Solving Saint Venant torsion problems for rectangular beams using single finite Fourier sine transform method

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Abstract. This research presents the single Fourier sine transform method (SFSTM) for solving the Saint Venant torsion problem of rectangular prismatic bars. The problem is a common theme in the theory of elasticity of unrestrained torsion which was previously expressed by Prandtl using Prandtl stress functions $\phi(x, y)$ as a Poisson type nonhomogeneous partial differential equation (PDE) called the stress compatibility equation. In this work the SFSTM was applied to the stress compatibility equation, converting the PDE to an easier to solve ordinary differential equation (ODE) in the transformed domain. The boundary conditions were used to find the integration constant and inversion was used to find the solution in the physical domain. The non vanishing stresses and torsional moments were thus found as a single series of infinite terms with rapid convergence. The maximum stresses and moments were found in standard form in terms of torsional parameters which were tabulated for various ratios of the cross-sectional dimensions. A comparison of the torsional parameters with previous results show that the present results are identical with previous results illustrating the accuracy of the SFSTM used. The sine kernel of the SFSTM satisfies the boundary conditions of the problem and contributed to the exact solution obtained. The SFSTM simplified the PDE to an ODE which is simpler to solve.

Keywords: single Fourier sine transform method, Saint Venant torsion problem, Prandtl stress function, torsional moment, stress compatibility equation.

1. Introduction

Torsion in beams result from the action of shear loads whose points of application do not coincide with the shear center of the beam cross section. Torsion would occur when a reinforced concrete encased I-section steel beam is subjected to a brick/block wall such that the weight of the wall acts at an eccentricity from the shear center of the cross-section. Torsion also arises when a reinforced concrete floor beam is cast in-situ with its supporting edge beams with the edge beams prone to torsional stresses upon the application of floor loads.

The analysis of torsional stresses and strains is thus an important aspect of structural analysis and design and is embodied as a requirement in codes of structural design.

Problems of torsional analysis are developed using the theory of elasticity. The basic assumptions used are kinematic equations, constitutive laws, equilibrium equations and compatibility relations [1-3].

Saint Venant formulated the problem of unrestrained torsion using theory of elasticity and Prandtl solved the Saint Venant torsion problem using Prandtl's stress function. Prandtl's stress function formulation of Saint Venant unrestrained torsion problem resulted in a Poisson partial differential equation (PDE); solvable using numerical or closed form mathematical methods [4, 5].

The numerical methods that have been used to solve the Saint Venant torsion problem are: Finite Difference Method (FDM), Finite Element Method (FEM), Finite Volume Method, Boundary Element Methods and Variational Methods of Ritz, Galerkin and their modifications.

The mathematical methods that have been used for closed form solutions include: the method of separation of variables, eigen function expansions, integral transformations, and Green functions.

Joy and Mokashi [6], and Abdelkadr et al. [7] used the finite element method to present Saint Venant torsional and bending solutions for prismatic bars with non-circular cross-sections. Their solutions were validated by favourable comparisons with previous results in the literature.

Chen [8] used the finite volume method to study Saint Venant's torsion problems.

Ike [9] used the Galerkin method to obtain closed form analytical solutions for the Saint Venant torsion of prismatic bars with rectangular cross-sections. The study used Prandtl's stress function formulation of the Saint Venant torsion problem. The study adopted trigonometric (cosine) shape functions which were exact shape functions that satisfied the boundary condition of the problem to derive the Galerkin integral equations which were minimized with respect to the undetermined parameters to give the exact solution for the Prandtl stress function and hence exact solutions for the stresses, torsional moments and the torsional constants.

Francu et al. [10] used analytical method to solve the problem of Saint-Venant torsion of non-circular bars with prismatic cross-sections, and obtained accurate solutions for the stresses, moments and torsional constants.

Ike and Oguaghamba [11] used the Double finite sine transform method (DFSTM) to obtain closed form mathematical solutions for the Saint-Venant torsional analysis of beams with rectangular cross-section. Agarana and Agboola [12] presented an analysis of Saint-Venant torsional problems of circular beams made with different engineering materials.

Romano et al. [13], Brice and Pickings [14] and Hughes et al. [15] have also studied the Saint-Venant's torsional problems of beams using analytical and experimental approaches.

Fogang [16] studied the Saint-Venant unrestrained torsional problem of beams using the Green's theorem and the finite difference method (FDM). The FDM adopted in the study is a numerical method for solving boundary value problems (BVPs) by expressing the governing differential equation (GDE) in finite difference form and discretizing the solutions domain by introducing grid points. The finite difference method then seeks a solution of the GDE at each of the grid points such that boundary conditions are satisfied. Fogang [16] used a two-dimensional grid network for the beam cross-section and additional grid/node points were introduced at the beam boundaries. These additional nodes at the boundaries permitted the solution of the GDE at the boundaries and the satisfaction of boundary conditions. The study considered beams with solid cross-sections and beams with multiply connected cross-sections. The study obtained results that were comparable with the closed form solutions for rectangular beams, and the accuracy of the FDM results increased with the refinements of the finite difference grids, though at the cost of increased computational rigour.

Noor and Robertson [17] presented a variational development of the governing differential equations (GDEs) via a mixed formulation approach for the Saint-Venant torsional analysis of heterogeneous, isotropic bars. The GDEs obtained were three first-order partial differential equations (PDEs) in terms of the shear stresses and warping function. The study discussed the basic merits of mixed formulation over the stress and displacement based approaches. A finite difference method was applied to the GDEs to obtain approximate solution which was validated by comparison with previous solutions in the literature.

Guendouz et al. [18] presented an analysis of torsion-bending problems of a composite beam with open cross-section using the advanced and refined theory of one-dimensional (1D) and three-dimensional (3D) beams and the 3D Saint-Venant's refined beam theory.

2. Theoretical framework

2.1. Basic assumptions

The basic assumptions are:

– The beam has an axis of twist about which the cross-section rotates approximately as a rigid body.

– The cross-section is constant along the length of the bar.

– The deformation of the bar consists of: (a) rotations of the cross-section about an axis that passes through the center of the twist of the bar, and (b) warping of the cross-section which is constant for all the cross-sections.

2.2. Displacement field

The displacement field components are:

$$\begin{aligned} u &= -\theta yz, \\ v &= \theta xz, \\ w &= \theta\psi(x,y), \end{aligned} \tag{1}$$

where u, v, and w are displacements in the x, y, and z coordinate directions respectively; $\psi(x,y)$ is the warping function; and θ is the angle of rotation of the cross-section at a distance z from the origin.

2.3. Strains

The normal strains $(\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz})$ and shear strains $(\gamma_{xy}, \gamma_{xz}, \gamma_{zy})$ are found using the small-displacement theory of elasticity as:

$$\begin{aligned} &\varepsilon_{xx} = \frac{\partial u}{\partial x} = 0, \quad \varepsilon_{yy} = \frac{\partial v}{\partial y} = 0, \quad \varepsilon_{zz} = \frac{\partial w}{\partial z} = 0, \\ &\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = \theta\left(\frac{\partial\psi}{\partial x} - y\right), \\ &\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0, \\ &\gamma_{zy} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = \theta\left(\frac{\partial\psi}{\partial y} + x\right). \end{aligned} \tag{2}$$

2.4. Stresses

The normal stresses σ_{xx}, σ_{yy}, σ_{zz} and shear stresses τ_{xz}, τ_{zy}, τ_{xy} are:

$$\begin{aligned} &\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = 0, \quad \tau_{xy} = 0, \\ &\tau_{zx} = G\gamma_{zx} = G\theta\left(\frac{\partial\psi}{\partial x} - y\right), \\ &\tau_{zy} = G\gamma_{zy} = G\theta\left(\frac{\partial\psi}{\partial y} + x\right), \end{aligned} \tag{3}$$

where G is the shear modulus.

2.5. Differential equations of equilibrium

The differential equations of equilibrium are:

$$\begin{aligned} &\frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\tau_{xy}}{\partial y} + \frac{\partial\tau_{xz}}{\partial z} + f_x = 0, \\ &\frac{\partial\tau_{yx}}{\partial x} + \frac{\partial\sigma_{yy}}{\partial y} + \frac{\partial\tau_{yz}}{\partial z} + f_y = 0, \end{aligned} \tag{4}$$

$$\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + f_z = 0,$$

where f_x, f_y, and f_z are body force components in the x, y, and z axes respectively.

The equations of equilibrium for the Saint-Venant torsion problem when body forces are ignored are:

$$\frac{\partial \tau_{xz}}{\partial z} = 0, \tag{5a}$$

$$\frac{\partial \tau_{yz}}{\partial z} = 0, \tag{5b}$$

$$\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} = 0. \tag{5c}$$

Integration of Eqs. (5a) and (5b) give:

$$\begin{aligned} \tau_{xz} &= \tau_{xz}(x, y), \\ \tau_{yz} &= \tau_{yz}(x, y). \end{aligned} \tag{6}$$

2.6. Prandtl stress function $\phi(x, y)$

Prandtl derived a scalar stress function $\phi(x, y)$ in terms of stresses that satisfy the equilibrium equations as:

$$\tau_{xz} = \frac{\partial \phi}{\partial y}, \tag{7a}$$

$$\tau_{zy} = -\frac{\partial \phi}{\partial x}. \tag{7b}$$

The stress compatibility equation is:

$$\frac{\partial \tau_{zx}}{\partial y} - \frac{\partial \tau_{zy}}{\partial x} = -2G\theta. \tag{8}$$

The strain compatibility equation is:

$$\frac{\partial \gamma_{zx}}{\partial y} - \frac{\partial \tau_{zy}}{\partial x} = -2\theta. \tag{9}$$

The strain compatibility equation is expressed in terms of Prandtl stress function $\phi(x, y)$ as:

$$\frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial y}\right) - \frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial x}\right) = -2G\theta, \tag{10}$$

$$\begin{aligned} &\nabla^2 \phi(x, y) = -2G\theta, \\ &\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \end{aligned} \tag{11}$$

where ∇^2 is the Laplacian.

2.7. Torsional moment, M_t

The torsional moment, M_t is:

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$$M_t = 2 \iint_{R_{xy}} \phi(x,y)dxdy, \tag{12}$$

where R_{xy} is the cross-sectional domain of the beam.

3. Methodology

The Saint Venant torsional problem is solved here for rectangular beam shown in Fig. 1.

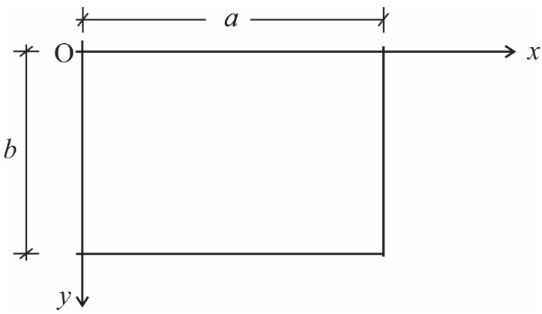

Fig. 1. Cross-section of the rectangular beam subjected to unrestrained torsion

The Saint Venant torsion equation in terms of Prandtl stress function is the stress compatibility equation:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + 2G\theta = 0, \tag{13}$$

where $0 \le x \le a$, $0 \le y \le b$.

The boundary conditions are:

$$\begin{aligned}\phi(x, y = 0) = \phi(x, y = b) = 0,\\ \phi(x = 0, y) = \phi(x = a, y) = 0.\end{aligned} \tag{14}$$

Applying the single Fourier sine transformation (SFST) to the stress compatibility equation yields the integral equation:

$$\int_0^a \left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + 2G\theta\right) \sin\frac{n\pi x}{a} dx = 0. \tag{15}$$

The single Fourier sine transform method (SFSTM) is a linear transformation. Using the linearity properties of the transformation, Eq. (15) is:

$$\int_0^a \frac{\partial^2\phi}{\partial x^2} \sin\frac{n\pi x}{a} dx + \int_0^a \frac{\partial^2\phi}{\partial y^2} \sin\frac{n\pi x}{a} dx + \int_0^a 2G\theta \sin\frac{n\pi x}{a} dx = 0. \tag{16}$$

Integrating by parts:

$$\begin{aligned}\int_0^a \frac{\partial^2\phi}{\partial x^2} \sin\frac{n\pi x}{a} dx = -\left(\frac{n\pi}{a}\right)((-1)^n \phi(x = a, y) - \phi(x = 0, y))\\ -\left(\frac{n\pi}{a}\right)^2 \int_0^a \phi(x, y) \sin\frac{n\pi x}{a} dx.\end{aligned} \tag{17}$$

From the boundary conditions, Eq. (14), $\phi(x = a, y) = \phi(x = 0, y) = 0$.
Hence:

$$\int_0^a \frac{\partial^2 \phi}{\partial x^2} \sin \frac{n\pi x}{a} dx = -\left(\frac{n\pi}{a}\right)^2 \int_0^a \phi(x, y) \sin \frac{n\pi x}{a} dx = -\left(\frac{n\pi}{a}\right)^2 \Phi(n, y), \tag{18}$$

where:

$$\Phi(n, y) = \int_0^a \phi(x, y) \sin \frac{n\pi x}{a} dx. \tag{19}$$

$\Phi(n, y)$ is the SFST of $\phi(x, y)$.
Similarly:

$$\int_0^a \frac{\partial^2 \phi}{\partial y^2} \sin \frac{n\pi x}{a} dx = \frac{\partial^2}{\partial y^2} \int_0^a \phi(x, y) \sin \frac{n\pi x}{a} dx = \frac{d^2}{dy^2} \Phi(n, y) \tag{20}$$

Also:

$$\int_0^a 2G\theta \sin \frac{n\pi x}{a} dx = 2G\theta \int_0^a \sin \frac{n\pi x}{a} dx = \frac{4G\theta a}{n\pi}, \tag{21}$$

where n is odd; $n = 1, 3, 5, 7, \ldots$.
Hence, the SFST of the stress compatibility equation is the ordinary differential equation ODE:

$$-\left(\frac{n\pi}{a}\right)^2 \Phi(n, y) + \frac{d^2}{dy^2} \Phi(n, y) = \frac{4G\theta a}{n\pi}. \tag{22}$$

4. Results

4.1. General solution

The homogeneous solution $\Phi_h(n, y)$ to Eq. (22) using the method of trial functions is:

$$\Phi_h(n, y) = c_1 \cosh \frac{n\pi y}{a} + c_2 \sinh \frac{n\pi y}{a}. \tag{23}$$

The particular solution $\Phi_p(n, y)$ is:

$$\Phi_p(n, y) = -\frac{4G\theta a^3}{(n\pi)^3}. \tag{24}$$

The general solution $\Phi_g(n, y)$ is:

$$\Phi_g(n, y) = c_1 \cosh \frac{n\pi y}{a} + c_2 \sinh \frac{n\pi y}{b} - \frac{4G\theta a^3}{(n\pi)^3}. \tag{25}$$

The unknown integrating constants c_1 and c_2 are found using the boundary conditions at $y = 0$ and $y = b$.

The application of FSTM on the boundary conditions give:

$$\int_0^a \phi(x, y = 0)\sin\frac{n\pi x}{a}dx = \Phi_g(n, y = 0) = 0, \tag{26a}$$

$$\int_0^a \phi(x, y = b)\sin\frac{n\pi x}{a}dx = \Phi_g(n, y = b) = 0. \tag{26b}$$

From Eq. (26a).
Thus:

$$\Phi_g(n, y = 0) = c_1 - \frac{4G\theta a^3}{(n\pi)^3} = 0. \tag{27}$$

Solving:

$$c_1 = \frac{4G\theta a^3}{(n\pi)^3}. \tag{28}$$

From Eq. (26b):

$$\Phi_g(n, y = b) = c_1\cosh\frac{n\pi b}{a} + c_2\sinh\frac{n\pi b}{a} - \frac{4G\theta a^3}{(n\pi)^3} = 0. \tag{29}$$

Simplifying Eq. (29) gives:

$$c_2\sinh\frac{n\pi b}{a} = \frac{4G\theta a^3}{(n\pi)^3} - \frac{4G\theta a^3}{(n\pi)^3}\cosh\frac{n\pi b}{a}. \tag{30}$$

Further simplification gives:

$$c_2\sinh\frac{n\pi b}{a} = \frac{4G\theta a^3}{(n\pi)^3}\left(1 - \cosh\frac{n\pi b}{a}\right). \tag{31}$$

Solving for c_2 gives:

$$c_2 = \frac{4G\theta a^3}{(n\pi)^3}\left(\frac{1 - \cosh\frac{n\pi b}{a}}{\sinh\frac{n\pi b}{a}}\right). \tag{32}$$

Hence:

$$\Phi_g(n, y) = \frac{4G\theta a^3}{(n\pi)^3}\cosh\frac{n\pi b}{a} + \frac{4G\theta a^3}{(n\pi)^3}\left(\frac{1 - \cosh\frac{n\pi b}{a}}{\sinh\frac{n\pi b}{a}}\right)\sinh\frac{n\pi y}{a} - \frac{4G\theta a^3}{(n\pi)^3}. \tag{33}$$

Simplifying gives:

$$\Phi_g(n,y) = \frac{4G\theta a^3}{(n\pi)^3}\left(\cosh\frac{n\pi b}{a} + \left(\frac{1-\cosh\frac{n\pi b}{a}}{\sinh\frac{n\pi b}{a}}\right)\sinh\frac{n\pi y}{a} - 1\right). \tag{34}$$

4.2. Solution in the physical space

By inversion of $\Phi_g(n,y)$ we have:

$$\Phi(x,y) = \frac{2}{a}\sum_{n=1}^{\infty}\Phi_g(n,y)\sin\frac{n\pi x}{a}. \tag{35}$$

Hence, using Eq. (34) gives:

$$\Phi(x,y) = \sum_{n=1}^{\infty}\frac{8G\theta a^2}{(n\pi)^3}\left(\cosh\frac{n\pi y}{a} + \left(\frac{1-\cosh\frac{n\pi b}{a}}{\sinh\frac{n\pi b}{a}}\right)\sinh\frac{n\pi y}{a} - 1\right)\sin\frac{n\pi x}{a}. \tag{36}$$

4.3. Shear stresses

The shear stresses are found using Eqs. (7a) and (7b) as:

$$\tau_{xz} = \frac{8G\theta a}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\left(\sinh\frac{n\pi y}{a} + \left(\frac{1-\cosh\frac{n\pi b}{a}}{\sinh\frac{n\pi b}{a}}\right)\cosh\frac{n\pi y}{a}\right)\sin\frac{n\pi x}{a}, \tag{37}$$

$$\tau_{yz} = -\frac{8G\theta a}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\left(\cosh\frac{n\pi y}{a} + \left(\frac{1-\cosh\frac{n\pi b}{a}}{\sinh\frac{n\pi b}{a}}\right)\sinh\frac{n\pi y}{a} - 1\right)\cos\left(\frac{n\pi x}{a}\right). \tag{38}$$

4.4. Torsional moment, M_t

The torsional moment M_t is given by Eq. (12).
Substituting the expression for $\phi(x,y)$ in Eq. (12) gives:

$$M_t = 2\int_0^b\int_0^a\sum_{n=1}^{\infty}\left\{\frac{8G\theta a^2}{(n\pi)^3}\left(\cosh\left(\frac{n\pi y}{a}\right) + \left(\frac{1-\cosh\left(\frac{n\pi b}{a}\right)}{\sinh\left(\frac{n\pi b}{a}\right)}\right)\sinh\left(\frac{n\pi y}{a}\right) - 1\right)\sin\left(\frac{n\pi x}{a}\right)\right\}dxdy. \tag{39}$$

Evaluating the integrals and simplifying gives:

$$M_t = \frac{64G\theta a^4}{\pi^5}\sum_{n=1}^{\infty}\left(\frac{\cosh\left(\frac{n\pi b}{a}\right)-1}{\sinh\left(\frac{n\pi b}{a}\right)} - \frac{n\pi b}{2a}\right). \tag{40}$$

Let:

$$\frac{a}{b} = r. \tag{41}$$

Then:

$$M_t = \frac{64G\theta a^3 br}{\pi^5}\sum_{n=1}^{\infty}\left(\frac{\cosh\left(\frac{n\pi b}{a}\right) - 1}{\sinh\left(\frac{n\pi b}{a}\right)} - \frac{n\pi b}{2a}\right). \tag{42}$$

M_t is expressed in general as:

$$M_t = G\theta J, \tag{43}$$

where:

$$J = \frac{64a^3 br}{\pi^5}\sum_{n=1}^{\infty}\left(\frac{\cosh\left(\frac{n\pi b}{a}\right) - 1}{\sinh\left(\frac{n\pi b}{a}\right)} - \frac{n\pi b}{2a}\right). \tag{44}$$

J can be expressed as:

$$J = F_t\left(\frac{a}{b}\right)a^3 b. \tag{45}$$

Then:

$$F_t\left(\frac{a}{b}\right) = \frac{64r}{\pi^5}\sum_{n=1}^{\infty}\left(\frac{\cosh\left(\frac{n\pi b}{a}\right) - 1}{\sinh\left(\frac{n\pi b}{a}\right)} - \frac{n\pi b}{2a}\right). \tag{46}$$

Let $a = 2H$, $b = 2B$, then for:

$$\tau_{\max} = \frac{M_t}{k_2(2H)^2(2B)}, \tag{47}$$

$$\theta = \frac{M_t}{k_1 G(2B)(2H)^3}, \tag{48}$$

where k_1 and k_2 are calculated for various values of B/H and shown in Table 1.Torsional moments are also presented for various values of a and b in Table 2 along with previous results.

Table 1. Torsional parameters for rectangular cross-sections

B/H	Present study k_1	Francu et al. [10]; Ike [9] k_1	Present study k_2	Francu et al. [10]; Ike [9] k_2
1.0	0.141	0.141	0.208	0.208
1.5	0.196	0.196	0.231	0.231
2	0.229	0.229	0.246	0.246
2.5	0.249	0.249	0.256	0.256
3	0.263	0.263	0.267	0.267
4	0.281	0.281	0.282	0.282
6	0.299	0.299	0.299	0.299
10	0.312	0.312	0.312	0.312
∞	0.333	0.333	0.333	0.333

Table 2. Torsional moments for cross-sections $2a \times 2b$

Cross-section		Present study	Timoshenko and Goodier [2]
a	b		Exact
1	1	2.2492	2.2492
2	1	7.3178	7.3178
3	1	12.6392	12.6392
4	1	17.9720	17.9720
5	1	23.3053	23.3053
6	1	28.6337	28.6337
7	1	33.9720	33.9720
8	1	39.3053	39.3053

5. Discussion

In this paper, exact analytical solutions have been obtained for the Saint Venant torsional problem of rectangular beams using the single Fourier sine transform method (SFSTM). The Saint Venant torsion problem is a Poisson type partial differential equation (PDE) expressed using Prandtl's stress function $\phi(x, y)$. The SFSTM converted the nonhomogeneous PDE to an ordinary differential equation ODE in the transformed domain which is easier to solve using methods for solving ODEs. Use of boundary conditions and inversion of the general solution gave the Prandtl stress function in the physical domain variables, from which the stresses and torsional moments were found. The stresses and torsional moments were expressed in terms of parameters k_1 and k_2 and presented in Tables 1 and 2 along with previously determined values by other researchers. Tables 1 and 2 illustrate that the present SFSTM solutions are identical with previous solutions by Ike [9] who used the Galerkin variational method and cosine basis functions; Francu et al. [10] and Timoshenko and Goodier [2].

6. Conclusions

This paper has presented a SFSTM for solving the Saint Venant torsion problem formulated as a Poisson type PDE in terms of the Prandtl stress function.

1) The SFSTM yielded analytical solutions to the Saint Venant torsion problem of prismatic bars with rectangular cross-section.

2) The SFSTM transformed the PDE to a more easily solved ODE in the transformed domain.

3) The kernel of the SFSTM satisfied the boundary conditions of the Saint Venant torsion problem and is the reason for the exact solution obtained.

4) The shear stresses, and torsional moments are convergent single series that give accurate solution with a few terms of the series.

5) The solutions obtained for the stresses, and torsional moments are identical to previous solutions obtained using Galerkin variational methods and the method of separation of variables.

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Data availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Conflict of interest

The authors declare that they have no conflict of interest.

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