Challenges to properly accounting for cyclic response in transversely isotropic elastic soil idealizations

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Abstract. The way natural soils are deposited gravitationally inherently leads to an anisotropic microfabric. The elasticity of such anisotropic soils has typically been idealized as being transversely isotropic (or "cross anisotropic"). The importance of elastic anisotropy in geotechnical engineering applications has particularly been invoked in conjunction with predicting ground deformations associated with underground structures such as deep excavations and tunnels. The development of elastic constitutive relations for transversely isotropic geomaterials is complicated by the fact that the elastic material parameters are usually not constant. This paper briefly reviews the issue of elasticity in soils. Following a short overview of isotropic elastic idealizations, the more relevant topic of transversely isotropic idealizations is discussed, with emphasis not only on monotonic but also on cyclic response.

Keywords: elasticity, soils, isotropic, transversely isotropic.

1. Introduction

The importance of elastic anisotropy in geotechnical engineering applications has been invoked in conjunction with predicting ground deformations associated with underground structures such as deep excavations and tunnels [1-4]. It is, however, equally relevant to simulating the small-strain response of any two- or three-dimensional geotechnical engineering problem. Indeed, different attempts have been made to improve the accuracy of soil deformations associated with various forms of underground construction by considering the anisotropic properties of soils in numerical analyses [1, 5].

This paper briefly reviews the issues elasticity in soils. Following a short overview of isotropic elastic idealizations, the more relevant topic of transversely isotropic (or "cross anisotropic") idealizations is discussed, with emphasis not only on monotonic but also on cyclic response.

2. Elasticity in soils

The state of stress in an elastic material depends only on the current strain state and is independent of the loading history. When it is loaded, the material stores all the energy attributed to deformation as strain energy. When unloaded, the material releases all this energy and reverts to its initial state without permanent strain. The response of elastic materials is also independent of the rate of loading.

In soils, which exhibit nonlinear and inelastic response, it natural to think that elastic response may be somewhat ambiguous [6]. If soils exhibit elastic response, then it would be expected to occur at very small strain levels. For example, when performing dynamic loading tests on normally consolidated clay specimens, at axial strain levels less than 0.01 % such soils exhibited only very small hysteretic damping, thus implying essentially elastic response [7]. From the results of numerous experimental studies performed on a wide range of geomaterials it has been concluded that such materials exhibit "imperfect elasticity" that was essentially rate-independent and nearly linear even at strains less than 0.001 % [8].

The quantification of material response at very small strain levels requires suitable experimental techniques. Initially, values of the elastic material parameters were almost exclusively determined in the laboratory by means of dynamic tests employing small amplitude excitations that generate longitudinal deformations typically around 0.0001 mm/mm or shearing deformations of 0.0001 radian or less [9]. The most common dynamic test is the resonant-column (RC) method, which was first used in the late 1930's [10] to investigate the propagation of waves in columns of sands subjected to torsional or longitudinal oscillations. Since the 1950's, the RC device has been rather widely used for both research and routine soil investigation [11-15]. In a typical RC test, a cylindrical specimen is subjected to torsional vibrations, and the resulting shearing deformations are recorded. Since the analysis of results from such tests assumes linear elastic material behavior, the test data are, however, strictly speaking, valid only for very small strains. Assuming isotropic elastic response, the shear modulus (G) is usually determined from torsional vibrations, while Young's modulus (E) is obtained from longitudinal or flexural vibrations.

Geophysical field tests have also been used to measure the velocity of propagation of compression, shear, and/or Rayleigh waves. Such tests involve seismic refraction surveys, Rayleigh wave techniques, or other surface methods [16], from which values of G can be determined for low strain conditions. At depth, the shear and compressional wave velocities can be measured using down-hole and cross-hole methods, as well as cyclic plate load tests.

3. The issue of elastic isotropy

Due to the way they are deposited, and considering their stress history and particle shapes, many natural soils are known to exhibit some degree of anisotropy [17-21]. Such soils have different properties in the direction of deposition as compared to that in planes oriented normal to this direction.

This notwithstanding, historically the elastic response of soils has been characterized as being isotropic. One reason for selecting such a material idealization was the lack of laboratory equipment that could accurately determine values for the parameters associated with an anisotropic elastic idealization. A second reason was simply the desire to keep the elastic formulations relatively simple analytically.

During the past 40 some years, rather significant progress in the development of laboratory and field equipment and techniques that allow for accurate determination of the elastic parameters has lessened the former constraint. Using such equipment, researchers have confirmed that at low levels of strain, soils indeed exhibit elastic response. Furthermore, this response was typically found to be anisotropic. As a result of such developments, anisotropic elastic material idealizations have become significantly easier to formulate.

4. Transversely isotropic elastic idealizations

Although the analytical details associated with a transversely isotropic elastic material idealization have been well established for some time now [18, 22-25], it required the aforementioned experimental advances, as well as more recent efforts [26-30] to facilitate the determination of values for the five elastic parameters associated with such an idealization.

Indeed, field seismic measurements have shown the existence of inherent anisotropy in the stiffness of natural soils [31-33]. Laboratory studies of shear stiffness have likewise identified varying levels of anisotropy that were attributed to inherent anisotropy, stress-induced anisotropy, or a combination of the two [34-37].

In the absence of initial stresses and strains, the constitutive equations for an anisotropic linear elastic material are written in the following manner:

$$\delta\varepsilon^e = A\delta\sigma', \tag{1}$$

where the symmetric matrix of compliance coefficients A has size ($N_{rowb} \times N_{rowb}$), where N_{rowb} is the number of strain and stress components [38]. In the case of three-dimensional analyses (N_{rowb}= 6), the infinitesimal elastic strain increments and effective stress increments are then given by:

$$\delta\varepsilon^e = \{\, \delta\varepsilon_{11}^e \quad \delta\varepsilon_{22}^e \quad \delta\varepsilon_{33}^e \quad \delta\gamma_{12}^e \quad \delta\gamma_{13}^e \quad \delta\gamma_{23}^e \,\}^T, \tag{2}$$
$$\delta\sigma' = \{\, \delta\sigma_{11}' \quad \delta\sigma_{22}' \quad \delta\sigma_{33}' \quad \delta\sigma_{12}' \quad \delta\sigma_{13}' \quad \delta\sigma_{23}' \,\}^T, \tag{3}$$

where engineering shear strains are used. In the above equations the superscript T denotes the operation of vector transposition.

In a transversely isotropic material, through all points there pass parallel planes of isotropy (Fig. 1). At every point in such a material there is thus a principal direction and, in a plane normal to the first direction, an infinite number of other principal directions [39].

In the present development, the local axes are assumed to coincide with the global (x, y, z) coordinate axes (Fig. 1). In addition, the global x-axis is assumed to coincide with the normal to the planes of isotropy. The global y and z axes are thus directed arbitrarily in such plane of isotropy.

The definition of a transversely isotropic material requires values for five elastic parameters, namely E_n, E_t, n_{tn}, n_{tt}, and G_{nt}. The parameter E_n represents the elastic (Young's) modulus that is associated with compression or tension in a direction normal to the plane of isotropy. The parameter E_t represents the elastic (Young's) modulus that is associated with compression or tension tangential to the plane of isotropy. Since the y-z plane is a plane of isotropy, n_{tn} represents the Poisson's ratio that is associated with the lateral contraction in a direction that is normal to the plane of isotropy when tension is applied within the plane. The quantity n_{tt} represents the Poisson's ratio that is associated with transverse contraction in the plane of isotropy when tension is applied within the same plane. Finally, G_{nt} is the modulus associated with shearing involving the shear strain and shear stress components with subscripts 13 and 12.

The shear modulus (G_{tt}), which is associated with shearing within a plane of isotropy, is computed in the following manner: $G_{tt} = \frac{E_t}{2(1+n_{tt})}$. From this expression it is apparent that G_{tt} is obviously not an additional independent material parameter.

For a transversely isotropic material idealization, the compliance matrix, given in equation (1), thus becomes:

$$A = \begin{bmatrix} 1/E_n & -\nu_{tn}/E_t & -\nu_{tn}/E_t & 0 & 0 & 0 \\ -\nu_{nt}/E_n & 1/E_t & -\nu_{tt}/E_t & 0 & 0 & 0 \\ -\nu_{nt}/E_n & -\nu_{tt}/E_t & 1/E_t & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{nt} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{nt} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu_{tt})/E_t \end{bmatrix}. \tag{4}$$

Since A is symmetric, $A_{12} = A_{21}$, implying that $\frac{n_{tn}}{E_t} = \frac{n_{nt}}{E_n}$ or $n_{tn} = n_{nt}\left(\frac{E_t}{E_n}\right)$. Kaliakin [41] has discussed various other important issues related to anisotropic elastic material idealizations for soils in general, and transversely isotropic idealizations in particular.

The values of the five independent elastic parameters associated with a transversely isotropic elastic material idealization can be obtained from either the results of laboratory experiments or from field tests. Laboratory experiments used for this task include torsional shear, RC, ultrasonic,

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and axisymmetric triaxial tests. Tests such as these must be appropriately modified to facilitate the determination of values for all the five elastic parameters. Such modifications include the use of bender elements (e.g., [28, 30, 43]) or other types of transducers suitable for measuring P and S-waves [44]. These laboratory tests permit either the direct (e.g., [26]) or the indirect determination (e.g., [27]) of all five parameters.

Commonly used field methods include in-situ seismic surveys consisting of down-hole and/or cross-hole tests, as well as the pressuremeter test. Due to differences in the shear deformations generated, the results obtained using such field tests can differ from values determined using laboratory experiments [42].

Additional details pertaining to the experimental determination of material parameter values for transversely isotropic material idealizations are outside the scope of the present paper. Such details are available in the pertinent references cited above.

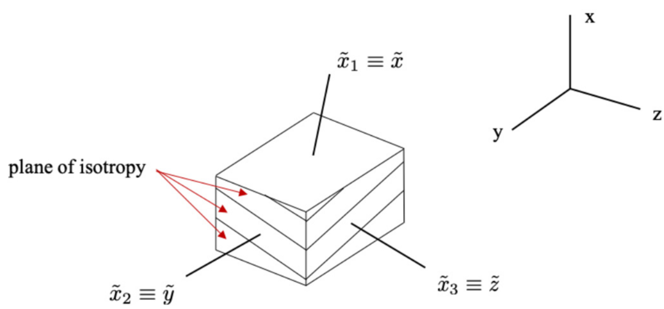

Fig. 1. Element of transversely isotropic material with local and global coordinates axes (after [40])

5. Functional forms used in transversely isotropic elastic idealizations

In general, the addition of a transversely isotropic elastic material idealization into inelastic constitutive models for soils necessitates a particular functional form that analytically describes the transversely anisotropic elastic idealization. Also required is a suitable empirical expression that accounts for the variation of the associated five independent material parameters. These topics have been discussed in greater detail by Kaliakin [40].

The development of elastic constitutive relations for transversely isotropic geomaterials is complicated by the fact that these material parameters are usually not constant. Indeed, test results [26] indicate that such parameters are a function of the density, stress state, and history of loading. The elastic strain increments will thus be related to the stress increments through compliance matrices (A) that are functions of the instantaneous stress state, the stress history and so forth. Since the elastic response is thus rendered stress path-dependent, only hypoelastic constitutive models are thought to be relevant [26].

A key assumption underlying the development of hypoelastic models is that the elastic modulus (E_i) associated with coordinate direction i is a function only of the normal stress acting in this direction and is unrelated to the normal stresses acting in the other two orthogonal directions [45, 46]. This important assumption has subsequently been supported by data from body wave velocity measurements [47-49] and by micromechanics-based simulations [42].

As an example of the functional forms used in a transversely isotropic elastic material idealization, consider the hypoelastic model of Yimsiri and Soga [50]. Based on the results of their tests on London clay, Yimsiri and Soga [50] proposed the following expressions for the elastic moduli:

$$E_n = 5.5F(e)(p')^{0.38}, \quad E_t = 12.0F(e)(p')^{0.38}. \tag{5}$$

The measured values of n_{tt} and n_{nt} exhibited some scatter but appeared to be independent of the confining pressure. Considering these findings, these Poisson's ratios were assumed to be constant, namely, $n_{nt} = 0.07$, $n_{tn} = 0.15$, $n_{tt} = 0.18$ [50]. The independent shear modulus was represented by the following expression:

$$G_{nt} = 4.25F(e)(p')^{0.38}. \quad (6)$$

Finally, using Eq. (5) gives $G_{tt} = E_t/[2(1 + n_{tt})] = 5.10F(e)(p')^{0.38}$. In the above expressions, p', and thus the elastic moduli, are expressed in units of MPa, and e is the void ratio. The void ratio function is assumed to be that proposed by Hardin and Black [7] for cohesionless soils, namely $F(e) = \frac{(2.973-e)^2}{1+e}$. The stiffness anisotropy at small strains can then be quantified as follows: $\frac{E_t}{E_n}$ = 12.0 / 5.5 = 2.18, $\frac{G_{tt}}{G_{nt}}$ = 1.20, and $\frac{n_{nt}}{n_{tt}}$ = 0.39.

A check the symmetry of the resulting compliance matrix A gives:

$$\nu_{tn}E_n = 0.825F(e)(p')^{0.38}, \quad \nu_{nt}E_t = 0.840F(e)(p')^{0.38}. \quad (7)$$

Thus, rendering A non-symmetric. Yimsiri and Soga [50] attributed this anomaly to the fact that the n_{nt} was deemed to be the most unreliable parameter in their study due to complications related to the alignment of the proximity transducer exactly with the in-situ vertical direction when testing horizontally cut specimens.

6. Stiffness degradation under cyclic loading

Of particular interest to the present discussion is an issue that was not investigated by Kaliakin [40], namely the response of transversely isotropic elastic idealizations under cyclic and/or more general vibrational environments. None of the functional forms discussed in [40] included any sort of degradation of the elastic parameters with cyclic load.

By contrast, such degradation has been rather extensively studied for isotropic elasticity. Although a rather large body of work in this area was performed on sands (e.g., [7, 15]), the present discussion is focused on cohesive soils.

Based on the results of numerous experimental studies, cyclically loaded cohesive soils may exhibit stiffness degradation [51-62]. This characteristic was found to rather strongly depend on the level of cyclic strain or stress that was applied to a specimen. Thus, cyclic loading that induces large strains tends to produce appreciable stiffness degradation, with substantial degradation typically occurring during the first few loading cycles.

Thiers and Seed [51] performed one of the first experimental studies of soil stiffness degradation in cohesive soils. They performed cyclic direct simple shear tests under strain-controlled conditions. They found that the isotropic elastic shear modulus G decreased by approximately 50 % to 80 % as the peak strain level increased from 0.5 % to 2 %. The change in G was, however, negligible for strains above 2 %. Thiers and Seed [51] also found that, in general, the largest changes in G occurred during the first 50 loading cycles.

To study the degradation in soil stiffness, Idriss et al. [52] performed both stress and strain-controlled cyclic triaxial tests on normally consolidated (NC) specimens. The degradation was found to be strongly dependent on the cyclic strain level applied to the specimen. Consequently, cyclic loading at high strain levels significantly degraded the soil stiffness.

Based on the results of their cyclic tests, Andersen et al. [53] also concluded that G decreased with increasing number of cycles. In addition, the higher the cyclic stress level, the more pronounced was this decrease. Interestingly, at low cyclic stress levels the stiffness degradation was negligible, indicating that G was thus essentially independent of the number of cycles. Comparing test results at the same cyclic deviator stress amplitude, Andersen et al. [53] found that overconsolidated (OC) specimens reached failure after a smaller number of cycles as compared to NC ones. Also, for both NC and OC specimens, the number of cycles to failure decreased with increasing cyclic deviator stress. Meimon and Hicher [56] found that for NC specimens failure

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occurred by reduction in the effective stresses, whereas for OC ones there was practically no evolution of the pore pressure during cyclic loading, even at failure.

Using cyclic direct simple shear tests, Vucetic and Dobry [57] studied soil stiffness degradation of anisotropically consolidated NC and OC clay specimens. To quantify the degradation of the undrained secant value of G during cyclic loading, they introduced the so-called "degradation index" δ, that is defined as follows:

$$\delta = \frac{G_{sN}}{G_{s1}}, \tag{8}$$

where G_{sN} and Gs_1 represent the secant shear modulus after N cycles and first cycle at constant shear strain amplitude, respectively. Small δ values thus correspond to high degree of stiffness degradation.

As evident from Fig. 2, δ is lower for NC specimens, meaning that G degrades more and faster than in the case of OC specimens. In addition, the controlled cyclic shear strain amplitude (γ_c) increases with overconsolidation ratio (OCR) (Fig. 2). Yasuhara et al. [60] also observed such response.

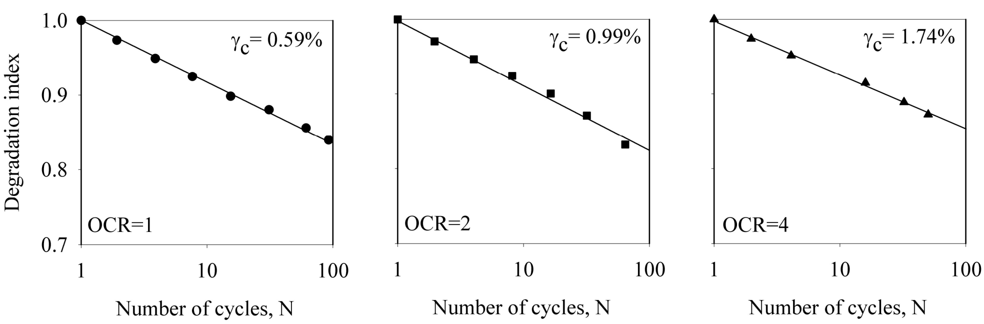

Fig. 2. Degradation index versus number of cycles for different OCR values (after [57])

Zhou and Gong [59] also quantified stiffness degradation through δ. They showed that such degradation was directly related to the cyclic stress ratio. Comparing δ values for the same number of cycles, it was found to be smaller with increasing the cyclic stress ratio, implying higher levels of soil degradation. In addition, δ values were lower for NC specimens, which confirmed that stiffness in such samples degrades more and faster than in OC ones and confirms that the OCR is an important factor in the study of soil degradation.

By contrast to the large body of work discussed above for isotropic elastic idealizations, there appears to be only one study of stiffness degradation for transversely isotropic elastic idealizations. Yu et al. [63] studied such degradation for Boom clay, which has been rather extensively studied as a potential deep underground disposal repository for nuclear waste. The degradation was mathematically accounted for by introducing a damage matrix that contained the damage variables D_1, D_2, and D_3 associated with the global coordinates $x_1 = x$, $x_2 = y$, and $x_3 = z$ shown in Fig. 1. For transversely isotropic elasticity, $D_2 = D_3$.

The following expressions for the elastic moduli were proposed by Yu et al. [63]:

$$\bar{E}_1 = (1 - D_1)^2 E_1, \quad \bar{E}_2 = \bar{E}_3 = (1 - D_3)^2 E_3, \tag{9}$$

where $E_1 = E_n$ and $E_2 = E_3 = E_t$ with respect to the notation used in the current development.

The shear modulus $G_{13} = G_{nt}$ is then modified as follows:

$$\bar{G}_{13} = (1 - D_1)(1 - D_3)G_{13}. \tag{10}$$

The following expression was next proposed for the Poisson ratio $n_{31} = n_{tn}$:

$$\bar{\nu}_{31} = \left(\frac{1 - D_3}{1 - D_1}\right)\nu_{31}. \tag{11}$$

The similar expression for $n_{23} = n_{tt}$ is:

$$\bar{\nu}_{23} = \left(\frac{1 - D_2}{1 - D_3}\right)\nu_{23}. \tag{12}$$

Since $D_2 = D_3$ it is, however, evident that n_{23} will be unaffected by the damage variables. Consequently, the following expression results for $G_{23} = G_{tt}$:

$$\bar{G}_{23} = \frac{(1 - D_3)^2}{2(1 + \nu_{23})}E_3. \tag{13}$$

The evolution of the degradation was mathematically accounted for through an elastic and a plastic damage law that were based on experimental results for Boom clay.

The work of Yu et al. [63] notwithstanding and compared to the body of work associated with isotropic elastic material characterizations, there exists a knowledge gap in the formulation of transversely isotropic elastic material idealizations as applied to other than static loading.

7. Conclusions

This paper briefly reviewed the issue of elasticity in soils. Following a short overview of isotropic elastic idealizations, the more relevant topic of transversely isotropic (or "cross anisotropic") idealizations was discussed, with emphasis not only on static but cyclic response.

Although progress in the development of transversely isotropic elastic models is notable, additional work is required before such models can be reliably used in the simulation of geomaterials. This conclusion is explained as follows:

First, not all transversely isotropic elastic models are complete. That is, they do not necessarily provide explicit functional forms for all five parameters associated with such idealizations. Whereas the Poisson's ratios n_{nt} and n_{tt} are commonly assumed to be constant, there is no consensus on the functional forms for E_n, E_t, and G_{nt}. In addition, some models render the compliance matrix A is non-symmetric.

Secondly, values of the parameters associated with such models are only available for select soils. No attempts have been made to apply a given model to more than one or two specific soils.

Finally, existing transversely isotropic elastic formulations have almost exclusively been developed for monotonic conditions. With one exception, these formulations have not investigated the degradation in the moduli with cyclic loading.

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Data availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Conflict of interest

The authors declare that they have no conflict of interest.

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References

[1] K. M. Lee and R. K. Rowe, "Deformations caused by surface loading and tunnelling: the role of elastic anisotropy," *Géotechnique*, Vol. 39, No. 1, pp. 125–140, Mar. 1989, https://doi.org/10.1680/geot.1989.39.1.125

[2] B. Simpson, J. H. Atkinson, and V. Jovicic, "The influence of anisotropy on calculation of ground settlements above tunnels," *Geotechnical Aspects of Underground Construction in Soft Ground*, Vol. 1, pp. 591–595, 1996.

[3] T. I. Addenbrooke, D. M. Potts, and A. M. Puzrin, "The influence of pre-failure soil stiffness on the numerical analysis of tunnel construction," *Géotechnique*, Vol. 47, No. 3, pp. 693–712, Jun. 1997, https://doi.org/10.1680/geot.1997.47.3.693

[4] J. Wongsaroj, K. Soga, and R. J. Mair, "Modelling of long-term ground response to tunnelling under St James's Park, London," *Géotechnique*, Vol. 57, No. 1, pp. 75–90, Feb. 2007, https://doi.org/10.1680/geot.2007.57.1.75

[5] B. Simpson and C. W. W. Ng, "Anisotropic analysis of settlement trough due to tunneling in an overconsolidated clay," in *Proceedings of the 10th Asian Regional Conference on Soil Mechanics and Foundation Engineering*, Vol. 2, pp. 166–16, 1995.

[6] G. T. Houlsby and C. P. Wroth, "The variation of shear modulus of a clay with pressure and overconsolidation ratio," *Soils and Foundations*, Vol. 31, No. 3, pp. 138–143, Sep. 1991, https://doi.org/10.3208/sandf1972.31.3_138

[7] B. O. Hardin and W. L. Black, "Closure to "Vibration modulus of normally consolidated clay"," *Journal of the Soil Mechanics and Foundations Division*, Vol. 95, No. 6, pp. 1531–1537, Nov. 1969, https://doi.org/10.1061/jsfeaq.0001364

[8] F. Tatsuoka, T. Uchimura, K. Hayano, H. Di Benedetto, J. Koseki, and M. S. A. Siddique, "Time-dependent deformation characteristics of stiff geomaterials in engineering practice," in *Proceedings of the Second International Conference on Pre-Failure Deformation Characteristics of Geomaterials*, Vol. 2, pp. 1161–1262, 2001.

[9] F. E. Richart, J. R. Hall, and R. D. Woods, *Vibrations of Soils and Foundations*. Englewood Cliffs, NJ: Prentice Hall, 1970.

[10] K. Lida, "The velocity of elastic waves in sand," *Bulletin of the Earthquake Research Institute, Japan*, Vol. 16, pp. 131–144, 1938.

[11] W. L. Shannon, G. Yamane, and R. J. Dietrich, "Dynamic Triaxial Tests on Sand," in *Proceedings of the First Panamerican Conference on Soil Mechanics and Foundation Engineering*, pp. 473–486, 1959.

[12] S. D. Wilson and R. J. Dietrich, "Effect of consolidation pressure on elastic and strength properties of clay," in *Proceedings of the ASCE Research Conference on Shear Strength of Cohesive Soils*, Vol. 1, pp. 419–435, 1960.

[13] B. O. Hardin and F. E. Richart, "Elastic wave velocities in granular soils," *Journal of the Soil Mechanics and Foundations Division*, Vol. 89, No. 1, pp. 33–65, Feb. 1963, https://doi.org/10.1061/jsfeaq.0000493

[14] B. O. Hardin and W. L. Black, "Vibration modulus of normally consolidated clay," *Journal of the Soil Mechanics and Foundations Division*, Vol. 94, No. 2, pp. 353–369, Mar. 1968, https://doi.org/10.1061/jsfeaq.0001100

[15] V. Drnevich, B. Hardin, and D. Shippy, "Modulus and damping of soils by the resonant-column method," *Dynamic Geotechnical Testing*, pp. 91–125, Jan. 1978, https://doi.org/10.1520/stp35673s

[16] W. M. Telford, L. P. Geldart, and R. E. Sheriff, "Applied geophysics," *International Journal of Rock Mechanics and Mining Sciences and Geomechanics Abstracts*, Vol. 15, No. 2, p. A46, Apr. 1978, https://doi.org/10.1016/0148-9062(78)91916-2

[17] A. Casagrande and N. Carrillo, "Shear failure of anisotropic materials," *Proceedings of the Boston Society of Civil Engineers*, Vol. 31, pp. 74–87, 1944.

[18] L. Barden, "Stresses and displacements in a cross-anisotropic soil," *Géotechnique*, Vol. 13, No. 3, pp. 198–210, Sep. 1963, https://doi.org/10.1680/geot.1963.13.3.198

[19] R. E. Gibson, "The analytical method in soil mechanics," *Géotechnique*, Vol. 24, No. 2, pp. 115–140, Jun. 1974, https://doi.org/10.1680/geot.1974.24.2.115

[20] R. N. Yong and B. R. Warkentin, *Developments in Geotechnical Engineering*. Amsterdam, The Netherlands: Elsevier, 1975, https://doi.org/10.1016/b978-0-444-41167-9.x5001-9

[21] C. M. Gerrard, "Background to mathematical modeling in geomechanics: the role of fabric and stress history," in *Finite Elements in Geomechanics*, London: John Wiley and Sons, 1977, pp. 33–120.

[22] D. J. Pickering, "Anisotropic elastic parameters for soil," *Géotechnique*, Vol. 20, No. 3, pp. 271–276, Sep. 1970, https://doi.org/10.1680/geot.1970.20.3.271

[23] J. H. Atkinson, "Anisotropic elastic deformations in laboratory tests on undisturbed London Clay," *Géotechnique*, Vol. 25, No. 2, pp. 357–374, Jun. 1975, https://doi.org/10.1680/geot.1975.25.2.357

[24] J. Graham and G. T. Houlsby, "Anisotropic elasticity of a natural clay," *Géotechnique*, Vol. 33, No. 2, pp. 165–180, Jun. 1983, https://doi.org/10.1680/geot.1983.33.2.165

[25] V. Sivakumar, I. G. Doran, J. Graham, and A. Johnson, "The effect of anisotropic elasticity on the yielding characteristics of overconsolidated natural clay," *Canadian Geotechnical Journal*, Vol. 38, No. 1, pp. 125–137, Feb. 2001, https://doi.org/10.1139/t00-074

[26] E. Hoque and F. Tatsuoka, "Anisotropy in elastic deformation of granular materials," *Soils and Foundations*, Vol. 38, No. 1, pp. 163–179, Mar. 1998, https://doi.org/10.3208/sandf.38.163

[27] M. L. Lings, D. S. Pennington, and D. F. T. Nash, "Anisotropic stiffness parameters and their measurement in a stiff natural clay," *Géotechnique*, Vol. 50, No. 2, pp. 109–125, Apr. 2000, https://doi.org/10.1680/geot.2000.50.2.109

[28] R. Kuwano and R. J. Jardine, "On the applicability of cross-anisotropic elasticity to granular materials at very small strains," *Géotechnique*, Vol. 52, No. 10, pp. 727–749, Dec. 2002, https://doi.org/10.1680/geot.2002.52.10.727

[29] M. Arroyo, D. Muir Wood, and P. D. Greening, "Source near-field effects and pulse tests in soil samples," *Géotechnique*, Vol. 53, No. 3, pp. 337–345, Apr. 2003, https://doi.org/10.1680/geot.2003.53.3.337

[30] S. Nishimura, "Assessment of anisotropic elastic parameters of saturated clay measured in triaxial apparatus: Appraisal of techniques and derivation procedures," *Soils and Foundations*, Vol. 54, No. 3, pp. 364–376, Jun. 2014, https://doi.org/10.1016/j.sandf.2014.04.006

[31] J. P. Sully and R. G. Campanella, "Evaluation of in situ anisotropy from crosshole and downhole shear wave velocity measurements," *Géotechnique*, Vol. 45, No. 2, pp. 267–282, Jun. 1995, https://doi.org/10.1680/geot.1995.45.2.267

[32] D. W. Hight, J. D. Bennell, B. Chana, P. D. Davis, R. J. Jardine, and E. Porovic, "Wave velocity and stiffness measurements of the Crag and Lower London Tertiaries at Sizewell," *Géotechnique*, Vol. 47, No. 3, pp. 451–474, Jun. 1997, https://doi.org/10.1680/geot.1997.47.3.451

[33] K. H. H. Stokoe and J. C. Santamaria, "Seismic-wave-based testing in geotechnical engineering," in *GeoEng 2000: An International Conference on Geotechnical and Geological Engineering*, Vol. 1, pp. 1490–1536, 2000.

[34] T. Thomann and R. Hryciw, "Laboratory measurement of small strain shear modulus under K conditions," *Geotechnical Testing Journal*, Vol. 13, No. 2, pp. 97–105, Jun. 1990, https://doi.org/10.1520/gtj10700j

[35] K. H. H. Stokoe, S. K. Hwang, J. N. K. Lee, and R. D. Andrus, "Effects of various parameters on the stiffness and damping of soils at small to medium strains," in *Proceedings of the International Symposium on Pre-Failure Deformation Characteristics of Geomaterials*, Vol. 2, pp. 785–816, 1994.

[36] D. S. Pennington, D. F. T. Nash, and M. L. Lings, "Anisotropy of G0 shear stiffness in Gault Clay," *Géotechnique*, Vol. 47, No. 3, pp. 391–398, Jun. 1997, https://doi.org/10.1680/geot.1997.47.3.391

[37] V. Fioravante, "Anisotropy of small strain stiffness of Ticino and Kenya sands from seismic wave propagation measured in triaxial testing," *Soils and Foundations*, Vol. 40, No. 4, pp. 129–142, Aug. 2000, https://doi.org/10.3208/sandf.40.4_129

[38] V. N. Kaliakin, *Approximate Solution Techniques, Numerical Modeling and Finite Element Methods*. New York: Marcel Dekker, 2001.

[39] S. G. Lekhnitskii, *Theory of Elasticity of an Anisotropic Body*. Moscow: Mir Publishers, 1981.

[40] A. Zhussupbekov, A. Sarsembayeva, and V. N. Kaliakin, *Smart Geotechnics for Smart Societies*. London: CRC Press, 2023, pp. 528–539, https://doi.org/10.1201/9781003299127

[41] V. N. Kaliakin, "Anisotropic elasticity for soils: a synthesis of some key issues," *Bulletin of L.N. Gumilyov Eurasian National University. Technical Science and Technology Series*, Vol. 127, No. 2, pp. 49–63, Jan. 2019, https://doi.org/10.32523/2616-7263-2019-127-2-49-63

[42] S. Yimsiri and K. Soga, "Micromechanics-based stress-strain behaviour of soils at small strains," *Géotechnique*, Vol. 50, No. 5, pp. 559–571, Oct. 2000, https://doi.org/10.1680/geot.2000.50.5.559

[43] M. L. Lings and P. D. Greening, "A novel bender/extender element for soil testing," *Géotechnique*, Vol. 51, No. 8, pp. 713–717, Sep. 2001, https://doi.org/10.1680/geot.2001.51.8.713

[44] R. Bellotti, M. Jamiolkowski, D. C. F. L. Presti, and D. A. O. 'Neill, "Anisotropy of small strain stiffness in Ticino sand," *Géotechnique*, Vol. 46, No. 1, pp. 115–131, Mar. 1996, https://doi.org/10.1680/geot.1996.46.1.115

[45] B. O. Hardin, "The nature of stress-strain behavior for soils," in *Proceedings of the Specialty Conference on Earthquake Engineering and Soil Dynamics*, Vol. 1, pp. 3–90, 1978.

[46] B. O. Hardin and G. E. Blandford, "Elasticity of particulate materials," *Journal of Geotechnical Engineering*, Vol. 115, No. 6, pp. 788–805, Jun. 1989, https://doi.org/10.1061/(asce)0733-9410(1989)115:6(788)

[47] M. Jamiokowski, S. Lerouell, and D. C. F. Lo Presti, "Design parameters from theory to practice," *International Journal of Rock Mechanics and Mining Sciences and Geomechanics Abstracts*, Vol. 30, No. 6, p. 343, Dec. 1993, https://doi.org/10.1016/0148-9062(93)91309-7

[48] D. C. F. Lo Presti and D. A. O. 'Neill, "Laboratory investigation of small strain modulus anisotropy in sands," in *Proceedings of the International Symposium on Calibration Chamber Testing*, Vol. 2, pp. 213–224, 1991.

[49] K. H. H. Stokoe, J. N. K. Lee, and S. H. H. Lee, "Characterization of soil in calibration chambers with seismic waves," in *Proceedings of the International Symposium on Calibration Chamber Testing*, Vol. 2, pp. 363–376, 1991.

[50] S. Yimsiri and K. Soga, "Cross-anisotropic elastic parameters of two natural stiff clays," *Géotechnique*, Vol. 61, No. 9, pp. 809–814, Sep. 2011, https://doi.org/10.1680/geot.9.p.072

[51] G. R. Thiers and H. B. Seed, "cyclic stress-strain characteristics of clay," *Journal of the Soil Mechanics and Foundations Division*, Vol. 94, No. 2, pp. 555–569, Mar. 1968, https://doi.org/10.1061/jsfeaq.0001110

[52] I. M. Idriss, R. Dobry, and R. D. Singh, "Nonlinear behavior of soft clays during cyclic loading," *Journal of the Geotechnical Engineering Division*, Vol. 104, No. 12, pp. 1427–1447, Dec. 1978, https://doi.org/10.1061/ajgeb6.0000727

[53] K. H. Andersen, W. F. Rosenbrand, S. F. Brown, and J. H. Pool, "Cyclic and static laboratory tests on drammen clay," *Journal of the Geotechnical Engineering Division*, Vol. 106, No. 5, pp. 499–529, May 1980, https://doi.org/10.1061/ajgeb6.0000957

[54] I. M. Idriss, Y. Moriwaki, S. G. Wright, E. H. Doyle, and R. S. Ladd, "Behavior of normally consolidated clay under simulated earthquake and ocean wave loading conditions," in *International Symposium on Soils under Cyclic and Transient Loading*, pp. 437–445, 1980.

[55] K. Ishihara and S. Yasuda, "Cyclic strengths of undisturbed cohesive soils," in *International Symposium on Soils under Cyclic and Transient Loading*, 1980.

[56] Y. Meimon and P. Y. Hicher, "Mechanical behaviour of clays under cyclic loading," in *International Symposium on Soils under Cyclic and Transient Loading*, pp. 77–87, 1980.

[57] M. Vucetic and R. Dobry, "Degradation of marine clays under cyclic loading," *Journal of Geotechnical Engineering*, Vol. 114, No. 2, pp. 133–149, Feb. 1988, https://doi.org/10.1061/(asce)0733-9410(1988)114:2(133)

[58] M. Vucetic, "Normalized behavior of clay under irregular cyclic loading," *Canadian Geotechnical Journal*, Vol. 27, No. 1, pp. 29–46, Feb. 1990, https://doi.org/10.1139/t90-004

[59] J. Zhou and X. Gong, "Strain degradation of saturated clay under cyclic loading," *Canadian Geotechnical Journal*, Vol. 38, No. 1, pp. 208–212, Feb. 2001, https://doi.org/10.1139/t00-062

[60] K. Yasuhara, S. Murakami, B.-W. Song, S. Yokokawa, and A. F. L. Hyde, "Postcyclic degradation of strength and stiffness for low plasticity silt," *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. 129, No. 8, pp. 756–769, Aug. 2003, https://doi.org/10.1061/(asce)1090-0241(2003)129:8(756)

[61] C.-J. Lee and S.-F. Sheu, "The stiffness degradation and damping ratio evolution of Taipei Silty Clay under cyclic straining," *Soil Dynamics and Earthquake Engineering*, Vol. 27, No. 8, pp. 730–740, Aug. 2007, https://doi.org/10.1016/j.soildyn.2006.12.008

[62] D. V. Okur and A. Ansal, "Stiffness degradation of natural fine grained soils during cyclic loading," *Soil Dynamics and Earthquake Engineering*, Vol. 27, No. 9, pp. 843–854, Sep. 2007, https://doi.org/10.1016/j.soildyn.2007.01.005

[63] H. D. Yu, W. Z. Chen, X. L. Li, and X. Sillen, "A transversely isotropic damage model for boom clay," *Rock Mechanics and Rock Engineering*, Vol. 47, No. 1, pp. 207–219, Jan. 2013, https://doi.org/10.1007/s00603-012-0346-7