Optimal design of protective clothing based on difference equation

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Abstract. The temperature distribution and thickness design of high temperature protective clothing are studied in this paper. Based on the data provided by China mathematical modeling competition in 2018. We establish the temperature distribution model and skin layer heat conduction and burn model. The interface continuous conditional difference method, differential iterative method, least squares method and the chasing method are used to solve the given temperature distribution on the protective clothing in the environment, and analyze protective clothing meeting the actual needs.

Keywords: heat transfer equation, heat exchange coefficient, least squares, the chasing method.

1. Introduction

When working in a high temperature environment, workers often need to wear adiabatic protective clothing, which can effectively prevent heatstroke, burns and other injuries, often have the characteristics of flame retardancy, liquid repellency, etc. Many researchers have carried out the material and thickness of the garment. Designed to increase the thermal insulation performance.

Lu studied the performance of protective clothing against liquid splash and its permeability and heat and moisture conductivity in high temperature environment, and used skin burn prediction model to predict the protection of thermal protective clothing [1]. Tian et al. applied numerical simulation to evaluate the performance of high temperature protective clothing, and summarized the characteristics and disadvantages of typical heat conduction model and skin burn model [2].

Zhang et al analyzed the development status and defects of temperature-adjustable protective clothing at home and abroad, and analyzed the mechanism of phase change materials in protective clothing, and explained its application in protective clothing [3].

For the application of phase change materials, Zhu and others conducted more in-depth research, analyzing from the aspects of reducing thermal stress, improving the comfort of protective clothing, preventing sudden changes in temperature, etc. [4]. Zhang Chao et al took into account human physiological indicators, used the dummy model to simulate the heated scene, and established a human-clothing-environmental thermal protective clothing evaluation standard [5]. Lin Jianbo studied the radiant heat penetration resistance of thermal protective clothing and selected two suitable thermal protective clothing materials [6]. M. J. Slapak contrasts PBI fibers with other insulating materials to highlight their superiority as a thermal protective clothing material [7].

Yang analyzed the research status of thermal protective clothing in the police field and introduced the development status of the new thermal protective material aerogel [8]. Li used 13 kinds of flame retardant fabrics as experimental samples, using statistical methods to obtain protective performance and fabric thickness, areal density, tpp value is positive related [9]. Shen Lanpin designed a double-layer thermal protective clothing fabric from the end use of flame retardant thermal protective clothing [10]. Wang Weiwei et al used the improved finite difference method to establish a heat transfer model and optimized the thickness of each layer of thermal protective clothing [11]. Lu Linzhen calculated the temperature value of each fixed contact surface with time, predicted the critical time of the burn level, and prevented skin burns [12].

In this paper, the temperature distribution outside the protective clothing is studied with time and distance. Lu Linzhen also carried out related research and established the heat transfer model of the protective clothing-air layer-skin system, and the influence of some parameters on the protective performance [13]. In addition, this paper optimizes the thickness of the garment in a specific environment to achieve the best protection.

2. Protective clothing temperature distribution model

The protective clothing studied in this paper is divided into four layers. A straight line of vertical protective clothing is selected as the research object. The outer side of the first layer is taken as the origin, and the horizontal direction is directed to the positive direction of the skin layer to establish the x-axis. The arrangement relationship of each layer is shown in Fig. 1. As shown, it is assumed that the layers are tightly connected and the density of the medium is uniform, wherein the thickness of the mth layer is recorded as l_m ($m = 1, 2, 3, 4$), and $L_m = \sum_{k=1}^{m} l_k$.

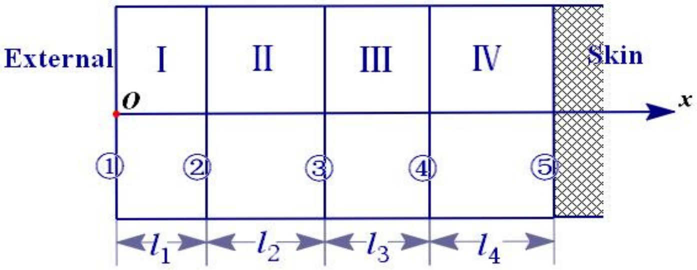

Fig. 1. The arrangement relationship of each layer

2.1. Generalized equation

$$\frac{\partial T_m}{\partial t} = {a_m}^2 \frac{\partial^2 T_m}{\partial x^2}, \quad m = 1,2,3,4, \tag{1}$$

the temperature distribution function of each layer is $T_m(x,t)$ ($m =$ 1, 2, 3, 4) where $a_m^2 = k_m / c_m \rho_m$, c_m is the specific heat capacity of the mth layer, ρ_m is the density of the mth layer, k_m is the thermal conductivity of the mth layer. The analysis of the conditions for the solution of the above generalized equations is as following.

2.2. Initial conditions

Assume that the initial temperature of each point of the protective suit is the same, so there are initial conditions:

$$T(x,0) = w_0, \tag{2}$$

where w_0 is the temperature of the protective clothing before entering the laboratory, the protective clothing has been worn before entering the laboratory, so it is assumed that the temperature is equal to the human body temperature.

2.3. Boundary conditions

For the actual situation of the protective layer and the contact surface, we use the third type of boundary conditions and interface continuous conditions.

According to the Heat Transfer theory, when there is heat exchange between the solid and the fluid, the third type of boundary condition is used:

$$-k\frac{\partial T}{\partial x}\Big|_{x\,=\,l} = h\big(T(l,\infty) - T(l,t)\big), \tag{3}$$

where $T(l,\infty)$ is a stable temperature maintained by the left boundary of the first layer after a long period of time, h is the heat exchange coefficient that is a constant determined by the material on both sides of the contact surface.

When the two objects are in full contact, the temperature of the contact surfaces of the two objects and the heat flux density values are equal:

$$T_1\big|_{x=l} = T_2\big|_{x=l},$$
$$-k_1\frac{\partial T_1}{\partial x}\Big|_{x\,=\,l} = -k_2\frac{\partial T_2}{\partial x}\Big|_{x\,=\,l}.$$

2.4. The left end boundary condition of the first layer

For the left boundary of the ith layer (1 in Fig. 1), the left side is air fluid, and the right side is the ith layer of clothing material, then:

$$-k_1\frac{\partial T_1}{\partial x}\Big|_{x\,=\,0} = h_1\big(w_1 - T(0,t)\big),$$

among them h_1 is the heat exchange coefficient between the ith layer and the air, w_1 is the outside temperature, $w_1 = 75$.

2.5. $x = L_m$ $(m = 1,2,3)$ Contact surface boundary condition

For $x = L_m$, two sides are solid clothing materials, there are temperature equations:

$$T_i\big|_{x=L_i} = T_{i+1}\big|_{x=L_i}, \quad (i = 1,2,3). \tag{4}$$

Heat flux density equation:

$$-k_i\frac{\partial T_i}{\partial x}\Big|_{x\,=\,L_i} = -k_{i+1}\frac{\partial T_{i+1}}{\partial x}\Big|_{x\,=\,L_i}, \quad (i = 1,2,3). \tag{5}$$

2.6. The IV layer right end boundary condition

For the right edge of the iv layer (5 in Fig. 1), there is a third type of boundary condition:

$$-k_4\frac{\partial T_4}{\partial x}\Big|_{x\,=\,L_4} = h_2\big(w_2 - T(L_4,t)\big), \tag{6}$$

where w_2 is body temperature.

A partial differential equation model of the temperature distribution of the protective suit is obtained from the above Eqs. (1)-(5).

3. Determination of temperature distribution

3.1. Difference method

To solve the differential equation of Eq. (1), convert the differential equation into a difference equation. The table represents the coordinate x on the horizontal axis and the time t on the vertical axis. The content is the temperature at the corresponding coordinate x and time t, temperature function $T(x,t)$. $T_{i,j}$ indicates the temperature corresponding to the j coordinate x at the i time t, as shown in Fig. 2.

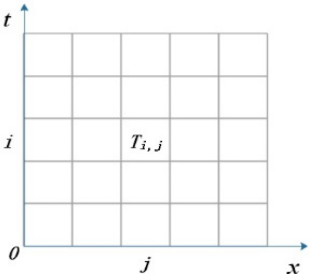

Fig. 2. Time-coordinate table

3.1.1. Heat conduction equation difference

For the solution of differential equations, it is often considered to differentiate the differential equations. The partial derivatives of the differential equations of Eq. (1) can be transformed.

From the first-order backward difference quotient and the second-order center difference:

$$c_m\rho_m\cdot\frac{T_{i,j}-T_{i-1,j}}{\Delta t}=k_m\frac{T_{i,j+1}-2T_{i,j}+T_{i,j-1}}{\Delta x_m^2}$$
$$-\lambda_m T_{i,j-1}+(1+2\lambda_m)T_{i,j}-\lambda_m T_{i,j+1}=T_{i-1,j},$$

among them $\lambda_m=k_m\Delta t/c_m\rho_m\Delta x_m^2$.

3.1.2. The third type of boundary condition difference method

For the third type of boundary conditions, the difference is used to discretize the boundary conditions, and the boundary temperature is calculated.

At the left and right edge:

$$\begin{cases}-k_1\dfrac{\partial u}{\partial x}+h_1T=h_1w_1,\\ k_4\dfrac{\partial u}{\partial x}+h_2T=h_2w_2,\end{cases}$$
$$\begin{cases}(1+M_1)T_{i,0}-T_{i,1}=M_1w_1,\\ -T_{i,n-1}+(1+M_2)T_{i,n}=M_2w_2,\end{cases}\tag{7}$$

among them $M_1=h_1/k_1\cdot\Delta x_1$, $M_2=h_2/k_4\cdot\Delta x_4$, n is the total number of columns.

3.1.3. Interface continuous condition difference

At the interface of different materials, because the parameters on both sides are different, and the heat conduction equation is different, the temperature value cannot be obtained by Eq. (3). In this paper, the difference in cross-section temperature and the equal heat flux density are considered, and the difference is symmetrized to obtain the interface. It is temperature function differentiation method:

$$\begin{gathered}k_m \frac{T_{i,j}-T_{i,j-1}}{\Delta x_m} = k_{m+1}\frac{T_{i,j+1}-T_{i,j}}{\Delta x_{m+1}}, \\ -N_m T_{i,j-1} + (N_m + N_{m+1})T_{i,j} - N_{m+1}T_{i,j+1} = 0,\end{gathered} \tag{8}$$

among them $N_m = k_m/\Delta x_m$.

Connect the Eqs. (6), (7) and (8) in parallel, transform into a linear system of equations, and use the chasing method to solve the equations.

3.2. Solution of heat exchange coefficient

The difference iterative method in 2.1 was used to obtain $T_{i,n}$, combining with the measured values T^*in annex 2, and the least square method was used to solve the following optimization problems:

$$\min \quad f(h_1, h_2) = \sum_{i=0}^{5400} \left(T_{i,n} - T^*\right)^2.$$

Finally, use the chasing method to obtain parameters that $h_1 = 135.094$ m²/s and $h_2 = 8.366$ m²/s, respectively.

The known parameters used (all have been converted to standard units): $w_1 = 75$, $w_2 = 37$, $l_1 = 6\times10^{-4}$, $l_2 = 6\times10^{-3}$, $l_3 = 3.6\times10^{-3}$, $l_4 = 5\times10^{-3}$.

3.3. Temperature distribution

Use the obtained parameters h_1, h_2, and the differential iteration method in 2.1, we find the temperature distribution of each point at each moment as shown in Fig. 3, Fig. 4 and the boundary temperature of each layer at each moment in Fig. 5.

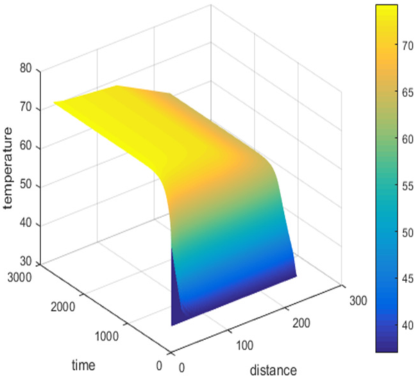

Fig. 3. Two-dimensional graph

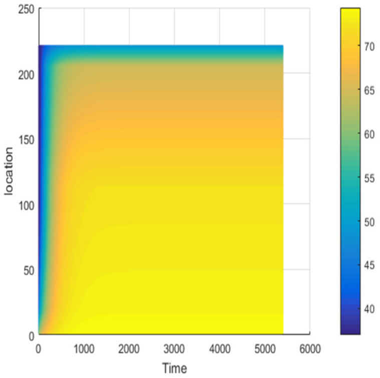

Fig. 4. Three-dimensional graph

According to Fig. 3 and Fig. 4, it can be found that the temperature of the protective clothing changes monotonously with the distance and time from the origin, and finally stabilizes, that is, the ambient temperature given by the problem is 75 °C, which basically conforms to the actual situation.

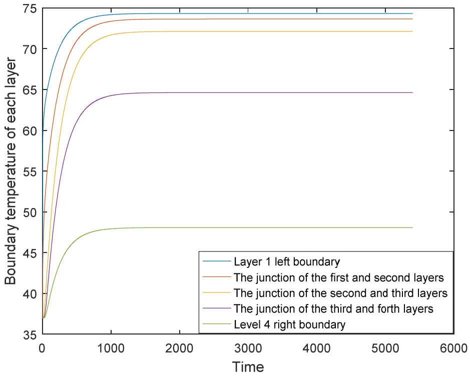

Fig. 5. The boundary temperature of each layer

4. Skin layer heat conduction and burn model

In order to further determine the optimal thickness of the thermal protective suit, it is first necessary to determine the temperature value of the skin layer, so it is considered to add a layer on the right side of the skin surface to be the skin layer. We analyze generalized equation, boundary conditions and initial conditions of the skin layer as follows.

4.1. Generalized equation

The Pennes biothermal transfer model introduced in the literature [13] was determined as:

$$\frac{\partial T_5}{\partial t} = \frac{k_5}{c_5\rho_5}\frac{\partial^2 T_5}{\partial x^2} - \frac{c_b\rho_b w_b}{c_5\rho_5}(T_r - T_5),$$

among them k_5 is the thermal conductivity of the skin, w_b is infusing blood, c_5, ρ_5 are the density and specific heat of the skin layer, $c_b\rho_b w_b(T_r - T_5)$ represents the energy of heat exchange between human cells and blood.

4.2. Boundary conditions

The left side of the skin layer is the interface between the air layer and the skin surface, and may involve thermal radiation, and the heat radiation gradually decays from the first layer toward the skin layer:

$$q_r = q_r\big|_{x=L_1} \cdot e^{-\eta x},$$

where η is Attenuation coefficient.

The right border is a constant temperature of 37 °C in the human body, $T_5\big|_{x=L_5} = 37$.

4.3. Initial conditions

At the initial moment, the temperature is 37 °C everywhere, $T_5|_{t=0} = 37$.

Establish the heat transfer model of the 5th layer:

$$\begin{cases} \dfrac{\partial T_5}{\partial t} = \dfrac{k_5}{c_5\rho_5}\dfrac{\partial^2 T_5}{\partial x^2} - \dfrac{c_b\rho_b w_b}{c_5\rho_5}(T_r - T_5), \\ T_5|_{\,t=0} = 37, \\ -k_4\dfrac{\partial T_4}{\partial x}\Big|_{x\,=\,L_4} = q_r - k_5\dfrac{\partial T_5}{\partial x}\Big|_{x\,=\,L_4}, \\ T_5|_{\,x=L_5} = 37. \end{cases}$$

The above equation is differentiated by the forward difference method, the skin surface temperature is calculated from the difference expression and the initial condition and the boundary condition, and the squared sum of difference of the theoretical value and the measured value is used to determine the parameter in the skin layer heat transfer model.

In order to make the research meet the actual needs, we analyze the degree of burns [15]:

$$\Omega(x,\tau) = \Omega(x,0) + \int_0^{\tau} Pe^{-\frac{\Delta E}{R(T(x,t)+273)}}dt,$$

among them ΔE is skin activation properties, P is the frequency disruption factor.

When $\Omega \leq 0.53$, no burns will occur. $0.53 < \Omega \leq 1$, a first-degree burn occurs, $0.53 < \Omega \leq 10^4$, second-degree burn occurs. $\Omega > 10^4$, third-degree burn occurs.

5. Sensitivity analysis

Since the data given by the accessory is measured, the measurement error generally shows a normal distribution. Sensitivity analysis is now performed on the model. Add the raw data to the measurement error with a mean of 0.01 and a variance of 0.01 and recalculate. When $t = 2000$, a temperature along with time t changing image is drawn. When $x = 200$, a temperature along with distance x changing image is drawn.

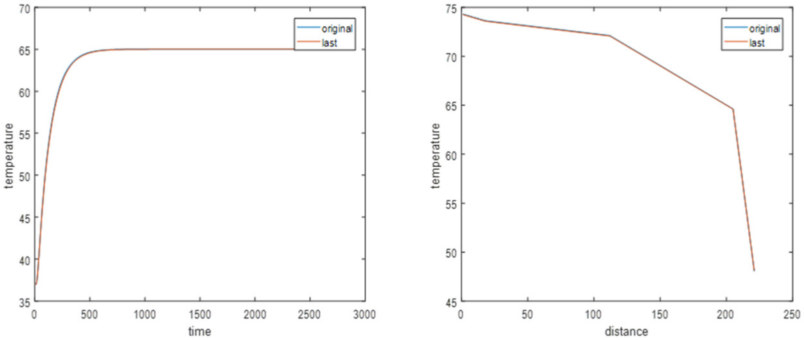

Fig. 6. Sensitivity analysis chart

It is known from the image that when considering the measurement error, the calculated temperature value is not much different from the temperature value calculated from the original measurement data, and only some random fluctuations are generated on the basis of the original data, and the overall trend of the image does not change much. Therefore, the model and algorithm we have established are highly stable.

6. Conclusions

In this paper, a temperature distribution model is established for thermal protective clothing at high temperature. The temperature distribution of each layer of thermal protection at a specific ambient temperature is studied, and the optimal thickness of each layer is obtained by combining specific temperature and thickness constraints. In the real environment, it is necessary to consider factors such as changes in the influence of human sweat and the moisture contained in the fabric itself, and it is possible to establish a heat and moisture transfer model under transient conditions in a multi-layer fabric combination state, and The radiant heat transfer between the layers is studied in depth, and the influence of the initial water content is analyzed, and the changes in the physical parameters affected by the water content in the model are analyzed.

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