Experimental study of the effect of the chamber radius on the bifurcation value of the drum rotation speed during self-excitation of monograin intrachamber filling auto-oscillations

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Abstract. The aim of the study is to experimentally establish the influence of the radius of the rotating drum chamber R on the value of the bifurcation rotation speed ω_{bif} during self-excitation of auto-oscillations of a single-grain intra-chamber filling. The criterion modeling method based on visualization of the filling behavior was applied. Eight variable drums with a chamber radius $R = 18.75$-212 mm were used. To represent the desired dependence in the criterion form, a bifurcation diagram (Re, Fr) was used in the axes of the dynamic criteria for the similarity of the flow on the cylindrical surface of the rotating drum chamber. An analogue of the Reynolds number was used based on the identified analogue of the granular kinematic viscosity of the liquefied granular filling. The influence of R on ω_{bif} was estimated by the position of the bifurcation line $Fr_{bif}(Re_{bif})$ on the diagram. The scientific novelty of the work lies in establishing the increase in the bifurcation value of the Froude number Fr_{bif} with increasing R. It was also found that Fr_{bif} decreases with increasing the degree of filling of the chamber. The results obtained allow us to predict the parameters of the implementation of innovative energy-efficient auto-oscillating processes for processing granular materials with a variation of the drum radius.

Keywords: drum-type machine, intra-chamber filling, mono-grain material, chamber radius, chamber filling degree, bifurcation rotation speed, self-excitation of auto-oscillations.

1. Introduction

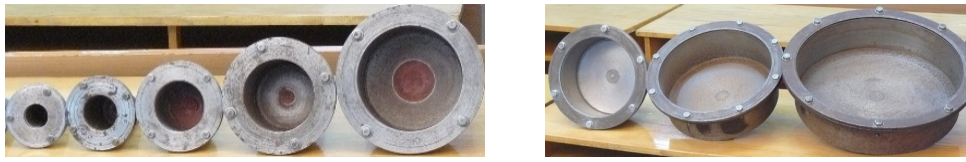
Every year, more than 100 billion tons of various granular materials are processed in the world in the chemical, mining, metallurgical, building materials, energy, mechanical engineering, biological and a number of other industries. The processing of such materials is carried out by drum machines by granulation, reaction, mixing, grinding, washing, size classification, tumbling, drying and heat treatment. The cross-industry class of drum-type technological machines has gained wide application due to a number of operational advantages. However, the problem of these machines is the high energy intensity of the processes [1]. This is due to the passivity of a significant part of the internal chamber filling. Self-excitation of auto-oscillations of granular filling of the rotating drum chamber allows to activate filling and increase the technological efficiency of drum machines. The first video recording of auto-oscillations of granular filling of the rotating drum was carried out in [2]. In [3], the mechanism of occurrence of oscillations of filling of self-grinding mills is revealed. Attempts to create models of self-excitation and the manifestation of instability in the motion of granular filling of a rotating drum were made in [4-9]. In [10], the conditions for the stability of the motion of the oscillatory system of a drum

machine are established. The manifestation of instability in the motion of a two-fraction granular filling of a rotating drum was elucidated in [11]. In [12], a model of the oscillatory stability of the motion of a two-fraction filling is constructed.

The defining parameter of the implementation of self-excitation of the auto-oscillating motion of the granular filling of the chamber is the lower bifurcation value of the drum rotation speed, which is recorded during a smooth increase in the angular velocity from the rest state. This value characterizes the transition from a continuous circulating to a periodic auto-oscillating mode of the filling motion. With an increase in the rotation speed, the instability of the continuous circulating mode of motion develops, which leads to the establishment of periodic self-oscillations. The task of predicting the influence of the chamber radius on the bifurcation value of the drum rotation speed seems to be quite relevant. Instead, in [11, 12] the lower bifurcation value of the rotation speed was established only for one value of the drum chamber radius at a filling degree of 0.25-0.5. The aim of the work is to establish the influence of the chamber radius on the bifurcation value of the drum rotation speed. This will allow predicting the effectiveness of the implementation of auto-oscillating processes for processing granular materials when the radius of the drum chamber varies. To achieve the aim, the following tasks were set: to perform experimental modeling of the lower bifurcation value of the rotation speed for different radii and degrees of filling of the drum chamber with mono-grained material; to assess the qualitative influence of the chamber radius on the bifurcation value of the rotation speed.

2. Materials and methods

The bifurcation values of the auto-oscillation parameters were determined by the criterion modeling method based on the visualization of the behavior of the granular filling of the chamber. Eight interchangeable drums with transparent end walls were used (Fig. 1). The radii R of the drum chambers were 18.75, 26.5, 37.5, 53, 75, 106, 150 and 212 mm.



a) b)

Fig. 1. Drums: a) R = 18.75, 26.5, 37.5, 53 and 75 mm; b) R = 106, 150 and 212 mm

As a mono-granular working material for filling the drum chamber, a non-cohesive granular material with particles of close to spherical shape with a size of d = 2.2 mm was used. The content of the drum chamber with working material was estimated by the volumetric degree of filling of the chamber $\kappa = w/(\pi R^2 L)$, where w is the volume of a portion of granular filling at rest, L is the length of the drum chamber. The discrete values of κ were 0.25, 0.35, and 0.45.

The experimental conditions are given in Table 1.

A 240 W collector electric motor was used to drive the filled drum, the rotation speed of which was changed by adjusting the supply voltage. The bifurcation value of the rotation speed ω_{bif} was determined during a smooth increase in the angular rotation speed ω until reaching the minimum value at which self-excitation of auto-oscillations of the granular filling of the rotating drum chamber occurred. In this case, the value of angular acceleration did not exceed 0.1 1/s^2.

To represent the dependence of the bifurcation value of the rotation speed ω_{bif} on the chamber radius R in the criterion form, a bifurcation diagram (Re, Fr) of dynamic flow similarity criteria on the cylindrical surface of the rotating drum chamber was used. The Reynolds number analogue for fluidized granular filling Re and the Froude number Fr were used as the coordinate axes of the diagram $Re = (\omega R^2)/\nu_{gr}$, $Fr = (\omega^2 R)/g$, where $\nu_{gr} \approx 10^{-3}$ m^2/s is an analogue of the granular kinematic viscosity of the fill [13], g is the gravitational acceleration. The expressions for the

coordinates of the points Re_{bif} and Fr_{bif} of the bifurcation line $Fr_{bif}(Re_{bif})$ on the bifurcation diagram (Re, Fr) have the form $Re_{bif} = (\omega_{bif}R^2)/\nu_{gr}$, $Fr_{bif} = (\omega_{bif}^2 R)/g$.

Table 1. Experiment conditions

Parameters		Designation	Units	Value
Main groups	Component elements			
Drum	Chamber radius	R	mm	18.75, 26.5, 37.5, 53, 75, 106, 150 and 212
	Chamber length	L	mm	100
	Angular speed of rotation	ω	rad/s	0-25
Granular filling of the drum chamber	Average particle size	d	mm	2.2
	Particle shape	–	–	close to spherical
	Bulk density of filling	ρ	kg/m^3	900
	Analogue of granular kinematic viscosity of granular fill	ν_{gr}	m^2/s	$\approx 10^{-3}$
	Degree of filling of the drum chamber with granular material	κ	–	0.25, 0.35 and 0.45
Video recording of auto-oscillations	Video capture frame rate	–	FPS	24
	Auto-oscillation frequency	f	Hz	1.4-6

3. Results

Sequential pictures of video recording of the movement of granular filling upon reaching the bifurcation value of the rotation speed ω_{bif} and self-excitation of auto-oscillations for one oscillation period, which were obtained at different values of R and κ, are shown in Fig. 2-4. (All photos taken by K. Deineka at the Drum-Type Machines Laboratory, National University of Water and Environmental Engineering, Rivne, Ukraine, on January 2026).

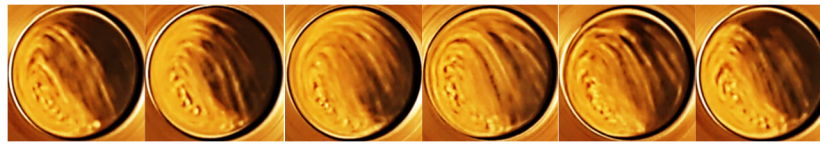


Fig. 2. Pictures of auto-oscillations at R = 18.75 mm, κ = 0.35, ω_{bif} = 20.8 rad/s, Re_{bif} = 7.31 and Fr_{bif}=0.826

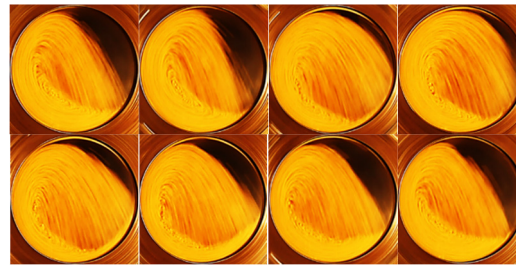


Fig. 3. Pictures of auto-oscillations at R = 53 mm, κ = 0.45, ω_{bif} = 13 rad/s, Re_{bif} = 36.4 and Fr_{bif} = 0.906

The results of the experimentally obtained bifurcation values of the auto-oscillation parameters are given in Table 2.

The data obtained in Table 2 allow us to construct the points of the bifurcation lines $Fr_{bif}(Re_{bif})$ on the bifurcation diagram (Re, Fr).

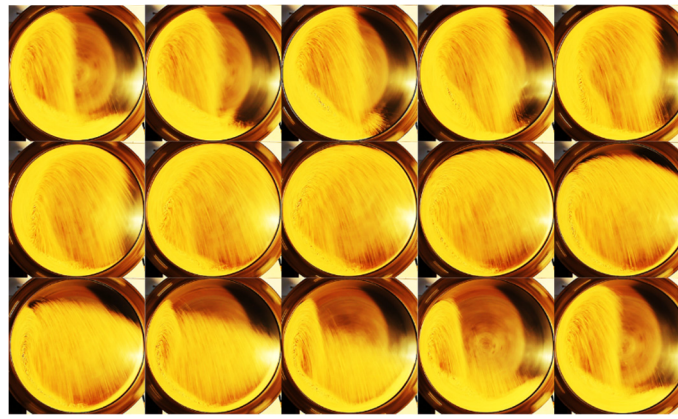


Fig. 4. Pictures of auto-oscillations at R = 212 mm, κ = 0.25, ω_{bif} = 7.94 rad/s, Re_{bif} = 357 and Fr_{bif} = 1.36

Table 2. Experimentally obtained bifurcation values of auto-oscillation parameters

Parameters	Designation	Measurement range		Value
Bifurcation value of the drum rotation angular velocity	ω_{bif}	R (mm)	18.75	19.5-21.8
			26.5	16.1-19
			37.5	14.8-17
			53	12.9-14.1
			75	11.1-12.9
			106	9.3-10.9
			150	8.5-9.7
			212	7.1-8
		κ	0.25	7.9-21.8
			0.35	7.7-20.8
			0.45	7.1-19.6
Coordinates of the points $Re_{bif}; Fr_{bif}$ of the bifurcation lines $Fr_{bif}(Re_{bif})$ on the bifurcation diagram (Re, Fr)	Re_{bif};Fr_{bif}	R (mm)	18.75	6.8; 0.72-7.7; 0.91
			26.5	11.3; 0.7-13.4; 0.98
			37.5	20.8; 0.83-23.9; 1.1
			53	36.3; 0.9-39.6; 1.08
			75	62.9 ;0.95-72.6; 1.27
			106	104.8; 0.94-121.8; 1.27
			150	192.2; 1.11-217.4; 1.43
			212	323.1; 1.11-356.9; 1.37
		κ	0.25	7.6; 0.9-356.9; 1.37
			0.35	7.3; 0.82-346.5; 1.29
			0.45	6.8; 0.72-323.1; 1.12

The graphs of the obtained bifurcation lines $Fr_{bif}(Re_{bif})$ on the bifurcation diagram (Re, Fr) are shown in Fig. 5.

The positions of the bifurcation lines $Fr_{bif}(Re_{bif})$ on the bifurcation diagram (Re, Fr) determine the dependence of the bifurcation value of the rotation speed ω_{bif} on the radius of the drum chamber R and the degree of filling of the chamber κ.

4. Discussion

The obtained results of experimental visualization of the behavior of granular filling of the rotating drum chamber allowed us to qualitatively and quantitatively assess the influence of the chamber radius R on the bifurcation value of the rotation speed ω_{bif}.

Visual qualitative analysis of the motion patterns in Fig. 2-4 clearly demonstrates the stable

nature of the manifestation of self-excitation of auto-oscillations of granular filling when the values of the radius of the rotating drum chamber vary. The number of patterns is inversely proportional to the bifurcation velocity ω_{bif}.

The established quantitative dependencies are determined by the position of the bifurcation lines $Fr_{bif}(Re_{bif})$ on the bifurcation diagram (Re, Fr) (Fig. 5) and (Table 2). The dependencies $Fr_{bif}(Re_{bif})$ of the detected bifurcation lines can be approximately approximated by step functions (Fig. 5).

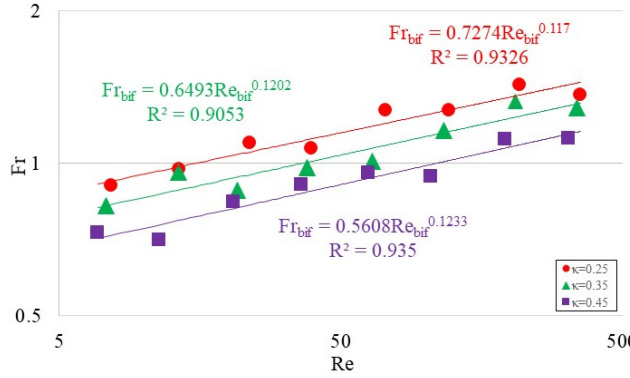


Fig. 5. Bifurcation lines $Fr_{bif}(Re_{bif})$ on the diagram (Re, Fr) for R = 18.75-212 mm and κ = 0.25-0.35

It is found that with an increase in R from 18.75 to 212 mm, and a corresponding increase in the bifurcation value of the Reynolds number Re_{bif} from 6.86 to 357, the bifurcation value of the Froude number Fr_{bif} increases from 0.704 to 1.43. This indicates an increase in the stability of the filling movement due to the influence of R, which is manifested in the corresponding increase in Fr_{bif}. This is due to the manifestation of the dynamic effect of weakening the dilatancy of the granular filling in the chamber of the rotating drum with increasing R [11, 12]. Such an increase in κ. The influence of the degree of filling of the chamber κ on the bifurcation velocity ω_{bif} was also revealed (Fig. 5) and (Table 2). It was established that with an increase in κ from 0.25 to 0.45, the bifurcation value of the Froude number Fr_{bif} decreases from 0.903 to 0.728 at R = 18.75 mm, and also from 1.36 to 1.12 at R = 212 mm. This indicates a decrease in the stability of the filling motion due to the influence of κ, which is manifested in the corresponding decrease in Fr_{bif}. This is due to the manifestation of the dynamic effect of weakening the filling dilatancy with increasing κ, which was first established in [11]. This decrease in Fr_{bif} is caused by the limitation of the space for possible free radial circulation of the filling in the chamber and increases with increasing κ. The revealed influence of the degree of filling κ on the bifurcation value of the Froude number Fr_{bif} complements and expands similar results of works [11, 12].

5. Conclusions

The influence of the chamber radius R on the lower bifurcation value of the drum rotation speed ω_{bif} consists in the increase in the bifurcation value of the Froude number Fr_{bif} of the (Re, Fr) diagram with increasing R. The influence of the degree of filling of the chamber with a granular medium κ on ω_{bif} consists in the decrease in the bifurcation value of the Froude number Fr_{bif} with increasing κ. An increase in the radius R increases the stability of the oscillatory motion of the granular filling in the chamber of the rotating drum, and an increase in the degree of filling κ reduces this stability.

For the size of a spherical particle of a single-grain filling d = 2.2 mm, with an increase in the radius R from 18.75 to 212 mm, the bifurcation value of the Froude number Fr_{bif} increases from

0.704 to 1.43 at a filling degree κ = 0.25-0.45. On the other hand, with an increase in κ from 0.25 to 0.45, the bifurcation value of the Froude number Fr_{bif} decreases from 0.903 to 0.728 at R = 18.75 mm, and also from 1.36 to 1.12 at R = 212 mm.

In the future, it is planned to determine the influence of the granulometric composition of the granular intra-chamber filling on the lower bifurcation value of the drum rotation speed ω_{bif} when varying the radius R and the degree of filling κ.

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Data availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Conflict of interest

The authors declare that they have no conflict of interest.

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