

Multiple root resonance of tandem mill

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Received 3 August 2019; received in revised form 1 March 2020; accepted 23 March 2020

DOI <https://doi.org/10.21595/jve.2020.20934>



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Abstract. The multiple-root resonance theory is introduced. The multiple-root resonance region is defined, and it includes the sub-multiple-root resonance region and complete-multiple-root resonance region. It is proved that the vibration mode shapes are identical if the natural frequencies are similar in size. The dynamic analysis of the 3-DOF model is investigated, and the results show that the variations of the mass ratio and stiffness ratio induce the natural frequencies approaching and forming a multiple-root resonance. The dynamic analysis of the tandem mill is made to study the multiple-root vibration, and the results show that with the increase in mass ratio, the 6th and 7th-order natural frequencies continue approaching one another and converge at the 5th-order natural frequency. The finite element analysis of the rolling mill shows that there are two similar natural frequencies in the rolling mill, and the vibration modes are not identical but have large vertical-component vibrations. The field test shows that the multiple-root resonance occurs in the tandem mill; after the foundation reconstruction, the multiple-root resonance is replaced by resonance with one dominant frequency, and the vibration is reduced.

Keywords: vibration, dynamic analysis, tandem mill, multiple-root region.

1. Introduction

Rolling mill vibration can reduce the life of equipment and can affect the product quality, which is a worldwide problem and has attracted the attention of many scholars. Yarita, Tlustý, etc. [1, 2] studied the rolling mill vibration theoretically and explored its vibration mechanism; Tamiya T., Furui K. and Lida H. [3] studied the flutter phenomenon in the cold rolling process; Roberts W. [4] deeply discussed the triple-frequency flutter of the rolling mill; Nessler [5] studied the five-frequency flutter mode of the four-high rolling mill; Johnson [6] considered the non-linear factors of the rolling mill vibration; Chinese scholar Yan Xiaoqiang found that the rolling mill vibration was induced by the combination of mechanical, electrical and hydraulic movements [7].

Absorber can effectively suppress vibration, and it has attracted a large amount of attention: Rao [8] thought that hybrid dampers were more effective and reliable than a single active damper or passive dampers; J. studied the efficiency of the Semi-active Absorber for Controlling Self-excited Vibration [9]. In the field of rolling vibration research, Wang Xinxin and Yan Xiaoqiang [10] used the active vibration suppressor on the rolling mill [11] and achieved a good vibration suppression effect.

Scholars have made many achievements in the rolling mill vibration mechanism and vibration suppression. However, when solving the inherent characteristics of the rolling mill, few people pay attention to the essence of the rolling mill natural frequency. In this paper, the entire tandem mill is taken as a research object, the foundation mass of the tandem mill is considered as a variable, and the relationship between the foundation mass of the tandem mill and the natural frequency of the system is dynamically observed. Then, it was found that the natural frequency of the resonance of the tandem mill was caused by the combined action of one or more groups of multiple-root natural frequencies, which implied that the essence of the resonance of the rolling mill was a multiple-root resonance. In terms of vibration suppression, it was proposed to consider the multiple-root resonance in the design stage to avoid the existence of two or more approximate natural frequencies in the tandem mill.

2. Theory

2.1. Definition of multiple-root resonance

According to the vibration theory [11], when the excitation frequency is included by the resonance region of the system, the resonance will occur. When there are two close natural frequencies ω_1 and ω_2 in a system, the resonance region of each natural frequency is described by the neighbourhood as follows:

$$U(\omega_i, \delta_i) = \{x | \omega_i - \delta_i < x < \omega_i + \delta_i\}, \quad (1)$$

where ω_i is the natural frequency of the system, and δ_i is the neighbourhood radius of the natural frequency ω_i , which is positive. When the excitation frequency ω is included by the resonance region $U(\omega_i, \delta_i)$ of a natural frequency of the system, the resonance will occur.

It is assumed that the two adjacent natural frequencies are ω_1 and ω_2 and $\omega_1 < \omega_2$. The neighbourhood of the two adjacent frequencies is shown in Eq. (1). Then, we have the intersection of these regions:

$$\begin{cases} U(\bar{\delta}) = \emptyset, & \omega_2 - \omega_1 > |\delta_1 + \delta_2|, \\ U(\bar{\delta}) = \{x | x = \omega_1 + \delta_1 = \omega_2 - \delta_2\}, & \omega_2 - \omega_1 = |\delta_1 + \delta_2|, \\ U(\bar{\delta}) = \{x | \omega_2 - \delta_2 < x < \omega_1 + \delta_1\}, & |\delta_1 - \delta_2| < \omega_2 - \omega_1 < |\delta_1 + \delta_2|, \\ U(\bar{\delta}) = \{x | x \in \text{MIN}[U(\omega_i, \delta_i)]\}, & \omega_2 - \omega_1 < |\delta_1 - \delta_2|, \end{cases} \quad (2)$$

where $U(\bar{\delta})$ is the intersection of two natural frequency resonance regions. If the excitation frequency $\omega \in U(\bar{\delta})$, the two resonance frequencies will be excited, and two natural modes will simultaneously occur. If two resonance frequencies are similar or equal, the corresponding natural modes are identical. In the case of $\omega \in U(\bar{\delta})$, it is known from the principle of linear superposition that the system will vibrate more violently than the resonance with a single natural mode. The vibration, where two or more natural modes are excited by the same source, is called as a “multiple-root resonance”, and the neighborhood intersection $U(\bar{\delta})$ is called as the “multiple-root region”.

If $\omega_1 \neq \omega_2$, but their values are similar, they are called as a pair of “sub-multiple roots”, and $U(\bar{\delta})$ is called as the “sub-multiple-root region”. If $\omega_1 = \omega_2$, the characteristic equation has multiple roots in the analytic solution, which are called as the “complete-multiple-root” here, and $U(\bar{\delta})$ is called as the “complete-multiple-root region”. This paper only studies the sub-multiple-root.

The definition of the “multiple-root region” of multiple frequencies is similar.

2.2. Vibration mode of “multiple-root resonance”

This section proves that when the two natural frequencies are similar enough by their value, their vibration modes are consistent. The matrix in the dynamic equation for the multi-DOF vibration system is as follows [11]:

$$(\mathbf{K} - \omega^2 \mathbf{M})\mathbf{u} = 0, \quad (3)$$

where \mathbf{K} is the stiffness matrix; \mathbf{M} is the mass matrix; \mathbf{u} is the eigenvector. When $\mathbf{F} = \mathbf{K} - \omega^2 \mathbf{M}$ is substituted, Eq. (3) is transformed to:

$$\mathbf{F}\mathbf{u} = \begin{bmatrix} f_{11}(\omega) & \cdots & f_{1(n-1)}(\omega) & f_{1n}(\omega) \\ \vdots & \ddots & \vdots & \vdots \\ f_{(n-1)1}(\omega) & \cdots & f_{(n-1)(n-1)}(\omega) & f_{(n-1)n}(\omega) \\ f_{n1}(\omega) & \cdots & f_{n(n-1)}(\omega) & f_{nn}(\omega) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = 0, \quad (4)$$

where u_i ($i = 1, 2, 3, 4$) is the element of \mathbf{u} ; $f_{ij}(\omega)$ is the element of matrix \mathbf{F} , and:

$$f_{ij}(\omega) = k_{ij} - \omega^2 m_{ij},$$

where k_{ij} , and m_{ij} are the corresponding elements of matrices \mathbf{K} and \mathbf{M} ; Eq. (4) has a nonzero solution, and matrix \mathbf{F} is not a full-rank matrix. According to the elementary transformation of matrix [12], matrix \mathbf{F} can be transformed into a trapezoidal matrix $\bar{\mathbf{F}}$:

$$\mathbf{F}\mathbf{u} = \bar{\mathbf{F}}\mathbf{u} = \begin{bmatrix} \bar{f}_{11}(\omega) & \bar{f}_{12}(\omega) & \bar{f}_{1(n-1)}(\omega) & \bar{f}_{1n}(\omega) \\ 0 & \bar{f}_{12}(\omega) & \bar{f}_{2(n-1)}(\omega) & \bar{f}_{2n}(\omega) \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & \bar{f}_{(n-1)(n-1)}(\omega) & \bar{f}_{(n-1)n}(\omega) \\ 0 & \cdots & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} = 0, \quad (5)$$

where $\bar{f}_{ij}(\omega)$ is the element of the transformed matrix $\bar{\mathbf{F}}$. For the convenience of discussion, it is assumed that the system does not have an analytic multiple root; so that only the n th row of the trapezoidal matrix $\bar{\mathbf{F}}$ is zero, and the first rows of Eqs. (4) and (5) are equal. The second lines of $\bar{\mathbf{F}}$ are:

$$\overline{ROW}_2 = ROW_1 \times \frac{-f_{21}(\omega)}{f_{11}(\omega)} + ROW_2, \quad (f_{11}(\omega) \neq 0), \quad (6)$$

where \overline{ROW}_i and ROW_i are elements of each row of $\bar{\mathbf{F}}$ and \mathbf{F} .

Every element, $f_{ij}(\omega)$, in the matrix \mathbf{F} has a quadratic continuous function for ω . From Eq. (6), we know that the operation of \mathbf{F} is a simple operation of addition, subtraction, multiplication and division in the trapezoidal transformation process. Therefore, according to the theorem of continuous function [13], it is known that $\bar{f}_{ij}(\omega)$ is a continuous function, so the following can be obtained:

$$\lim_{\sigma \rightarrow 0} \bar{f}_{ij}(\omega + \sigma) = \bar{f}_{ij}(\omega). \quad (7)$$

The linear equations of Eq. (5) are:

$$\begin{aligned} \bar{f}_{11}(\omega)u_1 + \bar{f}_{12}(\omega)u_2 + \cdots + \bar{f}_{1n}(\omega)u_n &= 0, \\ \vdots & \\ \bar{f}_{(n-2)(n-2)}(\omega)u_{n-2} + \bar{f}_{(n-2)(n-1)}(\omega)u_{n-1} + \bar{f}_{(n-2)n}(\omega)u_n &= 0, \\ \bar{f}_{(n-1)(n-1)}(\omega)u_{n-1} + \bar{f}_{(n-1)n}(\omega)u_n &= 0. \end{aligned} \quad (8)$$

According to the final equation of Eq. (8), the following can be obtained:

$$r_{n-1} = \frac{u_{n-1}}{u_n} = -\frac{\bar{f}_{(n-1)n}(\omega)}{\bar{f}_{(n-1)(n-1)}(\omega)}, \quad (9)$$

where r_{n-1} is the vibration mode. With substitution $u_n = 1$ and iteration, the corresponding mode

$\mathbf{r}^{(i)}$ of natural frequency ω_i is as follows:

$$\mathbf{r}^{(i)} = [r_1^{(i)} \ r_2^{(i)} \ \dots \ r_{n-1}^{(i)} \ 1]^T. \quad (10)$$

Suppose that the system has two natural frequencies: ω_1, ω_2 , and $\omega_2 = \omega_1 + \sigma$, where σ is infinitesimal. Their modes are:

$$\begin{aligned} \mathbf{r}^{(1)} &= [r_1^{(1)} \ r_2^{(1)} \ \dots \ r_{n-1}^{(1)} \ 1]^T, \\ \mathbf{r}^{(2)} &= [r_1^{(2)} \ r_2^{(2)} \ \dots \ r_{n-1}^{(2)} \ 1]^T. \end{aligned} \quad (11)$$

With Eqs. (7) and (9), the following can be obtained:

$$r_{n-1}^{(2)} = -\frac{\bar{f}_{(n-1)n}(\omega_2)}{\bar{f}_{(n-1)(n-1)}(\omega_2)} = \lim_{\sigma \rightarrow 0} -\frac{\bar{f}_{(n-1)n}(\omega_1 + \sigma)}{\bar{f}_{(n-1)(n-1)}(\omega_1 + \sigma)} = \frac{\bar{f}_{(n-1)n}(\omega_1)}{\bar{f}_{(n-1)(n-1)}(\omega_1)} = r_{n-1}^{(1)}. \quad (12)$$

Similarly, it can be proven that $r_i^{(1)} = r_i^{(2)}$. Therefore, when the natural frequencies, ω_1 and ω_2 , are similar by their value (the difference between them is tiny enough), the following can be obtained:

$$\mathbf{r}^{(1)} = \mathbf{r}^{(2)}. \quad (13)$$

Thus, when the system has two natural frequencies with very similar values, the mode shapes are identical. And if two natural frequencies are induced by an excitation frequency, the multiple-root resonance occurs, and the superposition of the identical mode shapes amplifies the vibration.

2.3. Dynamics study of multiple-root vibration

In this section, the “device-foundation” dynamic model is introduced to study the multiple-root vibration. This model is shown in Fig. 1.

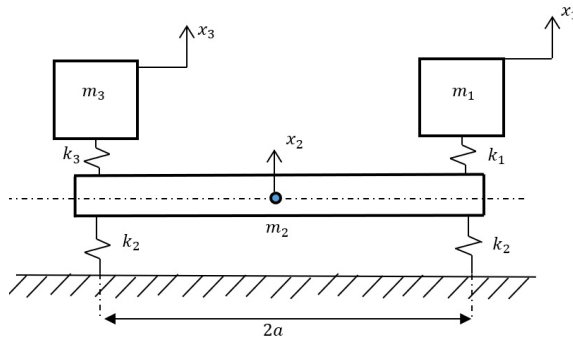


Fig. 1. Dynamic model of “device-foundation” system

It is assumed that the foundation does not swing. For Fig. 1, the dynamic equations are as follows:

$$\begin{cases} m_1 \ddot{x}_1 + k_1 x_1 - k_1 x_2 = 0, \\ m_2 \ddot{x}_2 - k_1 x_1 + (k_1 + k_3 + 2k_2) x_2 - k_3 x_3 = 0, \\ m_3 \ddot{x}_3 - k_3 x_2 + k_3 x_3 = 0. \end{cases} \quad (14)$$

The characteristic equation of Eq. (14) is:

$$|\Delta(\omega^2)| = \begin{vmatrix} k_1 - \omega^2 m_1 & -k_1 & 0 \\ -k_1 & k_1 + k_3 + 2k_2 - \omega^2 m_2 & -k_3 \\ 0 & -k_3 & k_3 - \omega^2 m_3 \end{vmatrix} = 0. \quad (15)$$

The changing mass and stiffness have a great significance in discussing the “multiple-root”.

Thus the mass ratio β is considered as a variable, and $m_1 = m_3 = 1$ kg, $k_1 = k_2 = k_3 = 1$ N/m, $m_2 = \beta m_1$ is substituted, the multiple-root natural frequency of the equipment when the mass ratio changes is discussed; and then the stiffness ratio γ is considered as a variable, and substituted $m_1 = m_3 = 1$ kg, $k_1 = k_3 = 1$ N/m, $m_2 = 2m_1$, $k_2 = \gamma k_1$, then the “multiple-root” of the system is discussed when the stiffness varies. The relationships among the mass, stiffness and natural frequency are shown in Fig. 2, where a_1 is the approaching degree between ω_1 and ω_2 ; a_2 is the approaching degree between ω_2 and ω_3 , and $a_i = (\omega_{max} - \omega_{min})/\omega_{min}$ is defined. Fig. 2 shows that when the mass ratio β increases, the 2nd-order natural frequency and 3rd-order natural frequency continue approaching, and when $\beta > 11$, the approaching factor $a_2 < 0.1$, when the stiffness ratio γ increases, the 1st-order natural frequency ω_1 and 2nd-order natural frequency ω_2 are gradually approaching one another, and when $\gamma = 40$, the approaching factor $a_1 = 0.013$, and will continue to decrease with the increase in stiffness ratio.

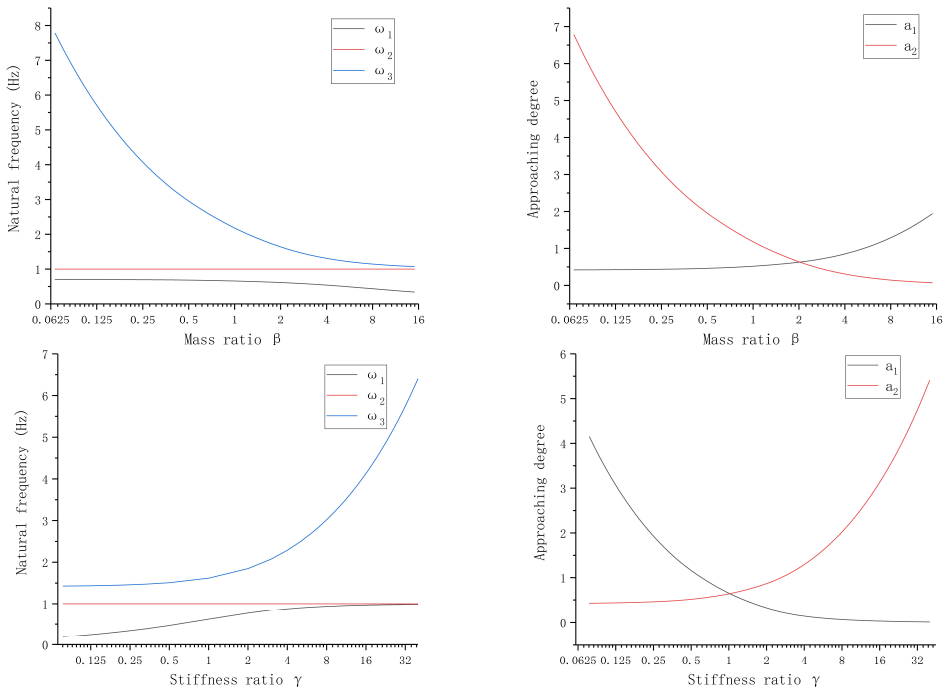


Fig. 2. Relationship among mass ratio, stiffness ratio and root natural frequency

3. Multiple-root resonance of tandem mill

Tandem mills, including hot tandem mills and cold tandem mills, are used for steel strip production, as shown in Fig. 3.

The tandem mill is a typical “multi-device-foundation” system as shown in Fig. 1, which has 5 stands installed on the foundation.

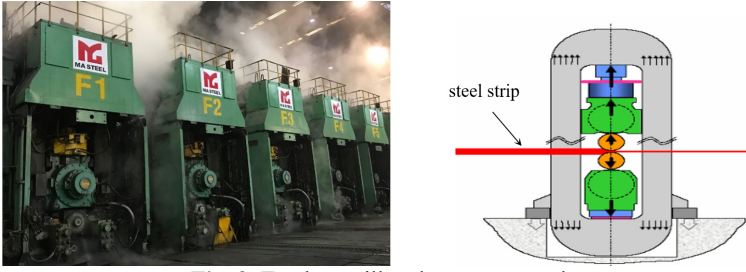


Fig. 3. Tandem mill and separate stand

3.1. Dynamics analysis of tandem mill

According to Fig. 3, a simplified diagram of the tandem mill is shown in Fig. 4, where x_i ($i = 1, 2, 3, 4, 5$) is the DOF of each mill, and x_6 is the DOF of the foundation; q_i ($i = 1, 2, 3, 4, 5$) is the DOF of each connect region, and $q_3 = x_6$; m_i ($i = 1, 2, 3, 4, 5$) is the quality of each stand; m_6 is the quality of the foundation; I is the moment of inertia of the foundation; k_1 is the connection stiffness of each mill and foundation; k_2 is the stiffness of the foundation and earth; a micro-pendulum motion around the center of the foundation is considered in the system, and θ is the pendulum angle; F is the harmonic excitation, and $F = F_0 \sin(\omega t)$, F_0 is the amplitude; a is the distance between two stands. There are seven DOFs in the system, and vector \mathbf{x} is defined as the DOFs of the system:

$$\mathbf{x} = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ q_5]^T. \quad (16)$$

The dynamics equations of Fig. 4 are:

$$\begin{cases} m_1 \ddot{x}_1 + k_1 x_1 - 2k_1 x_6 + k_1 q_5 = 0, \\ m_2 \ddot{x}_2 + k_1 x_2 - \frac{1}{2} k_1 x_6 + \frac{1}{2} k_1 q_5 = 0, \\ m_3 \ddot{x}_3 + k_1 x_3 - k_1 x_6 = 0, \\ m_4 \ddot{x}_4 + k_1 x_4 - \frac{1}{2} k_1 x_6 - \frac{1}{2} k_1 q_5 = 0, \\ m_5 \ddot{x}_5 + k_1 x_5 - k_1 q_5 = F \sin(\omega t), \\ m_6 \ddot{x}_6 - \frac{I(\ddot{q}_5 - \ddot{x}_6)}{4a^2} - 2k_1 x_1 - \frac{1}{2} k_1 x_2 - k_1 x_3 \\ \quad - \frac{1}{2} k_1 x_4 + \frac{11}{2} (k_1 + k_2) x_6 - 2(k_1 + k_2) q_5 = 0, \\ \frac{I(\ddot{q}_5 - \ddot{x}_6)}{4a^2} + k_1 x_1 + \frac{1}{2} k_1 x_2 - \frac{1}{2} k_1 x_4 - k_1 x_5 - 2(k_1 + k_2) x_6 + \frac{5}{2} (k_1 + k_2) q_5 = 0. \end{cases} \quad (17)$$

The mass matrix \mathbf{M} is:

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_6 + \frac{I}{4a^2} & -\frac{I}{4a^2} \\ 0 & 0 & 0 & 0 & 0 & -\frac{I}{4a^2} & \frac{I}{4a^2} \end{bmatrix}. \quad (18)$$

The stiffness matrix \mathbf{K} is:

$$\mathbf{K} = \begin{bmatrix} k_1 & 0 & 0 & 0 & 0 & -2k_1 & k_1 \\ 0 & k_1 & 0 & 0 & 0 & -\frac{1}{2}k_1 & \frac{1}{2}k_1 \\ 0 & 0 & k_1 & 0 & 0 & -k_1 & 0 \\ 0 & 0 & 0 & k_1 & 0 & -\frac{1}{2}k_1 & -\frac{1}{2}k_1 \\ 0 & 0 & 0 & 0 & k_1 & 0 & -k_1 \\ -2k_1 & -\frac{1}{2}k_1 & -k_1 & -\frac{1}{2}k_1 & 0 & \frac{11}{2}(k_1 + k_2) & -2(k_1 + k_2) \\ k_1 & \frac{1}{2}k_1 & 0 & -\frac{1}{2}k_1 & -k_1 & -2(k_1 + k_2) & -\frac{5}{2}(k_1 + k_2) \end{bmatrix}. \quad (19)$$

The characteristic equation is:

$$\Delta(\omega^2) = \mathbf{K} - \omega^2 \mathbf{M} = 0. \quad (20)$$

From the literature [14] and field measurement, we have: $k_1 = 4.152 \times 10^9$ N/m, $k_2 = 2.36 \times 10^7$ N/m. The inertia of the foundation is:

$$I = \frac{m_6}{12} [(4a)^2 + h^2], \quad (21)$$

where h is the height of the foundation, and a is the length between two stands. From the design, it is possible to obtain $h = 9.6$ m, $a = 5.5$ m. Because the quality of each stand of the tandem mill approaches 444000 kg according to the field data, it is supposed $m_1 = m_2 = m_3 = m_4 = m_5 = 444000$ kg. The mass of the foundation is about 20 times that of the rolling mill according to the field data, thus, it is supposed as $m_6 = 8880000$ kg.

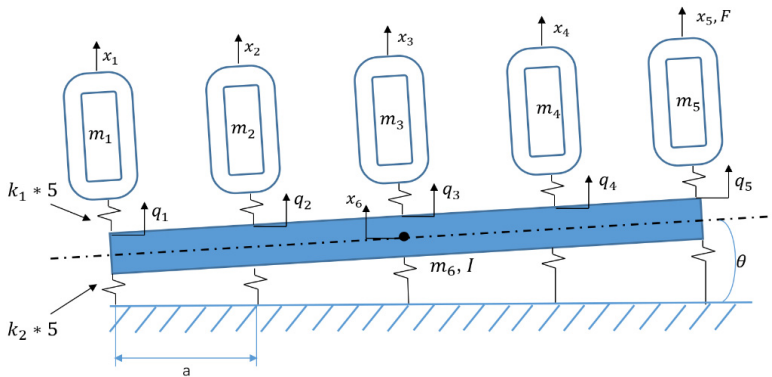


Fig. 4. Simplified diagram of tandem mill

3.2. Multiple-root resonance of tandem mill

When the mass of the foundation, m_6 , is considered as a variable, $m_6 = \beta m_1$ (β is the mass ratio) is defined and $k_1, k_2, m_1, m_2, m_3, m_4, m_5$ and m_6 are substituted to Eq. (20); when the stiffness is considered as a variable, $k_1 = \gamma k_2$ (γ is the stiffness ratio) is defined, and the above parameters are substituted to Eq. (20). The relationship among β, γ and the natural frequencies of the system are obtained by numerical calculation as shown in Fig. 5, where a_1 is the approaching degree between ω_5 and ω_6 ; a_2 is the approaching degree between ω_5 and ω_7 ; a_3 is the

approaching degree between ω_6 and ω_7 , and the equation is defined as follows: $a_i = (\omega_{max} - \omega_{min})/\omega_{min}$. Fig. 5 shows that when β is considered as a variable, the 6th- and 7th-order frequencies of the mill gradually approach and converge to the 5th-order frequency, and the approaching degree of the 6th and 7th-order frequencies, a_3 , is minimal; when γ is considered as a variable, the proximity factor has little variation, thus the mass ratio β is only considered.

The foundation mass of the tandem mill in one steel factory is 20 times of that of a single mill ($\beta = 20$). Then, the 6th- and 7th-order natural frequencies are 147 Hz and 150 Hz, and the proximity factor between them is 2.4 %. It is easy to form the multiple-root region and initiate the multiple-root resonance.

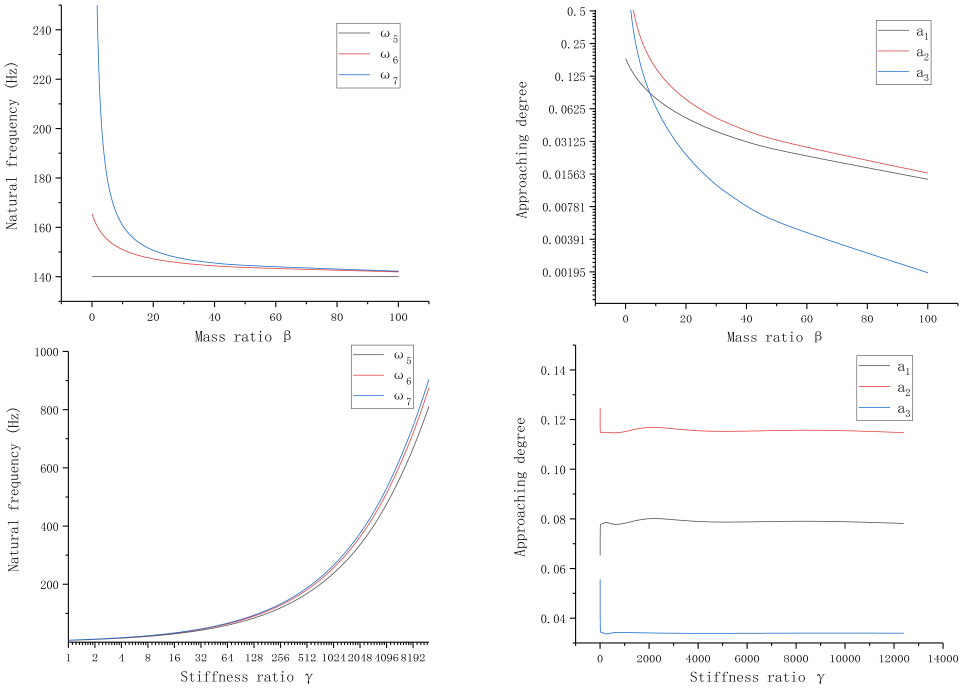


Fig. 5. Relationship among natural frequency, mass ratio and stiffness ratio of rolling mill

3.3. Modal analysis of multiple-root resonance

The modal analysis of the tandem mill is performed with ANSYS workbench. The results are shown in Fig. 6.

Fig. 6 shows that the tandem mill has two natural frequencies with similar value: 142.93 Hz and 144.33 Hz, and the corresponding vibration modes are different but have large vertical components: the maximum vibration amplitudes occur on F5 and F4, respectively. Therefore, if the tandem mill is excited with a frequency approach to 142 Hz and 144 Hz, two natural modes will be simultaneously excited and initiate the multiple-root resonance.

3.4. Field test of multiple-root resonance of tandem mill

The beat vibration is the superposition of two or more signals with similar frequencies and amplitudes, and these signals are characterized with strong-weak cycles, therefore, the spectrum of the beat vibration shows two or more dominant frequencies approach each other. Many studies show that, one of these dominant frequencies in the beat vibration is the excitation frequency. However, in the study of tandem mill vibration, it is found that the two dominant frequencies approach each other due to that they are both natural frequencies, and make a pair of

“sub-multiple-root”. To verify that the “sub-multiple-root” exists in the tandem mill, the mill vibration was tested before and after the foundation modification which reduced the mass of the whole foundation and turned the mass ratio of the foundation and the single mill from 20 to 17.5. Magnetolectric vibration velocity transducers and data collector were used in the field test, and the parameters of the sensor and collector are shown in Table 1 and Table 2, respectively.

Fig. 7 shows the test position in field.

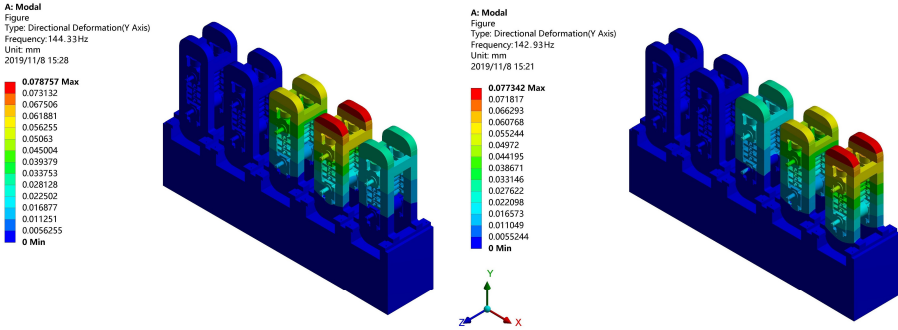


Fig. 6. Mode of continuous rolling mill

Table 1. Sensor parameters

Technical specifications	Value
Frequency response range	1 Hz-500 Hz
Sensitivity	28.2 mV/(mm/s)

Table 2. Data collector parameters

Technical specifications	Value
Bandwidth	1 Hz-10 kHz
Maximum analysis frequency	100 kHz
Rotation rate range	0.01-400.00 rpm

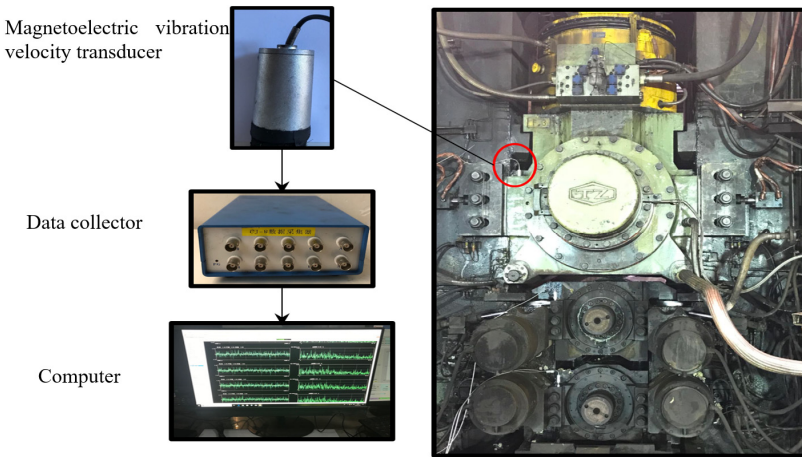


Fig. 7. Field test site and testing instruments

The vibrations of F4 and F5 before and after the reconstruction of foundation are shown in Fig. 8 and Fig. 9, respectively.

Fig. 8 shows that both F4 and F5 stands have an obvious double-beat vibration with the frequencies of 142 Hz and 144 Hz, which are almost identical to the two natural frequencies in the simulation in section 3.3. The excitation frequency is 138 Hz, which approaches two dominant frequencies of F5 stand. From Fig. 8, it can be seen that F5 stand has a larger vibration than F4.

Fig. 9 shows that after the foundation reconstruction, F4 and F5 stands have one dominant frequency of 132 Hz. The excitation frequency of 138 Hz remains in the spectrum of F5.

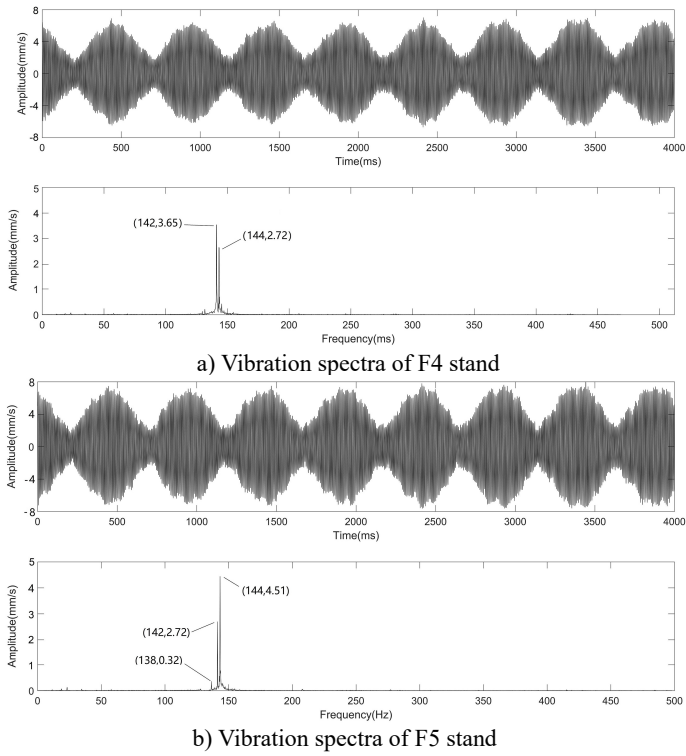


Fig. 8. Vibration spectrum before foundation reconstruction

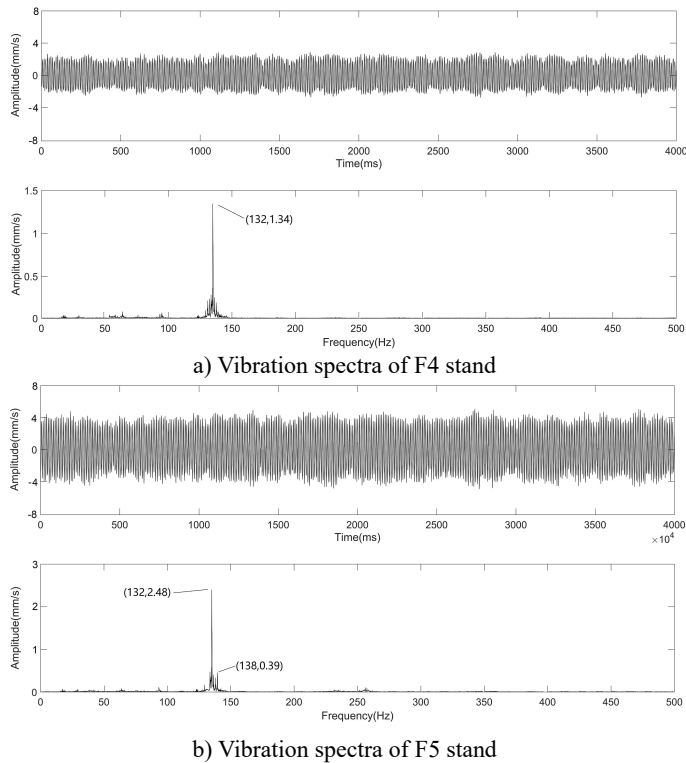


Fig. 9. Vibration spectrum after foundation reconstruction

Comparing the time domain waveforms of Fig. 8 and Fig. 9, it can be seen that the multiple-root resonance of the mill is much more violent than that of the mill with the resonance as one dominant frequency. After the foundation reconstruction, the mass ratio is changed, and the gap between the two natural frequencies, which are a pair of “sub-multiple-root” before the reconstruction, widens. Then, only one natural mode is induced by the excitation, thus, the vibration is reduced.

4. Conclusions

1) The theory of “multiple-root resonance” is introduced, and the “multiple-root region” is defined. Vibration mode of the “multiple-root resonance” is analyzed, and it is concluded that when the system has two natural frequencies with very similar value, and the corresponding modes are identical.

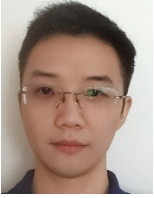
2) “Device-foundation” dynamic model is introduced to analyze the relationship among the mass ratio, stiffness ratio and the natural frequency, the results give the basis to conclude that the natural frequencies vary when the mass ratio or stiffness ratio varies, in some case the natural frequencies approach each other and result to “multiple-root”.

3) A theoretical analysis, finite element simulation and field test show that the multiple-root resonance existed in the tandem mill, which has never been noticed in the past. By comparing with the vibration spectra before and after the foundation modification, it was concluded that the multiple-root resonance of the mill is much more violent than the mill with the resonance as one dominant frequency. Thus, it was suggested that in the design stage of the tandem mill, it was necessary to consider the mass ratio of the foundation and single mill to avoid the multiple-root resonance, which was very instructive to the design of tandem mills.

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