Vibrational analysis of orthotropic rectangular plate having combination of circular thickness and parabolic temperature

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Abstract. The objective of this study is to investigate the natural vibration of a rectangular plate made of orthotropic material with circular thickness (two dimensions) and temperature variation on the plate is parabolic (two dimensions) in nature. The solution to the problem is obtained by utilizing the Rayleigh-Ritz technique and the first four frequency modes are obtained under clamped edge conditions. The study aims to provide numerical data that demonstrate how circular variation in tapering parameters of plate can effectively control and optimized vibrational frequencies of the plate. Orthotropic rectangular plate, thermal gradient, circular tapering, aspect ratio.

Keywords: orthotropic rectangular plate, thermal gradient, circular tapering, aspect ratio.

1. Introduction

To design structures or understand system characteristics, it becomes vital to investigate the vibrational properties of plates. Many systems and structures such as bridges, buildings, and aircraft wings consist of plates of various shapes. The vibration characteristics of a plate are influenced by plate parameters such as tapering, non-homogeneity (in the case of nonhomogeneous materials), and thermal gradient. A considerable number of studies in the literature have focused on various values of plate parameters.

The approach outlined in [1] was utilized to amalgamate solutions for plates with different geometries (such as circular, annular, circular sector, and annular sector plates) under various boundary conditions. In [2], the wave-based method (WBM) was utilized to forecast the flexural vibrations of orthotropic plates. In [3], a solution based on two-variable refined plate theory of Levy type was developed for free vibration analysis of orthotropic plates. In [4], a new analytical solution utilizing a double finite sine integral transform technique was introduced for the vibration response of plates reinforced by orthogonal beams. In [5], the Rayleigh Ritz method was utilized to determine the frequency of an orthotropic rectangular plate, whereas in [6], the time period of transverse vibration of a skew plate with different edge conditions was assessed. In [7], the influence of temperature on the frequencies of a tapered plate was discussed, while [8] investigated a non-uniform triangular plate subjected to a two-dimensional parabolic temperature distribution. The investigation of time period of rectangular plates with varying thickness and temperature was examined in [9]. Time period analysis of isotropic and orthotropic visco skew plate having circular variation in thickness and density at different edge conditions is discussed in [10] and [11].

It is noticeable from the literature that most of the authors have investigated either linear or parabolic variations in tapering parameters, but no one has focused on circular variation in tapering
parameter. This study aims to fill this research gap by exploring the influence of two dimension circular thickness on the vibrational frequency of an orthotropic rectangular plate under a two dimension parabolic temperature profile. The circular variation examined in this paper results in a reduction in the variation in frequency modes, as shown in the numerical results section.

2. Problem geometry and analysis

Taking into account that the nonhomogeneous rectangular plate shown in Fig. 1 with sides $a$, $b$ and thickness $l$.

![Fig. 1. Orthotropic rectangular plate with 2D circular thickness](image)

The formulation of the kinetic energy and strain energy for plate vibration is given below, similar to the approach presented in [12]:

$$T_s = \frac{1}{2} \omega^2 \int_0^a \int_0^b l \Phi^2 d\psi d\zeta,$$  \hspace{1cm} (1)

$$V_s = \frac{1}{2} \int_0^a \int_0^b \left[ D_\zeta \left( \frac{\partial^2 \Phi}{\partial \zeta^2} \right)^2 + D_\psi \left( \frac{\partial^2 \Phi}{\partial \psi^2} \right)^2 + 2\nu_\zeta D_\psi \frac{\partial^2 \Phi}{\partial \zeta^2} \frac{\partial^2 \Phi}{\partial \psi^2} + 4D_{\zeta\psi} \left( \frac{\partial^2 \Phi}{\partial \zeta \partial \psi} \right)^2 \right] d\psi d\zeta,$$  \hspace{1cm} (2)

where $\Phi$ is deflection function, $\omega$ is natural frequency, $D_\zeta = E_\zeta l^3/12(1-\nu_\zeta\nu_\psi)$, $D_\psi = E_\psi l^3/12(1-\nu_\zeta\nu_\psi)$, $D_{\zeta\psi} = E_\zeta l^3/12(1-\nu_\zeta\nu_\psi)$. Here, $D_\zeta$ and $D_\psi$ is flexural rigidity in $\zeta$ and $\psi$ directions respectively and $D_{\zeta\psi}$ is torsional rigidity.

In order to address the investigated problem, the Rayleigh-Ritz method is utilized, which necessitates:

$$L = \delta(V_s - T_s) = 0.$$  \hspace{1cm} (3)

Using Eqs. (1), (2), we have:

$$L = \frac{1}{2} \int_0^a \int_0^b \left[ D_\zeta \left( \frac{\partial^2 \Phi}{\partial \zeta^2} \right)^2 + D_\psi \left( \frac{\partial^2 \Phi}{\partial \psi^2} \right)^2 + 2\nu_\zeta D_\psi \frac{\partial^2 \Phi}{\partial \zeta^2} \frac{\partial^2 \Phi}{\partial \psi^2} + 4D_{\zeta\psi} \left( \frac{\partial^2 \Phi}{\partial \zeta \partial \psi} \right)^2 \right] d\psi d\zeta$$

$$-\frac{1}{2} \omega^2 \int_0^a \int_0^b \Phi^2 d\psi d\zeta = 0.$$  \hspace{1cm} (4)

Proposing non-dimensional variable as $\zeta_1 = \zeta/a$, $\psi_1 = \psi/a$ along with two dimension circular thickness as:

$$l = l_0 \left( 1 + \beta_1 \left( 1 - \sqrt{1 - \zeta_1^2} \right) \right) \left( 1 + \beta_2 \left( 1 - \sqrt{1 - \frac{\alpha^2}{b^2} \psi_1^2} \right) \right),$$  \hspace{1cm} (5)
where $l_0$ is thickness at origin and $\beta_1, \beta_2 \leq 1$ are tapering parameters.

The two-dimensional parabolic temperature distribution, as presented in Eq. (6):

$$\tau = \tau_0 (1 - \zeta_1^2) \left(1 - \frac{a^2 \psi_1^2}{b^2}\right)$$

(6)

where $\tau$ and $\tau_0$ represent the temperature at a given point and at the origin respectively.

For orthotropic materials, modulus of elasticity is evaluated by:

$$E_\zeta = E_1 (1 - \gamma \tau), \quad E_\psi = E_2 (1 - \gamma \tau), \quad G_{\zeta\psi} = G_0 (1 - \gamma \tau),$$

(7)

where $E_\zeta$ and $E_\psi$ are the Young’s modulus in $\zeta$ and $\psi$ directions, $G_{\zeta\psi}$ is shear modulus and $\gamma$ is called slope of variation.

Using Eq. (6), Eq. (7) becomes:

$$E_\zeta = E_1 \left(1 - \alpha (1 - \zeta_1^2) \left(1 - \frac{a^2 \psi_1^2}{b^2}\right)\right), \quad E_\psi = E_2 \left(1 - \alpha (1 - \zeta_1^2) \left(1 - \frac{a^2 \psi_1^2}{b^2}\right)\right),$$

$$G_{\zeta\psi} = G_0 \left(1 - \alpha (1 - \zeta_1^2) \left(1 - \frac{a^2 \psi_1^2}{b^2}\right)\right),$$

(8)

where $\alpha = \gamma \tau_0 \ (0 \leq \alpha < 1)$ is called thermal gradient.

Using Eqs. (5), (8) and non-dimensional variable, the functional in Eq. (4) become:

$$L = \frac{D_0}{2} \int_0^b \int_0^a \left[1 - \alpha (1 - \zeta_1^2) \left(1 - \frac{a^2 \psi_1^2}{b^2}\right)\right]\left(1 + \beta_1 \left(1 - \sqrt{1 - \zeta_1^2}\right)\right)^3$$

$$\left(1 + \beta_2 \left(1 - \frac{a^2 \psi_1^2}{b^2}\right)\right)^3 \left(\frac{\partial^2 \Phi}{\partial \zeta_1^2}\right)^2 + \frac{E_2}{E_1} \left(\frac{\partial^2 \Phi}{\partial \psi_1^2}\right)^2 + 2\nu_{\psi} \frac{E_2}{E_1} \left(\frac{\partial^2 \Phi}{\partial \zeta_1^2}\right) \left(\frac{\partial^2 \Phi}{\partial \psi_1^2}\right)$$

$$+ 4 \frac{G_0}{E_1} (1 - \nu_{\psi}) \left(\frac{\partial^2 \Phi}{\partial \zeta_1 \partial \psi_1}\right)^2 \right] d\psi_1 d\zeta_1 - \lambda^2 \int_0^1 \int_0^b \left[1 + \beta_1 \left(1 - \sqrt{1 - \zeta_1^2}\right)\right]^3$$

$$\left(1 + \beta_2 \left(1 - \frac{a^2 \psi_1^2}{b^2}\right)\right) \Phi^2 d\psi_1 d\zeta_1 = 0,$$

(9)

where $D_0 = \frac{1}{2} \left(\frac{E_1 \rho_0^2}{12 (1 - \nu_{\psi})^2}\right)$ and $\lambda^2 = \frac{12 a^4 \rho_0^2 (1 - \nu_{\psi})^2}{E_1 h_0^2}.$

The deflection function that meets all the edge conditions is taken as in [13]:

$$\Phi(\zeta, \psi) = \left[(\zeta_1)^{\delta}(\psi_1)^{\delta}(1 - \zeta_1)^{\delta} \left(1 - \frac{a \psi_1}{b}\right)^{\delta}\right] \times \sum_{i=0}^{n} \Psi_i \left[(\zeta_1)(\psi_1)(1 - \zeta_1) \left(1 - \frac{a \psi_1}{b}\right)^{\delta}\right]^{i},$$

(10)

where $\Psi_i, i = 0, 1, 2, \ldots n$ are unknowns and the value of $e, f, g, h$ can be 0, 1 and 2, corresponding to given edge condition.

Eq. (10) can be minimized by imposing the following condition:
\[ \frac{\partial L}{\partial w_i} = 0, \quad i = 0, 1, \ldots, n. \]  

(11)

Solving Eq. (11), we have frequency equation:

\[ |P - \lambda^2 Q| = 0, \]  

(12)

where \( P = [p_{ij}]_{i=0,1,n} \) and \( Q = [q_{ij}]_{i=0,1,n} \) are square matrix of order \((n + 1)\).

3. Numerical results and discussion

In this study, the first four natural frequencies of a clamped orthotropic rectangular plate with two dimension circular thickness and two dimension parabolic temperature variations are investigated corresponding to various plate parameters aspect ratio \(a/b\), tapering parameters \(\beta_1\) and \(\beta_2\), and thermal gradient \(\alpha\). The numerical calculations are based on the subsequent parameter values:

\[ E = 0.04, \quad E_1 = 1, \quad E_2 = 0.32, \quad G = 0.09, \quad \rho = 2.80 \times 10^3 \text{ kg/m}^3, \quad \nu = 0.345. \]

| Table 1. Modes of frequency of clamped orthotropic rectangular plate corresponding to \(\beta_1\) |
| \hline
| \(\beta_2\) | \(\alpha = 0.2\) & \(\alpha = 0.4\) & \(\alpha = 0.6\) |
| \hline
| \(\lambda_1\) & \(\lambda_2\) & \(\lambda_3\) & \(\lambda_4\) & \(\lambda_1\) & \(\lambda_2\) & \(\lambda_3\) & \(\lambda_4\) |
| 0.0 & 17.002 & 65.265 & 146.471 & 335.761 & 17.085 & 64.734 & 145.409 & 344.526 & 17.173 & 64.058 & 144.568 & 353.913 |
| 0.2 & 17.747 & 67.704 & 151.913 & 351.645 & 17.848 & 67.198 & 151.149 & 359.343 & 17.951 & 66.545 & 150.178 & 372.482 |
| 0.6 & 19.400 & 73.032 & 163.844 & 388.066 & 19.531 & 72.556 & 163.396 & 397.071 & 19.660 & 71.918 & 162.620 & 412.912 |

| Table 2. Modes of frequency of clamped orthotropic rectangular plate corresponding to \(\beta_2\) |
| \hline
| \(\alpha\) & \(\beta_1 = \beta_2 = 0.2\) & \(\beta_1 = \beta_2 = 0.4\) & \(\beta_1 = \beta_2 = 0.6\) |
| \hline
| \(\lambda_1\) & \(\lambda_2\) & \(\lambda_3\) & \(\lambda_4\) & \(\lambda_1\) & \(\lambda_2\) & \(\lambda_3\) & \(\lambda_4\) & \(\lambda_1\) & \(\lambda_2\) & \(\lambda_3\) & \(\lambda_4\) |
| 0.0 & 18.539 & 70.998 & 150.217 & 364.788 & 20.306 & 76.727 & 172.199 & 403.590 & 22.224 & 82.942 & 186.104 & 449.270 |
| 0.4 & 16.912 & 64.226 & 144.209 & 338.017 & 18.665 & 69.816 & 156.984 & 378.585 & 20.571 & 75.821 & 170.826 & 425.369 |
| 0.6 & 16.027 & 60.520 & 136.028 & 323.885 & 17.772 & 66.023 & 148.867 & 365.262 & 19.660 & 71.917 & 162.636 & 412.879 |
| 1.0 & 14.050 & 52.161 & 118.026 & 293.397 & 15.777 & 57.482 & 130.883 & 337.433 & 17.620 & 63.109 & 144.730 & 386.914 |

| Table 3. Modes of frequency of clamped orthotropic rectangular plate corresponding to \(\alpha\) |
| \hline
| \(\beta_1 = \beta_2 = 0.2\) & \(\beta_1 = \beta_2 = 0.4\) & \(\beta_1 = \beta_2 = 0.6\) |
| \hline
| \(\lambda_1\) & \(\lambda_2\) & \(\lambda_3\) & \(\lambda_4\) & \(\lambda_1\) & \(\lambda_2\) & \(\lambda_3\) & \(\lambda_4\) & \(\lambda_1\) & \(\lambda_2\) & \(\lambda_3\) & \(\lambda_4\) |
| 0.0 & 18.539 & 70.998 & 150.217 & 364.788 & 20.306 & 76.727 & 172.199 & 403.590 & 22.224 & 82.942 & 186.104 & 449.270 |
| 0.4 & 16.912 & 64.226 & 144.209 & 338.017 & 18.665 & 69.816 & 156.984 & 378.585 & 20.571 & 75.821 & 170.826 & 425.369 |
| 0.6 & 16.027 & 60.520 & 136.028 & 323.885 & 17.772 & 66.023 & 148.867 & 365.262 & 19.660 & 71.917 & 162.636 & 412.879 |

Table 1 presents the modes of frequency (first four modes) corresponding to \(\beta_1\). Specifically, the values of \(\beta_2 = \alpha\) chosen were 0.2, 0.4, and 0.6 respectively. Based on the results presented in Table 1, it can be inferred that:

1. The frequency modes increase in all four modes as \(\beta_1\) rises from 0.0 to 1.0.
2. As both \(\beta_1\) and \(\alpha\) increase from 0.2 to 0.6 (i.e., \(\beta_2 = \alpha = 0.2\) to \(\beta_2 = \alpha = 0.6\)), the modes of frequency also show an increment.
3. The modes of frequency (rate of increment) are predominantly influenced by $\beta_1$, as opposed to the $\alpha$ and $\beta_2$.

Table 2 presents the modes of frequency (first four modes) corresponding to $\beta_2$, with fixed values of tapering parameter $\beta_1$ and thermal gradient $\alpha$ i.e., $\beta_1 = \alpha = 0.2$, $\beta_1 = \alpha = 0.4$, and $\beta_1 = \alpha = 0.6$, respectively. Based on the findings in Table 2, it is evident that:

1. The modes of frequency exhibit an increase as $\beta_2$ varies from 0.0 to 1.0.
2. The modes of frequency decrease as $\beta_1$ and $\alpha$ increase from 0.2 to 0.6 (i.e., $\beta_1 = \alpha = 0.2$ to $\beta_1 = \alpha = 0.6$), when $\beta_2$ changes from 0.0 to 0.6. However, the modes of frequency increase as $\beta_1$ and $\alpha$ increase from 0.2 to 0.6 (i.e., $\beta_1 = \alpha = 0.2$ to $\beta_1 = \alpha = 0.6$), when $\beta_2$ varies from 0.8 to 1.0.
3. In terms of the rate of change in modes of frequency, $\beta_2$ exerts a stronger influence compared to $\alpha$ and $\beta_1$.
4. The tables 1 and 2 indicate that the modes of frequency are primarily influenced by $\beta_2$ as compared to $\beta_1$.

Table 3 presents the modes of frequency (first four modes) for various values of $\alpha$, while keeping both $\beta_1$ and $\beta_2$ fixed at 0.2, 0.4, and 0.6, respectively. By examining Table 3, the subsequent key findings can be observed:

1. The frequency modes also increase with an increase in $\beta_1$ and $\beta_2$ from 0.0 to 1.0, while the modes of frequency decrease with an increase in the value of $\alpha$ from 0.0 to 1.0.
2. Compared to $\beta_1$ and $\beta_2$, $\alpha$ has a greater influence on the modes of frequency (i.e., rate of change in the modes of frequency).

4. Conclusions

The above observation suggests that the plate parameters have a significant impact on the frequency modes of the plate. The selection of appropriate plate parameters can enable the manipulation of frequency modes and their variations as per the system requirements. Thus, it can be inferred that circular variation in plate parameters can be effective in minimizing and regulating frequency modes and their variations.

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Data availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Conflict of interest

The authors declare that they have no conflict of interest.

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