of a coordinator with a narrow field of view which measures small deviations relative to the boresight:

$$\mathbf{r}(x, t) = \mathbf{S}^{(l)}(x, \varepsilon^{(l)}, t) + \mathbf{N}^{(l)}(x, t), \quad (9)$$

where $\mathbf{N}^{(l)}(x, t)$ is the spatial-time-varying noise of white Gaussian type with spectral densities $C_0^{(l)}$ in the case of stationary measurements noise, and $C_0^{(l)}(t)$, in the case of nonstationary noise depending on $D^{(l)}(t)$. The form of the signal which is a nonlinear function of the estimated Markov process $\varepsilon(t)$ is determined by antenna pattern with the beam width of $3^\circ-7^\circ$. The changes

of structure and the corresponding algorithms occur by the Markov process law with the two states $l = \overline{1,2}$ and the transition intensities $\nu_{12}(t)$, and $\nu_{21}(t)$, taking into account the values of estimation errors of a LOS angle. In a special case the system changes the mode of tracking when the estimation error exceeds the threshold value and the moments of the structure changes are known. There is a special case of the $\varepsilon(t)$ coordinate jumping disturbance, when the system noise $W_\varepsilon(t)$ is a function of a structure $W_\varepsilon^{(l)}(t)$, particularly, when the disturbance is a hit.

The purpose of this part of the present scientific research is to create reliable nonlinear information processing algorithms for a target tracking system as a part of a guidance-navigation complex which provide suboptimal estimation of the range from an aircraft to a target $D(t)$, the line of sight angle $\varepsilon(t)$, the bearing angle, and the guidance law parameters for different cases and conditions.

The adaptive filtering schemes are considered for 1) the proposed in this paper approach considering the navigation constant N_c as a parameter to be identified, which changes its structure according to various conditions in contrast to all known previous solutions including [5] where the PN guidance law $\dot{\theta}(t) = N_c \dot{\varepsilon}(t)$ with the traditionally used navigation constant values N_c , interpreted as the constant values received by experimental approach was applied; 2) the proposed derived guidance law, in which the navigation coefficient N_c is not a constant, (it is a function of the state estimates as an analytical expression); 3) the direct guidance law.

The estimate of the bearing angle was obtained for its state dynamics as a constant value and in the presence of noise.

The behavior of the system has been investigated with comparison for different first proposed conditions: 1) with adaptive switching from one to the other mode of operation using the information artificial aging techniques on the required parts of the trajectories (when the moment of the structure change depends on the estimation errors value); 2) with changes of the corresponding guidance law parameters, and 3) with switching from one guidance law to the other during tracking.

The adaptation mechanism of the proposed estimation algorithms represents an effective means of switching the field of view of a radar tracker or an optical (infrared image) tracker, and ensures the system operation, particularly, for a wide dynamic range of target maneuvers.

2. Conclusions

The proposed algorithms in comparison with the traditional estimation algorithms first show a relationship between the a posteriori probability density maximum criterion, the covariance matrix minimum criterion, and the cross-correlation function maximum criterion.

The proposed expansion of mathematical description based on the nonlinear filtering theory of processes and the systems with random structure optimization provides adaptive features for the correlation-extremum systems by generating the probabilistically weighted average of the state estimates. The correlation-extremum methods combined with the theory of Markov processes and systems with random structure ensure the stochastic dynamic systems reliability and operation in different conditions and under environment influences.

These algorithms allow to solve the filtering and identification problems for complex stochastic dynamic systems equipped with image sensors (e.g., radar, optics, and etc.) using information of spatial-time-varying signals of different nature fields.

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References

- [1] **Baklitski V., Yuriev A.** Correlation-Extremum Methods in Navigation. Radio and Communication, Moscow, 1982, (in Russian).
- [2] **Chang C., Athans M.** State estimation for discrete systems with switching parameters. IEEE Transactions on Electronic Systems, Vol. 14, Issue 3, 1978, p. 418-425.
- [3] **Kaminskas V., Sakalauskas E.** Identification algorithm for linear systems with distributed parameters. Engineering Cybernetics, Vol. 3, 1987, p. 184-189, (in Russian).
- [4] **Kolosovskaya T.** Spatial-time-varying signals processing algorithms in systems with random structure. Mechanical Engineering and Machine Reliability Problems, Vol. 5, 1995, p. 105-112, (in Russian).
- [5] **Shukla U., Mahapatra P.** Generalized linear solution of proportional navigation. IEEE Transactions on Aerospace and Electronic Systems, Vol. 24, Issue 3, 1988, p. 231-237.
- [6] **Kolosovskaya T.** The application of an extended Kalman filter to synthesis and analysis of estimation algorithms for magnetic navigation systems. 14-th International Conference on Aviation and Cosmonautics, Moscow, 2015, p. 306-309.