A mathematical model and microcontroller-based method for measuring dielectric permittivity and discharge characteristics with Arduino ATmega 328: a case study in a physics laboratory

Valentinus Galih Vidia Putra¹, Ngadiyono²

¹Basic and Applied Science Research Group in Theoretical and Plasma Physics, Politeknik STTT Bandung, Bandung, Indonesia
²Department of Textile Engineering, Politeknik STTT Bandung, Bandung, Indonesia

¹Corresponding author
E-mail: ¹galih_vidia@yahoo.com, ²dion@kemenperin.go.id

Received 10 October 2023; accepted 31 October 2023; published online 23 November 2023
DOI https://doi.org/10.21595/mme.2023.23695
Copyright © 2023 Valentinus Galih Vidia Putra, et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract. A microcontroller-based measuring instrument and a new mathematical model are used to investigate capacitor permittivity and dielectric materials' charge-discharge characteristics. In this study, a prototype capacitive permittivity measurement apparatus for dielectric materials was carefully developed using an Arduino microcontroller, a resistor, and a capacitor. The experimental setup comprises a capacitor-resistor circuit, wherein a 5-volt power supply sourced from the microcontroller interfaces with a computer. During the charging process, a comprehensive evaluation of model-data alignment was performed, yielding values of 0.54252156, 0.9951, and 111.2508701 for the sum of squares error (SSE), the coefficient of determination (R-squared), and the sum of squares total (SST), respectively. Similarly, the analysis extended to the discharging process, unveiling values of 5.10174756, 0.962805684, and 137.1647082 for SSE, R-squared, and SST, respectively. These findings confirm the accuracy of the microcontroller that was programmed by incorporating a model in precisely measuring the relative permittivity of dielectric materials and capacitance values, with an R-squared value above 0.95 following capacitor literature benchmarks. The novelty of this study is that this configuration enabled the precise assessment of both permittivity and the charge-discharge characteristics of the dielectric materials within the capacitor. This methodology made it possible to accurately measure the permittivity and charge-discharge characteristics of dielectric materials within a capacitor. The scientific significance of this research lies in its ability to provide a carefully developed instrument capable of investigating the permittivity of dielectric materials and capacitor capacitance measurements. Scholars, international engineering communities, and academics can use this technological breakthrough to advance research into dielectric material properties and capacitor characteristics.

Keywords: resistor, capacitor, permittivity, mathematical model.

1. Introduction

Measuring the permittivity and dielectric properties of materials and the charge-discharge process in capacitors is a crucial aspect of electrical engineering and material science [1]. Several scholars have applied physics principles to advance various aspects of electronics [2, 3]. According to some researchers, understanding materials’ permittivity and dielectric properties is crucial for designing electronic devices [4]-[10]. Capacitors store electrical energy, and their performance depends on the dielectric material between the plates. Engineers can choose the most suitable dielectric material by accurately measuring permittivity and optimize the capacitor's efficiency and performance [11]-[15]. In industrial applications, it is essential to ensure the consistency and quality of dielectric materials used in capacitors. Accurate measurement tools and
models help quality control by verifying that the dielectric properties meet the required specifications. This ensures capacitors operate reliably and meet safety standards [16]-[18]. Permittivity and dielectric measurements also play a crucial role in material science. Researchers use these measurements to understand the electrical properties of various materials, not just for capacitors but also for applications in sensors, insulators, and semiconductors [17, 18]. For instance, Allagui, Elwakil, Fouda, and Radwan [1] have extensively explored the integration of physics in supercapacitors as reservoirs of stored energy. Arshad, Khan, Alam, Tasnim, Gunawan, Ahmad, and Nataraj [2] developed a system for monitoring senior citizen activity levels using capacitive sensing technology. Mita and Boufaida [3] have devised innovative capacitor circuits to optimize energy consumption. Dean, Bell, and Baty pioneered the innovative use of capacitance enhancement techniques [4]. Yang and Chenming [5] focused on quantifying capacitance in thin dielectrics characterized by elevated leakage. A novel microcontroller-based approach to capacitance measurement has been developed by Putra, Wijayono, Purnomosari, Ngadiono, and Irwan [6]. Researchers have extensively documented a wide range of research encompassing the application of microcontrollers for measuring physical attributes, specifically capacitance [7]-[18]. Permittivity, a fundamental parameter in electromagnetism that describes electric polarizability, has far-reaching implications. Higher permittivity materials exhibit a higher capacity for polarization in response to an applied electric field, denoted as E, resulting in the accumulation of increased energy within the electric field. The representation of permittivity typically includes the relative permittivity, absolute permittivity, and ratio relative to the vacuum permittivity, all of which serve as crucial descriptors in the permittivity characterization [19]. Measurement of permittivity and dielectric properties of materials, such as for the fabrication of electromagnetic shielding materials, is critical in electrical engineering and material science. Recognizing the critical role of permittivity in determining the capacitance of a capacitor within the context of electrostatics, Putra, Ngadiono, Wijayono, and Purnomosari [6], as well as Westerlund & Ekstam [7] demonstrated the indispensable importance of permittivity. Their research clearly showed the utility of physics in quantifying the capacitance of various materials using microcontrollers. It is worth noting that while their investigations accurately characterized capacitance during the charging and discharging processes, a thorough analysis of permittivity calculation for the capacitor materials was left out of their discussion. In this regard, an equation with more detailed analysis and validation of the model is required to study the characteristics of a dielectric material and the charge-discharge processes. This study aims to develop a microcontroller-based measuring instrument and a new mathematical model for studying capacitor permittivity and the charge-discharge characteristics of dielectric materials. The study has significant scholarly implications, providing a novel tool for studying dielectric material permittivity and capacitor capacitance. This innovative tool, designed to aid researchers and academics, enables sophisticated research within these comprehensive areas.

2. Research methods

2.1. Materials

A capacitor (330 μF), a resistor (51 kΩ), a DC source voltage (5 volts), several cables, and a project board comprised the experimental apparatus. These parts were purchased in Bandung, Indonesia's traditional market. The dielectric material under consideration was an ideal cylindrical capacitor with an unspecified dielectric type and permittivity value. We determined the permittivity of the dielectric material for the ideal capacitor using the charge-discharge resistor-capacitor (RC) method and a 5 V Arduino source voltage.

2.2. Instrumentations

The power supply connected the microcontroller, the capacitor-resistor circuit, and the
computer. A capacitor-resistor circuit interfaced with a microcontroller ATmega 328 was used in this study performed at Politeknik STTT Bandung, Indonesia. This study investigated the permittivity and capacitance of dielectric materials.

2.3. Methods

The experiment scheme is depicted in Fig. 1 and Fig. 2. In Fig. 3, it shows the capacitance measurement using the Arduino ATmega 328 microcontroller.

![Charging scheme of the cylinder capacitor](image1)

**Fig. 1.** Charging scheme of the cylinder capacitor

![Discharging scheme of the capacitor](image2)

**Fig. 2.** Discharging scheme of the capacitor

The following are the procedures for measuring capacitance and permittivity: On a computer, an Arduino program was created and transferred to the Arduino ATmega 328 microcontroller on the project board, with resistors and capacitors configured as shown in Fig. 1 and Fig. 2. The computer was connected to the microcontroller, and the program was uploaded to it. After that, the capacitor was gradually charged up to a maximum voltage of 5 volts or until it stabilized. Following that, discharging was done from 5 volts to 0 volts or until steadiness was achieved. Microsoft Office Excel was used to compare experimental and theoretical results.

![Measuring capacitance using a microcontroller](image3)

**Fig. 3.** Measuring capacitance using a microcontroller

3. Mathematical model

A new mathematical equation was constructed to quantify the capacitance and permittivity of dielectric materials. Notably, this model was constrained because the dielectric substance was
A MATHEMATICAL MODEL AND MICROCONTROLLER-BASED METHOD FOR MEASURING DIELECTRIC PERMITTIVITY AND DISCHARGE CHARACTERISTICS WITH ARDUINO ATMEGA 328: A CASE STUDY IN A PHYSICS LABORATORY. VALENTINUS GALIH VIDIA PUTRA, NGADIYONO

located between two conducting plates that served as positive and negative electrodes. The plates, which had a cylindrical shape, kept a specific distance from the dielectric material. The dielectric material then aligned with the two electrodes. In this experimental setup, an RC circuit was constructed, comprising a 330 μF capacitance in series with a 51,000 ohms resistor powered by a 5-volt input voltage source. A microcontroller device facilitated the charging and discharging of capacitors for obtaining permittivity and capacitance measurements. The capacitor voltage was monitored experimentally every two seconds, commencing from the initial voltage of 0 and increasing to 5 volts during the charging phase. This voltage data was then compared with theoretical predictions to determine capacitance and relative permittivity. The same procedure was repeated during the capacitor discharge phase. The measured voltage during the closed-circuit charging operation of the capacitor can be expressed as Eq. (1):

\[ V = \frac{Q}{C} = V_s \left(1 - e^{-\frac{t}{RC}}\right), \]  

(1)

where \( V \) represents the capacitor voltage, \( Q \) is the capacitor charge, \( C \) represents capacitance, \( V_s \) is the microcontroller source voltage, \( t \) is time, and \( R \) is resistance. Eq. (2) can be used to calculate the voltage for the capacitor discharge process:

\[ V = V_s e^{t/RC}. \]  

(2)

To determine the voltage theoretically, we can use Eq. (3):

\[ \beta = \frac{1}{RC}, \]  

(3)

where \( \beta \) is a constant value determined by the capacitor and resistor values using the microcontroller and resistor-capacitor circuit. We control five-volt DC for the supply voltage and use Eq. (3) to calculate the capacitor voltage, \( V \), as shown in Eq. (4):

\[ V_c = 5 \left(1 - e^{-t\beta}\right). \]  

(4)

It can be used \( R^2 \) to know the accuracy value of our prediction from the curve using Eq. (5) as shown below:

\[ R^2 = 1 - \frac{\text{SSE}}{\text{SST}}. \]  

(5)

SSE represents the sum of squares error, \( R^2 \) is the coefficient of determination, and the sum of squares total for the SST. The capacitance, \( C \), can be measured using Eq. (6):

\[ C = \frac{1}{R\beta}. \]  

(6)

In the cylinder capacitor (Fig. 1), it has been found that for the diameter of the inner cylinder of \( r = a \) and the diameter of the outer cylinder of \( b \) with height \( L \), volume charge density, \( \rho \), permittivity, \( \varepsilon \), and the electric field, \( E \), in the area \( r < a \) can be written in Eqs. (7) to (8):

\[ \int E \cdot \hat{n} \, ds = \int \frac{\rho}{\varepsilon} \, dV, \quad \int \int E |\hat{r} \cdot |r \, d\varphi \, dz|\hat{r} = \int \int \frac{\rho}{\varepsilon} r \, dr \, d\varphi \, dz, \]  

(7)

\[ \varepsilon \varepsilon_0 2\pi r L = \rho \pi r^2 L. \]  

(8)
Hence, we get Eq. (9):

\[ E = \frac{\rho r^2 L}{\varepsilon 2 \pi r L} = \frac{\rho r^2}{\varepsilon 2 r} = \frac{\rho r}{2 \varepsilon} \]  

(9)

The electric field on \( a < r < b \) can be written as Eq. (10) to (12):

\[ \int E \cdot d\mathbf{s} = \int \frac{\rho}{\varepsilon} dV, \quad \int \int |E| r \cdot |r| d\varphi dz = \int \int \int \frac{\rho}{\varepsilon} r dr d\varphi dz. \]  

(10)

\[ E \varepsilon 2 \pi r L = \rho \pi a^2 L. \]  

(11)

\[ E = \frac{\rho a^2 L}{2 \pi r} = \frac{Q a^2}{\pi a^2 L 2 \pi r} = \frac{Q}{2 \pi r L}. \]  

(12)

The electric field, \( E \), on \( r > b \) is shown in Eq. (13):

\[ E = \frac{\rho a^2}{2 \pi r} - \frac{\rho a^2}{2 \pi r} = 0. \]  

(13)

The voltage and capacitance can be written as Eq. (16) and Eq. (18):

\[ \Delta \varphi = \int_1^2 E \cdot d\mathbf{r} = \frac{\int_0^{r=a} \rho r dr d\varphi dz}{C}, \]  

(14)

\[ \Delta \varphi = \int_a^b E \cdot d\mathbf{r} = \frac{Q}{C}, \]  

(15)

\[ \Delta \varphi = \int_a^b \frac{Q}{2 \pi r L} dr = \frac{Q}{2 \pi \varepsilon L} \ln \left( \frac{b}{a} \right), \]  

(16)

\[ C = \frac{\Delta \varphi}{\Delta \varphi} = \frac{\ln \left( \frac{b}{a} \right)}{2 \pi \varepsilon L}, \quad \varepsilon = C \frac{\ln \left( \frac{b}{a} \right)}{2 \pi b L} = \frac{1}{R \beta} \frac{b \ln \left( \frac{b}{a} \right)}{2 \pi b L}, \]  

(17)

\[ \varepsilon = \frac{Cb \ln \left( \frac{b}{a} \right)}{2 \pi b L}, \quad \varepsilon = C \frac{b \ln \left( \frac{b}{a} \right)}{2 \pi b L} = \frac{1}{R \beta} \frac{b \ln \left( \frac{b}{a} \right)}{2 \pi b L}, \]  

(18)

with \( d^* = b \ln \left( \frac{b}{a} \right) \), \( b = 0.7 \) cm, \( a = 0.5 \) cm, \( L = 1.28 \) cm, and \( A = 2 \pi b L \) is the area of the cylinder capacitor. The capacitance and dielectric permittivity values can be calculated using Eqs. (6) and (18).

4. Results and discussions

In this study, we have chosen a constant value determined by the capacitor and resistor values \( \beta \) of 0.05 both for the charging and discharging process. After comparing the experimental results with the model using Eqs. (1), (4), (5), and (6), we demonstrated the relationship between experimental data and the model during the charging process of a 330 \( \mu \)F capacitor and a 51,000 ohms resistor. Fig. 4 depicts the fit level between the model and data during the charging process, with values for the sum of squares error (SSE), coefficient of determination (R-squared), and sum of squares total (SST) of 0.54252156, 0.9951, and 111.2508701, respectively. Using the model, we discovered that the charging process’s \( R^2 \) was 0.99512344. The curve's result is depicted in Fig. 4.
Based on Fig. 4, the capacitance and dielectric permittivity of the material can be calculated. Eq. (6) can be used to calculate capacitance, \( C \), and Eq. (18) to calculate the permittivity of dielectric material, resulting in Eqs. (19) to (24):

\[
\beta = \frac{1}{RC} = 0.05, \quad (19)
\]

\[
C = \frac{1}{R\beta} = 0.000342157, \quad (20)
\]

\[
C = 342.15 \mu F, \quad (21)
\]

\[
\varepsilon = \frac{\left(b \ln \left(\frac{b}{a}\right)\right)}{\left(2\pi bL\right)}, \quad (22)
\]

\[
\varepsilon = \frac{\left(0.000342157 \times 0.7 \times 10^{-2} \times \ln \left(\frac{0.7}{0.5}\right)\right)}{\left(2 \times 3.14 \times 0.7 \times 10^{-2} \times 1.28 \times 10^{-2}\right)}, \quad (23)
\]

\[
\varepsilon = 0.0016414946 = 1.64 \times 10^{-3} \text{ (F/m)} = 0.16 \times 10^{-2} \text{ (F/m)}, \quad (24)
\]

The dielectric permittivity value of the material is obtained by using Eq. (22) and then substituting the following values into the equation: \( b = 0.7 \text{ cm}, \ a = 0.5 \text{ cm}, \ L = 1.28 \text{ cm}, \) and \( A = 2\pi bL \) is the area of the cylinder capacitor, and also \( C = 342.157 \mu F \). According to the model result in Eq. (21) and (24), the capacitance is 342.157 \mu F, while the actual capacitance result is 330 \mu F. According to the model, the material's dielectric permittivity is 0.16×10^{-2} (F/m), which is generally in the range of dielectric values for the capacitor types used. We also showed the experimental data for the discharging process of capacitor 330 \mu F based on the experimental results. Furthermore, we measured the level of fit between the model and data in the testing process during the discharging process, and the values for the sum of squares error (SSE), the coefficient of determination (R-squared), and the sum of squares total (SST) were 5.10174756, 0.962805684, and 137.1647082, respectively. From Fig. 5, it can be shown that the \( R^2 \) of the discharging process of the capacitor is 0.9628.

Based on experimental results and models, the capacitance and permittivity values can be calculated using Eqs. (6) and (18). The capacitance measured in this experiment was 342.157 \mu F (the actual capacitance was 330 \mu F), and the capacitance permittivity was 0.0016414946 (F/m). \( R^2 > 0.95 \) was obtained for the charging and discharging process results, indicating that the correlation between predicted and experimental results is good. The capacitance values during this experiment's charging and discharging processes show the concordance between theory and experiment, as shown in Fig. 4 and Fig. 5. In this study, the dielectric materials within the capacitor were precisely assessed for permittivity as well as charge-discharge characteristics. This methodology made it possible to accurately measure the permittivity and charge-discharge characteristics of dielectric materials within a capacitor. In addition to providing a carefully
developed instrument for measuring capacitor capacitance, this research is scientifically significant for its ability to investigate the permittivity of dielectric materials. The findings of this technological breakthrough can be used to advance research into dielectric material properties and capacitor characteristics by scholars, international engineering communities, and academics.

5. Conclusions

This study developed a method for accurately determining a capacitor's relative permittivity and capacitance value. A capacitor permittivity measurement prototype was created in this study using an Arduino microcontroller device. The capacitor permittivity was measured using a series of capacitor resistors with a 5-volt source from the microcontroller's voltage. A capacitor-resistor circuit with a 5-volt voltage source from a microcontroller connected to a computer device was used to measure the permittivity of the capacitor. It was found that the method used in this study was quite successful in measuring the capacitor's permittivity. It was concluded that microcontroller devices could accurately determine the relative permittivity of dielectric materials and capacitance values with the $R^2$ above 0.95 to the literature value on the capacitor. We measured the level of fit between the model and data in the experimental process during the charging process, with values of 0.54252156, 0.9951, and 111.2508701 for the sum of squares error (SSE), the coefficient of determination (R-squared), and the sum of squares total (SST), respectively. Furthermore, we also measured the level of fit between the model and data in the experimental process during the discharging process, and the values for the sum of squares error (SSE), the coefficient of determination (R-squared), and the sum of squares total (SST) were 5.10174756, 0.962805684, and 137.1647082, respectively. Based on our findings, we discovered that the capacitance and permittivity values could be calculated using Eqs. (6) and (18). The result of the capacitance in this experiment was 342.157 $\mu$F, while the actual capacitance was 330 $\mu$F, with the permittivity of dielectric materials of capacitance, $\varepsilon$, measured at 0.0016414946 (F/m).

Acknowledgements

The study was funded by Politeknik STTT Bandung's Research and Community Service Division (UPPM) and the Republic of Indonesia's Ministry of Industry. The authors sincerely thank everyone who helped and contributed to this study. Their encouragement and motivation have influenced our decision to pursue this project.

Data availability

The datasets generated during and/or analyzed during the current study are available from the
corresponding author on reasonable request.

Author contributions

Valentinus Galih Vida Putra conducted the experiment and the calculations. Valentinus Galih Vida Putra and Ngadiyono wrote and revised the manuscript. All authors agreed to the final version of this manuscript.

Conflict of interest

The authors declare that they have no conflict of interest.

References

A MATHEMATICAL MODEL AND MICROCONTROLLER-BASED METHOD FOR MEASURING DIELECTRIC PERMITTIVITY AND DISCHARGE CHARACTERISTICS WITH ARDUINO ATMEGA 328: A CASE STUDY IN A PHYSICS LABORATORY. VALENTINUS GALIH VIDIA PUTRA, NGADIYONO


Assoc. Prof. Dr. Valentinus Galih Vida Putra, S.Si., M.Sc., is an associate professor of physics at Politeknik STTT Bandung, the Ministry of Industry of the Republic of Indonesia. He was born in Klaten on March 4, 1987. He received his Bachelor’s degree from the Department of Physics, Universitas Gadjah Mada, in 2010. In 2012, he received a Master of Science in Applied Physics, and in 2017, a Doctor of Science in Theoretical Physics from Universitas Gadjah Mada, both with cum-laude predicate. Between 2017 and 2022, he researched mainly at the Department of Textile Engineering, Politeknik STTT Bandung, Indonesia.

Ngadiyono, A.Md., S.T, is a senior educational laboratory institution and guest lecturer of physics at Politeknik STTT Bandung. He received his Associate's Degree from the Department of Electrical Engineering, Politeknik Negeri Semarang, and his Bachelor's degree from the Department of Electrical Engineering at Sekolah Tinggi Teknologi Mandala. In 2015, Ngadiyono became a guest lecturer at Politeknik STTT Bandung and the author of several independent scientific papers, mainly on applied physics and engineering topics. He researched mainly at the Department of Textile Engineering, Politeknik STTT Bandung, Indonesia.