Wave propagation in different theories of fractional thermoelasticity

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Abstract. In the present paper, the theories of fractional thermoelasticity with derivative and integral fractional orders are employed to study the homogeneous plane waves and the Rayleigh surface waves. The governing equations of homogeneous and isotropic generalized fractional thermoelasticity are solved for plane wave solutions and a dispersive velocity equation is obtained. There exists one transverse and two coupled longitudinal waves in a two-dimensional model of fractional thermoelastic medium where the speeds of coupled longitudinal waves are found to be dependent on the derivative and integral fractional orders. The Rayleigh waves is also studied along the traction-free surface of a half-space of a generalized fractional thermoelastic solid. The governing equations are solved for the general surface wave solutions which follow the decaying conditions in the half-space. A Rayleigh wave secular equation is obtained for thermally insulated surface. For a particular example of the present model, the numerical values of the speeds of coupled longitudinal waves and the Rayleigh wave are computed and graphically illustrated to visualize the effects of derivative and integral fractional orders and the circular frequency on the wave speeds.

1. Introduction

Biot [1] formulated the hyperbolic-parabolic field equations for coupled theory of thermoelasticity. The coupled theory of thermoelasticity was generalized by Lord and Shulman [2] and Green and Lindsay [3] by using the hyperbolic field equations. A detailed analysis and applications of these theories were presented by Ignaczak and Ostoja-Starzewski [4] and Hetnarski and Ignaczak [5].

In last few years, the fractional calculus was employed for improving various existing physical process models. The fractional order derivatives and integrals are found more useful and economical in solving certain physical problems as compared to the classical approach. Various materials like colloids, glassy, amorphous, porous, polymers, etc exist where fractional heat conduction models are appropriate instead of classical Fourier type heat conduction models as classical models break down in such materials. During the second half of 19th century, the fractional derivatives and integrals theories came into existence. Caputo and Mainardi [6, 7] and Caputo [8] made an important connection of fractional derivatives with the linear viscoelasticity theory and their outcomes were verified with the experimental results. Using the fractional heat conduction equation, Povstenko [9] formulated a quasi-static theory of uncoupled thermoelasticity. Making use of the heat conduction equation involving the Caputo time-fractional derivative, Povstenko [10] also analysed the thermal stresses in an infinite body containing a circular cylindrical hole. Sherif et al. [11] and Youssef [12] introduced alternative thermoelastic models using fractional calculus. Ezzat [13, 14] developed a novel fractional heat conduction equation by employing the Taylor series expansion of time-fractional order developed by Jumarie [15]. Using the fractional derivatives and integrals, El-Karamany and Ezzat [16] modified the Cattaneo heat equation [17] to formulate the two models of two-temperature fractional

thermoelasticity. Thereafter, various theories of fractional thermoelasticity have been developed with additional parameters.

The propagation of waves has wide uses in different scientific areas including mineral and oil exploration, geophysical exploration and seismology. Particularly, the surface waves are more useful in the fields of geophysics, earthquake and geodynamics. Lord Rayleigh [18] was first to explore the existence of surface waves known as Rayleigh waves propagating on the free surface of an isotropic solid half-space. In contrast to the body waves, the Rayleigh waves have slower energy attenuation and due to this reason these cause destruction to the structure. These surface waves can propagate along the thick solid surface with one wave length penetration and behave differently for surface defects. These waves are mainly applied for characterization of material and to investigate the structural and mechanical properties of the objects. In context of thermoelastic theories, the propagation of Rayleigh wave finds numerous uses in different engineering fields and future technologies. The surface waves in context of thermoelastic theories were analyzed by different investigators. Few prominent research works are highlighted. For example, the thermal effects on the Rayleigh wave speed were noticed by Lockett [19]. The Rayleigh wave in a prestressed thermoelastic medium was considered by Flavin [20]. The Rayleigh wave along both isothermal and insulated surfaces was examined by Chadwick and Windle [21]. Thereafter, various problems on the Rayleigh waves in thermoelasticity with additional parameters were studied [22-33]. The present work is motivated by the theoreis of fractional thermoelasticity given by Sherief et al. [11] and Youssef [12]. In this paper, the propagation of the homogeneous plane waves and the Rayleigh waves in an isotropic generalized fractional thermoelastic medium is explored. To observe the effects of circular frequency and fractional orders, the speeds of coupled longitudinal waves and Rayleigh surface wave are graphically illustrated.

2. Governing equations

Following Lord and Shulman [2], Sherief [11] and Youssef [12], the governing equations of linear, homogenous and isotropic fractional thermoelasticity without body forces and heat sources, are.

(a) Constitutive equations:

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{1}$$
$$\sigma_{ij} = 2\mu e_{ij} + (\lambda e_{kk} - \gamma T)\delta_{ij}. \tag{2}$$

(b) Equations of motion:

$$\mu u_{i,jj} + (\lambda + \mu)u_{j,ij} - \gamma T_{,i} = \rho \ddot{u}_i. \tag{3}$$

(c) Heat Equations.
(i) Sherief model:

$$KT_{,ii} = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^{\alpha+1}}{\partial t^{\alpha+1}}\right)(\rho c_E T + \gamma T_0 e_{ii}), \tag{4}$$

where α is a constant such that $0 < \alpha \le 1$. In the limiting case $\alpha \to 1$, the Eq. (4) reduces to the heat conduction equation obtained by Lord and Shulman [3] with single relaxation time. In the limiting case $\alpha \to 0$, the Eq. (4) reduces to the heat conduction equation for the coupled theory of thermoelasticity when $\tau_0 = 0$.
(ii) Youssef model:

$$KI^{\nu-1}T_{,ii} = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)(\rho c_E T + \gamma T_0 e_{ii}), \qquad 0 < \nu \le 2, \tag{5}$$

where $I^{(\nu)}$ is Riemann-Liouville fractional integral operator given by:

$$I^{(\nu)}g(t) = \frac{1}{\Gamma(\nu)}\int_0^t (t-\tau)^{\nu-1}g(\tau)d\tau, \ 0 < \nu \le 2, \qquad I^0 g(t) = g(t), $$

and $0 < \nu < 1$ for weak conductivity, $\nu = 1$ for normal conductivity and $1 < \nu \le 2$ for strong conductivity. σ_{ij} are the stress tensor components, e_{ij} are the strain tensor components, u_i are the displacement components, δ_{ij} is the Kronecker delta, λ, μ are Lame's constants, ρ is the density of the medium, T is change in temperature with reference temperature T_0, K is the thermal conductivity, c_E is the specific heat at constant strain, τ_0 is a non-negative parameter called as the relaxation time, $\gamma = (3\lambda + 2\mu)\alpha_0$ and α_0 is the thermal expansion coefficient. The overlayed dot symbolizes the time partial differentiation. The subscripts given after a comma symbolizes space partial differentiation.

A thermally conducting isotropic and homogeneous elastic material is considered at reference temperature T_0 in the unstrained state. A Cartesian system of axes is considered with the origin at plane surface $y = 0$ of the half-space $y \ge 0$. The positive y-axis is taken normal into the half-space. The surface $y = 0$ is assumed as stress free and without any heat transfer across the surface. The propagation direction of elastic waves is selected along the x-axis with equal displacement of particles on a line parallel to z-axis. Then, all the field quantities will not dependent on z-coordinates. Using the Helmholtz's decomposition given below:

$$u_1 = \phi_{,1} + \psi_{,2}, \ \ u_2 = \phi_{,2} - \psi_{,1}, \tag{6}$$

the governing Eqs. (3) and (4) are specialized in x-y plane as under:

$$\mu(\psi_{,11} + \psi_{,22}) = \rho\ddot{\psi}, \tag{7}$$
$$(\lambda + 2\mu)(\phi_{,11} + \phi_{,22}) - \gamma T = \rho\ddot{\phi}. \tag{8}$$

(i) Sherief model:

$$K(T_{,11} + T_{,22}) = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^{\alpha+1}}{\partial t^{\alpha+1}}\right)[\rho c_E T + \gamma T_0(\phi_{,11} + \phi_{,22})]. \tag{9}$$

(ii) Youssef model:

$$KI^{\nu-1}(T_{,11} + T_{,22}) = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)[\rho c_E T + \gamma T_0(\phi_{,11} + \phi_{,22})]. \tag{10}$$

3. Plane waves

The following type of plane wave solutions of Eqs. (7) to (10) are sought:

$$\{\psi, \phi, T\} = \{A, B, C\}e^{\iota k(x\sin\theta + y\cos\theta) - \iota\omega t}, \tag{11}$$

where θ is the propagation angle, k is the complex wavenumber and ω is the circular frequency.

Using Eq. (11) into Eqs. (7) to (10), the following equation in velocity is derived:

$$(V^2 - c_2^2)[V^4 - (K^* + \epsilon^* + c_1^2)V^2 + K^* c_1^2] = 0, \tag{12}$$

where:

$$V^2 = \frac{\omega^2}{k^2}, \quad \epsilon^* = \frac{\gamma^2 T_0}{\rho^2 c_E}, \quad c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad c_2^2 = \frac{\mu}{\rho}, \tag{13}$$

and (a) for Sherief model:

$$K^* = \frac{K}{\rho c_E \tau_1}, \quad \tau_1 = \tau_0(-i\omega)^{\alpha-1} + \frac{i}{\omega}. \tag{14}$$

(a) for Youssef model:

$$K^* = \frac{K}{\rho c_E \tau_2} \left(\frac{i}{\omega}\right)^{\nu-1}, \quad \tau_2 = \tau_0 + \frac{i}{\omega}. \tag{15}$$

The Eq. (12) has following three roots:

$$V_1^2 = \frac{1}{2}[(K^* + \epsilon^* + c_1^2) + \sqrt{(K^* + \epsilon^* + c_1^2)^2 - 4K^*c_1^2}], \tag{16}$$

$$V_2^2 = \frac{1}{2}[(K^* + \epsilon^* + c_1^2) - \sqrt{(K^* + \epsilon^* + c_1^2)^2 - 4K^*c_1^2}], \tag{17}$$

$$V_3^2 = c_2^2, \tag{18}$$

which shows that there exists two coupled longitudinal waves with distinct speeds $v_1 = Re(V_1)$ and $v_2 = Re(V_2)$ and one transverse wave with speed $v_3 = V_3$ in an isotropic and homogeneous fractional thermoelastic medium [34]. Here, the phase speeds v_1 and v_2 of the coupled longitudinal waves are dependent on the derivative fractional order in Sherief model or integral fractional order in Youssef model, respectively.

4. Rayleigh surface wave

The Rayleigh wave propagation is considered in the x-direction and decaying in the y-direction with wave number $k(>0)$ and velocity $v(>0)$. The appropriate displacement and temperature potential functions for propagation of Rayleigh waves along the surface $y = 0$, are selected as:

$$\phi(x, y, t) = B_1 e^{-\eta k y} e^{ik(x-vt)},$$
$$T(x, y, t) = B_2 e^{-\eta k y} e^{ik(x-vt)}, \tag{19}$$
$$\psi(x, y, t) = B_3 e^{-\eta k y} e^{ik(x-vt)},$$

where η is a complex coefficient with $Im(\eta) > 0$ to satisfy the decay conditions in the half-space $y > 0$. The unknowns B_1, B_2 and B_3 are the polarization vectors.

Making use of Eq. (19) in Eqs. (7) to (10), the appropriate solutions in half-space $y > 0$ for surface wave are obtained as:

$$\phi(x, y, t) = \sum_{j=1}^{2} B_{1j} e^{-\eta_j k y} e^{ik(x-vt)},$$

$$T(x, y, t) = \sum_{j=1}^{2} r_j B_{1j} e^{-\eta_j k y} e^{ik(x-vt)}, \tag{20}$$

$$\psi(x, z, t) = B_3 e^{-\eta_3 k y} e^{ik(x-vt)},$$

where the unknowns B_{1j} and B_3 are amplitudes of displacement potentials and temperature and (a) for Sherief case:

$$\eta_1^2 + \eta_2^2 = \left(1 - \frac{\rho v^2}{\lambda + 2\mu}\right) + \left(1 - \frac{1}{K_1^*}\right) - \frac{\epsilon^*}{K_1^*},$$
$$\eta_1^2 \eta_2^2 = \left(1 - \frac{\rho v^2}{\lambda + 2\mu}\right)\left(1 - \frac{1}{K_1^*}\right) - \frac{\epsilon^*}{K_1^*}, \tag{21}$$
$$\eta_3^2 = \left(1 - \frac{\rho v^2}{\mu}\right),$$
$$r_j = \frac{k^2(\lambda + 2\mu)(a_1 + \eta_j^2)}{\gamma} = \frac{k^2 \gamma T_0 (1 - \eta_j^2)}{\rho c_E (a_2 + K_1^* \eta_j^2)}, \qquad (j = 1,2), \tag{22}$$

and (b) for Youssef case:

$$\eta_1^2 + \eta_2^2 = \left(1 - \frac{\rho v^2}{\lambda + 2\mu}\right) + \left(1 - \frac{v^2}{K_2^*}\right) - \frac{\epsilon^* v^2}{K_2^*},$$
$$\eta_1^2 \eta_2^2 = \left(1 - \frac{\rho v^2}{\lambda + 2\mu}\right)\left(1 - \frac{v^2}{K_2^*}\right) - \frac{\epsilon^* v^2}{K_2^*}, \tag{23}$$
$$\eta_3^2 = \left(1 - \frac{\rho v^2}{\mu}\right),$$
$$r_j = \frac{k^2(\lambda + 2\mu)(a_1 + \eta_j^2)}{\gamma} = \frac{\gamma T_0 (1 - \eta_j^2) \omega^2}{\rho c_E (a_3 + K_2^* \eta_j^2)}, \qquad (j = 1,2), \tag{24}$$

where:

$$a_1 = \frac{\rho v^2}{\lambda + 2\mu} - 1, \qquad a_2 = 1 - K_1^*, \qquad a_3 = v^2 - K_2^*, \tag{25}$$
$$K_1^* = \frac{K\omega^2}{\rho c_E \tau_3 v^2}, \qquad K_2^* = \frac{K}{\rho c_E \tau_2}\left(\frac{i}{\omega}\right)^{\nu - 1}, \qquad \tau_3 = \iota\omega[1 + \tau_0(-\iota\omega)^\alpha]. \tag{26}$$

The required boundary conditions at thermally insulated stress-free surface $y = 0$ are vanishing of the tangential stress component, normal stress component and the normal heat flux component at $y = 0$, i.e.:

$$\sigma_{12} = 0, \quad \sigma_{22} = 0, \quad \frac{\partial T}{\partial y} = 0, \tag{27}$$

where:

$$\sigma_{12} = \mu(\psi_{,11} - \psi_{,22} + 2\phi_{,12}),$$
$$\sigma_{22} = \lambda(\phi_{,11} + \phi_{,22}) + 2\mu(\phi_{,22} + \psi_{,12}) - \gamma T. \tag{28}$$

The appropriate potential functions given by Eq. (20) satisfy the conditions Eq. (27) and a homogeneous system of three equations in B_{11}, B_{12} and B_3 are obtained after using first equations of Eq. (22) and Eq. (24). The non-trivial solution of the homogeneous system require the vanishing of the determinant of the coefficients matrix, i.e.:

$$\left(\frac{\rho v^2}{\mu} - 2\right)^2 \left[1 - \frac{\rho v^2}{\lambda + 2\mu} - \eta_1^2 - \eta_2^2 - \eta_1\eta_2\right] + 4\eta_1\eta_2\eta_3(\eta_1 + \eta_2) = 0. \tag{29}$$

The Eq. (29) is dispersion equation of Rayleigh waves along the stress-free thermally insulated surface of an isotropic and homogeneous fractional thermoelastic solid half-space. In absence of thermal effects, the Eq. (29) reduces to:

$$\left(\frac{\rho v^2}{\mu} - 2\right)(1 + \eta_3^2) + 4\eta_1^*\eta_3 = 0, \tag{30}$$

where:

$$\eta_1^* = \sqrt{1 - \frac{\rho v^2}{\lambda + 2\mu}}. \tag{31}$$

The Eq. (30) is the secular equation of the Rayleigh wave along the free surface of an isotropic elastic solid half-space as obtained by Lord Rayleigh [18].

5. Numerical results and discussion

For numerical illustrations of the speeds of homogeneous plane waves and the Rayleigh surface wave, the following relevant parameters of copper material at $T_0 = 300$ K are taken $\rho = 2.7 \times 10^3$ Kg.m^{-3}, $\lambda = 5.775 \times 10^{10}$ N.m^{-2}, $\mu = 2.646 \times 10^{10}$ N.m^{-2}, $K = 0.00492 \times 10^2$ W.m^{-1}.deg^{-1}, $c_E = 2.36 \times 10^2$ J.Kg^{-1}.deg^{-1}, $\tau_0 = 0.005$ s.

The velocity Eq. (12) is solved numerically to illustrate the effects of derivative fractional order α, integral fractional order ν and circular frequency ω on the speeds v_1 and v_2 of the coupled longitudinal waves. As shown in Fig. 1(a) and 1(b), a significant effect of fractional order parameter α on wave speeds of coupled longitudinal waves is observed in higher frequency range.
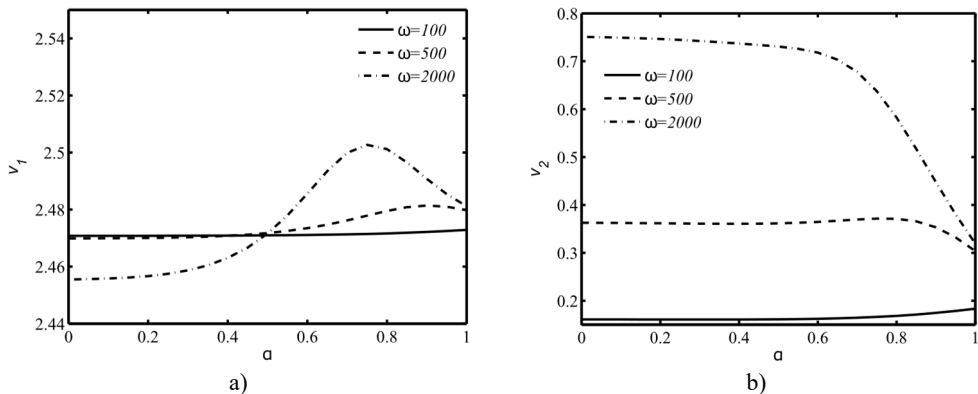

Fig. 1. The speed variations of coupled longitudinal waves against
the derivative fractional order α for distinct values of ω

The speed variations in Figs. 2(a) and 2(b) show that the speeds v_1 or v_2 of the coupled longitudinal waves increases or decreases sharply in weak conductivity region ($0 < \nu < 1$) and the effect of circular frequency ω on wave speeds is more prominent in weak conductivity region as compared to the strong conductivity region ($1 < \nu < 2$).

With the use of functional iteration method, the dispersion Eq. (29) is numerically solved for the dimensionless speed $v_R = Re\{\sqrt{\rho v^2/\mu}\}$ of the Rayleigh wave. The impacts of derivative

fractional order α, integral fractional order ν and the circular frequency on the wave speed v_R of the Rayleigh wave are illustrated in Figs. 3 and 4. It is observed that the effects of fractional orders α and ν on the speed are found more prominent in higher frequency range.
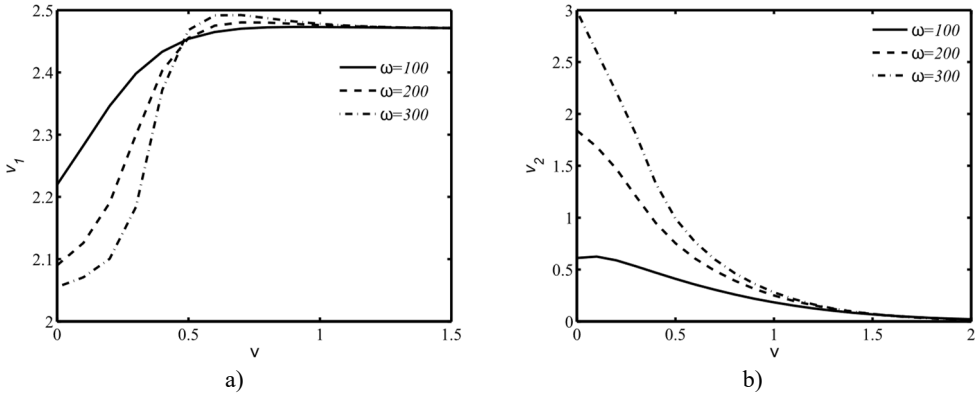

a)

b)

Fig. 2. The speed variations of coupled longitudinal waves against
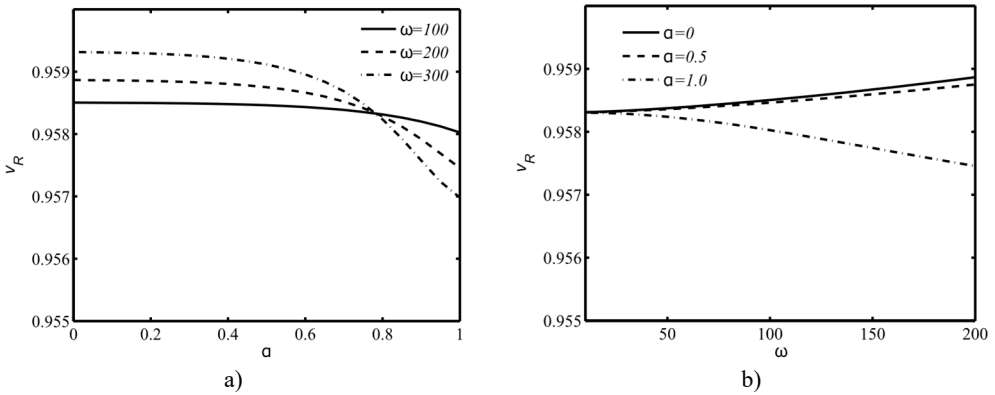the integral fractional order ν for distinct values of ω

a)

b)

Fig. 3. The influences of derivative fractional parameter α and
the circular frequency ω on the normalized Rayleigh wave speed v_R
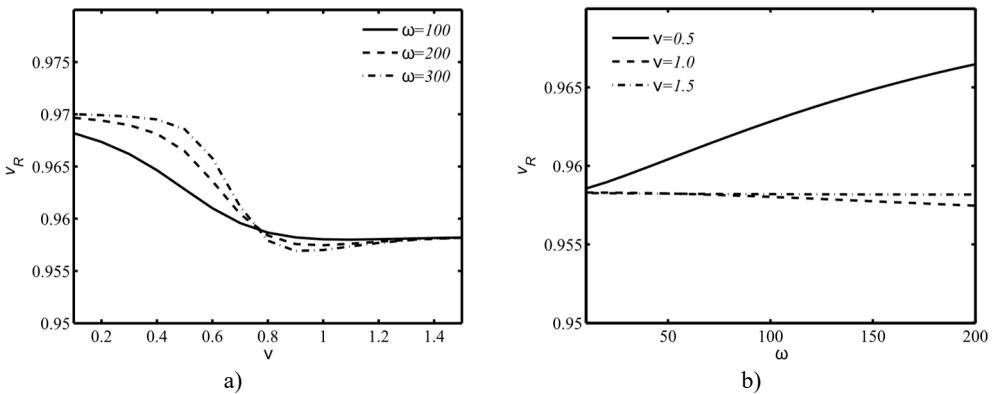

a)

b)

Fig. 4. The influences of integral fractional parameter ν and
the circular frequency ω on the normalized Rayleigh wave speed v_R

The speed v_R of the Rayleigh wave as shown in Fig. 5 increases linearly with the non-

dimensional elastic parameter $\delta = \mu/(\lambda + 2\mu)$ and the effects of fractional orders α and ν are found more prominent at larger values of δ. The thermal effect on the wave speed v_R against δ is also illustrated in Fig. 6 where in absence of thermal disturbances, the speed enhances at each δ.
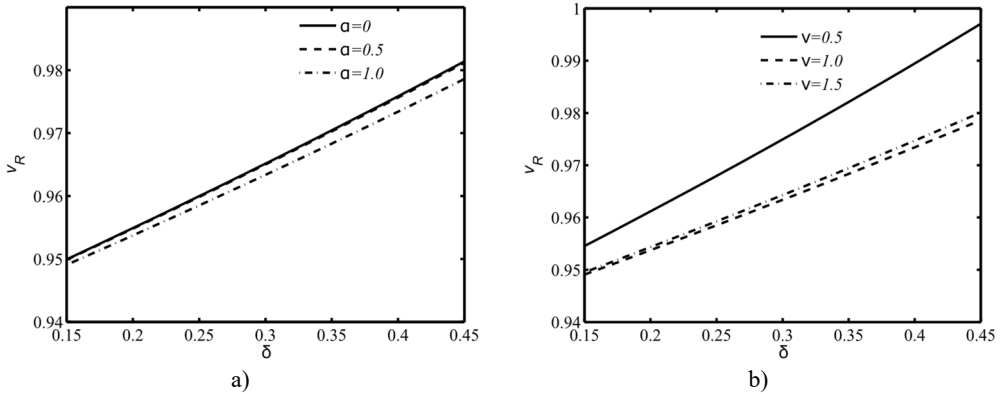
Fig. 5. The Rayleigh wave speed against the non-dimensional material parameter δ for different values of derivative fractional parameter α and integral fractional parameter ν
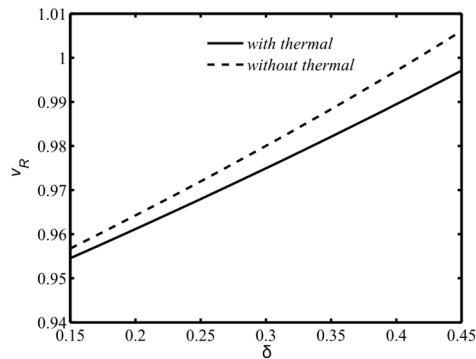
Fig. 6. Thermal effects on the Rayleigh wave speed against the non-dimensional material parameter δ

6. Conclusions

Sherief and Youssef models of fractional thermoelasticity are applied to study the homogeneous plane waves and the Rayleigh surface wave in a generalized fractional thermoelastic medium. It is found that there exists two coupled longitudinal waves and one transverse wave in a plane of a fractional thermoelastic medium. In both Sherief and Youssef models, the coupled longitudinal waves are found affected by thermal fields and fractional order parameters. A dispersion equation of Rayleigh wave along a stress-free thermally insulated surface of a halfspace of a fractional thermoelastic material. The numerical computations and illustrations of wave speeds of plane and Rayleigh wave based on a particular material show that the thermal field, derivative fractional order and integral fractional order changes significantly the speeds of both plane and surface waves. The effects of fractional orders are observed more significant in high frequency range. The present theoretical predictions on plane and surface waves in both Sherief and Youssef models of fractional thermoelasticity may be applied in possible experimental studies on the heat conduction in materials like colloids, glassy, amorphous, porous, polymers, etc.

Acknowledgements

Data availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Conflict of interest

The authors declare that they have no conflict of interest.

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